THE PUBLISHING HOUSE
OF THE ROMANIAN ACADEMY

PROCEEDINGS OF THE ROMANIAN ACADEMY, Series A,
Volume12, Number 3/2011, pp. 203–208

CLUSTER-DECAY TRAJECTORY

Mihail MIREA[1], Aureliu SĂNDULESCU[2,3], Doru-Sabin DELION[1,2]

[1] "Horia Hulubei" National Institute for Physics and Nuclear Engineering, P.O. Box MG–6, Bucharest, Romania
[2] Academy of Romanian Scientists, RO-050094, Bucharest, Romania
[3] Institute for Advanced Studies in Physics, Bucharest, Romania
E-mail: mirea@nipne.ro

The dynamical path for cluster decay is determined in the frame of the macroscopic-microscopic model for the ^{24}Ne emission from ^{232}U. The inertia is computed in the cranking approximation. A nuclear shape parametrization characterized by five degrees of freedom is used. The single particle energies and the nucleon wave functions are obtained within the superasymmetric Woods-Saxon two center shell model. It is evidenced that the cluster decay follows a potential valley that begins from the ground state of the parent and reach a configuration of two touching nuclei at scission. A comparison within cold fission is realized.

Key words: Woods-Saxon two center shell model, Cluster decay, Cold fission.

1. INTRODUCTION

The cluster decay was predicted in the 80 [1–6] and experimentally evidenced in 1984 [7,10]. Since that time, the spontaneous emission of heavy fragments was investigated intensively. A phenomenological unified approach of cluster radioactivity, cold fission and alpha decay as well as many body theories were used. Few time later, a fine structure in cluster emission was also anticipated. An elaborated study of the fission dynamics in a wide range of mass asymmetries could help us to understand better the underlying physics. In this work, our aim is to develop the macroscopic-microscopic approach to treat in a unitary manner the cluster decay and the fission process. For this purpose a fission like theory will be used to determine the best action trajectory of the cluster decay in a configuration space spanned by five degrees of freedom: elongation, necking, mass-asymmetry and deformation of fragments. In this context, the minimal action principle will be used. Two ingredients are needed: the deformation energy of the disintegrating system and the nuclear inertia. Such trajectories were previously obtained for the fission of Th [11,12], U [13–17], Np [18] and Cf [19,20]. The potential barrier obtained for cluster emission will be discussed in correlation with that of the fission process.

2. MODEL

The calculation addresses the ^{24}Ne cluster emission from of ^{232}U. The microscopic-macroscopic model [21] is exploited dynamically, by determining the least action trajectory. The dynamical analysis of a fissioning nucleus requires at least the knowledge of the deformation energy and the effective mass. For simplicity, it is considered that these quantities depend upon the shape coordinates. So, in our analysis, the basic ingredient is the nuclear shape parametrization. The nuclear shape parametrization used in the following is given by two ellipsoids of different sizes smoothly joined by a third surface obtained by the rotation of a circle around the axis of symmetry. Five degrees of freedom characterize this nuclear shape parametrization, that are: the elongation given by the inter-nuclear distance $R = z_2 - z_1$ between the centers of the ellipsoids, the two deformations of the nascent fragments denoted by the eccentricities $\varepsilon_i = [1-(b_i/a_i)^2]^{1/2}$ (i = 1,2), the mass asymmetry obtained within the ratio of major semi-axis as $\eta = a_2/a_1$ and the necking

parameter related to the curvature of the intermediate surface $C = s/R_3$. The quantity C is used for swollen shapes in the median region while R_3 is used for necked shapes. The meaning of the geometric symbols can be understood by inspecting the Fig. 1. The determination of the fission trajectory can be obtained through a minimization of the action integral [22] in our five-dimensional configuration space, beginning with the ground state of the system and ending in the exit point of the barrier.

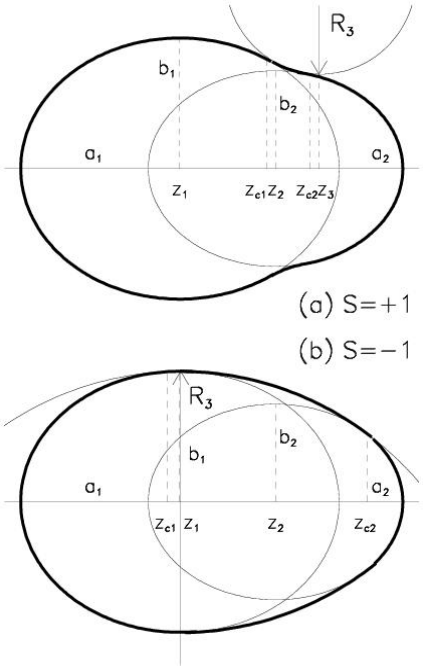

Fig. 1 – Nuclear shape parametrization.

The probability P of the fission process is determined by an exponential factor in the WKB approximation.

$$P = \exp\left\{-\frac{2}{\hbar}\int_{Ri}^{Rf}\sqrt{2V(R,C,\varepsilon_1,\varepsilon_2,\eta)B(R,C,\varepsilon_1,\varepsilon_2,\eta,\frac{\partial C}{\partial R},\frac{\partial \varepsilon_1}{\partial R},\frac{\partial \varepsilon_2}{\partial R},\frac{\partial \eta}{\partial R})}\mathrm{d}R\right\}. \tag{1}$$

The exponent of the previous relation gives the classical action integral of a fixed energy along a trajectory in our multidimensional configuration space. Our aim is to determine the path for cluster decay. In the present work, this energy is considered as the ground state energy of the parent nucleus. The trajectory connects the ground state configuration R_i to the exit point of the barrier R_f. For this purpose, two ingredients are required: the deformation energy V and the tensor of the effective mass B.

The deformation energy V was obtained by summing the liquid drop energy E_{LDM} with the shell and the pairing corrections δE:

$$V = E_{LDM} + \delta E. \tag{2}$$

The macroscopic energy E_{LDM} is obtained in the framework of the Yukawa – plus-exponential model [23] extended for binary systems with different charge densities [24] as detailed in Ref. [25]:

$$E_{LDM} = E_n + E_C + E_V, \tag{3}$$

where

$$E_n = -\frac{a_2}{8\pi^2 r_0^2 a^4}\iint_V\left(\frac{r_{12}}{a}-2\right)\frac{\exp\left(-\frac{r_{12}}{a}\right)}{\frac{r_{12}}{a}}\mathrm{d}^3r_1\mathrm{d}^3r_2, \tag{4}$$

is the nuclear term, and

$$E_C = \frac{1}{2}\iint_{\infty} \frac{\rho_e(r_1)\rho_e(r_2)}{r_{12}} \mathrm{d}^3 r_1 \mathrm{d}^3 r_2, \tag{5}$$

is the Coulomb energy, and E_V is the volume energy. In the previous definitions ρ_e are charge densities and $r_{12} = |\mathbf{r}_1 - \mathbf{r}_2|$. Another degree of freedom must be introduced here, namely the charge asymmetry. It is considered that ρ_e is charge density of the parent nucleus for an elongation smaller than $0.7 \times R_t$, where R_t is the elongation that characterizes the configuration of two touching fragments, and varies linearly up to the final values of the two nascent nuclei at scission.

The shell effects δE are obtained as a sum between the shell δU and the pairing δP microscopic corrections. In this context, the Strutinsky procedure [22] was used. These corrections represent the varying parts of the total binding energy caused by the shell structure. The single particle level diagrams are computed within the Woods-Saxon superasymmetric two-center shell model [26]. In calculating the pair corrections, the blocking effects are taken into account.

The deformation energy is a function of the collective parameters and gives the generalized forces that act on the nuclear shape. For a complete description of the fission process, it is therefore required to know how the nucleus reacts to these generalized forces. This information is contained in the effective mass of the system [22]. The most used approach to calculate the inertia is the cranking model. Recently, the cranking model was generalized by taking into account the intrinsic excitation produced during the fission process itself [27]. Along the minimal action path for the ^{232}U fission, the mass parameters are evaluated microscopically with the cranking model.

$$M_{ij}(q_i) = \frac{2}{\hbar^2} \sum_{\nu,\mu} \frac{\left\langle \nu \left| \frac{\partial H}{\partial q_i} \right| \mu \right\rangle \left\langle \mu \left| \frac{\partial H}{\partial q_j} \right| \nu \right\rangle}{\left(E_\nu + E_\mu\right)^3} \left(u_\nu v_\mu + u_\mu v_\nu\right)^2 + P_{ij}, \tag{6}$$

where ν and μ denote the single particle wave functions, E_ν, u_ν and v_ν are the quasiparticle energy, the vacancy and occupation amplitudes of the state ν, respectively, in the BCS approximation, and P_{ij} is a correction that depends on the variation of the pairing gap Δ and the Fermi energy λ as function of the deformation coordinates q_i. This correction amount up to 10% of the total value of the inertia. The inertia B along a trajectory in the configuration space spanned by the generalized coordinates q_i (i = 1,n) can be obtained within the formula

$$B = \sum_{i=1}^{n}\sum_{j=1}^{n} M_{ij} \frac{\partial q_i}{\partial R} \frac{\partial q_j}{\partial R}. \tag{7}$$

Results concerning the inertia concerning the fission process can be found in [28,29]. To calculate the inertia, we need a microscopic potential. The microscopic potential must be constructed to be consistent within our nuclear shape parametrization. The simplest way it to use a semi-phenomenological Woods-Saxon potential. In order to take into account nuclear deformations going over to separate shapes and obtain two separated fragments, a two-center shell model with a Woods-Saxon potential was developed recently [26]. The mean field potential is defined in the frame of the Woods-Saxon model:

$$V_0(\rho, z) = \frac{V_c}{1 + \exp\left[\frac{\Delta(\rho, z)}{a}\right]}, \tag{8}$$

where $\Delta(\rho,z)$ represents the distance between a point (ρ,z) and the nuclear surface. This distance is measured only along the normal direction on the surface and it is negative if the point is located in the interior of the nucleus. V_c is the depth of the potential while a is the diffuseness parameter. In our work, the depth is $V_c = V_{0c}[1 \pm \kappa\,(N_0 - Z_0)/(N_0 + Z_0)]$ with plus sign for protons and minus sign for neutrons, V_{0c}= 51 MeV, a = 0.67 fm, κ = 0.67. Here A_0, N_0 and Z_0 represent the mass number, the neutron number and the charge number of the parent, respectively. This parametrization, referred as the Blomqvist-Walhlborn one, is adopted because

it provides the same radius constant r_0 for the mean field and the pairing field. That ensures a consistency of the shapes of the two fields at hyperdeformations, i.e., two tangent ellipsoids. The Hamiltonian is obtained by adding the spin-orbit and the Coulomb terms to the Woods-Saxon potential. The eigenvalues are obtained by diagonalization of the Hamiltonian in the semi-symmetric harmonic two center basis [30, 31]. In this work, the major quantum number used is N_{max}=12. The two center Woods-Saxon model will be used to compute shell and pairing corrections together with inertia in this work. The two center shell model represents a valuable instrument to investigate the role of individual orbitals for the treatment of a wide variety of superasymmetric disintegration processes, pertaining to cluster- and alpha-decays [32–34] or superheavy elements [35, 36].

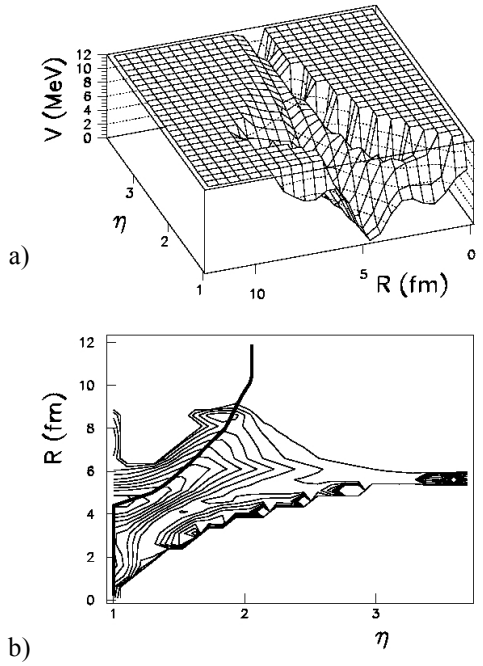

Fig. 2 – a) Potential energy surface V as function of the elongation R and the mass asymmetry η; b) contour plot of the potential energy surface. The step between two equipotential lines is 1 MeV. The variations of the coordinates ε_1, ε_2, and C follow the least action path as function of R. The least action trajectory is plotted within a thick curve.

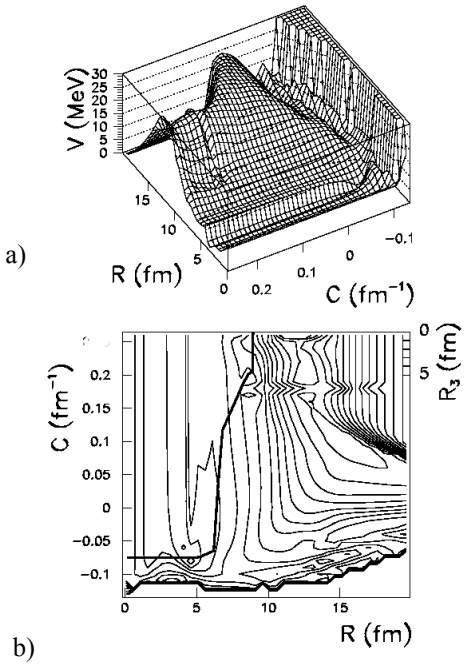

Fig. 3 – Same as Fig. 1 for a representation in the C and R generalized coordinates. For C greater that 0.2 fm^{-1} the left scale for R_3=1/C apply.

3. RESULTS AND DISCUSSION

It is not possible to minimize the functional (1) directly due to the time required to compute the values of the potential energies and of the effective masses. So, in the relevant configuration space, a small number of potential energies and inertia are computed and their interpolated values are used in the numerical minimization program. First of all, a grid of 576 000 deformation values was fixed in our five-dimensional configuration space: 20 values of R between 0 fm and 20 fm, 8 values of the eccentricities ε_i between 0 and 0.75, 15 values of the ratio $\eta = a_1/a_2$ in the interval 1 and 3 and 25 values for C between –0.115 fm^{-1} and 0.115 fm^1 (5 values for R_3 between 0 and 5 fm are also added). The deformation energy and the elements of the inertia tensor were computed in these selected points. In this way, the pertinent region in the configuration space that includes the possible fission trajectories between the ground state and the exit point from the barrier was spanned. A first minimal action trajectory was obtained in this selected region by interpolating the calculated masses and the energies within a method initiated in Ref. [22] and used extensively to describe the fission process. The trajectory of the decaying system is obtained simultaneously as a function of the five generalized coordinates. This trajectory emerges by minimizing numerically the action functional K that defines the quantum penetrability $\exp(-2K/\hbar)$ in the semi-classical Wentzel-Kramers-Brillouin approximation. The two turning points R_i and R_f denote the elongation that characterize

the first well and the exit point of the barrier, respectively. The functions $C = f(R)$, $\varepsilon_i = f(R)$ and $\eta = f(R)$ are approximated with a spline functions of n variables C_j, ε_{ij} and η_j in fixed mesh points R_j comprised in the interval $[R_i, R_f]$. A numerical function for the WKB functional that depends only on the the parameters C_j , ε_{ij} and η_j is obtained. This expression is minimized numerically for different input parameters. Different local minima are obtained. The best values are retained. After this first minimization, a new array of deformation values and inertia parameters is calculated in the identified region of the second barrier. A new minimization is made, by varying only the necking collective parameter. The remaining generalized coordinates are kept unchanged.

The results are plotted in Fig. 2 and Fig. 3. In Fig. 2 the potential energy surface is represented as function of the mass asymmetry parameter η and the elongation R. The ground state of the parent is located at an elongation $R = 4.6$ fm and at a mass asymmetry $\eta = 1$, that is the nuclear shape is reflection symmetric. The mass asymmetry is changed abruptly when the nucleus begins to deform, i.e., when the elongation is increased. It is clearly evidenced that the nuclear system follows a well behaved valley in the potential energy surface up to the scission configuration. This scission configuration is approximated by two tangent spherical fragments (^{208}Pb and ^{24}Ne), This scission configuration is located at an elongation R close to 10 fm. It is interesting to note that in the case of the fission phenomena, the situation is very different. Information about the fission can be obtained if the mass asymmetry is kept at a value close to 1 where two fragments of comparable sized are formed. It can be seen on the upper plot of Fig. 2 that a double barrier is formed. The nucleus initially in the ground state disintegrates in fission by penetrating a first barrier located at $R = 6.5$ fm and reaches a second well at $R = 7$ fm. The situation is completely different for cluster decay: the system follows an energy valley in the deformation energy.

As displayed in Fig. 3, the parent ground state is located at $R = 4.6$ fm while the necking parameter $C = 0.075$ fm^{-1}. So, the ground state is characterized by a swollen shape in the median region. These swollen shapes are preserved up to $R = 6$ fm. From this value, the necking parameter starts to vary abruptly and the shapes become very necked producing a rupture at $R \approx 10$ fm. At scission, the configuration of nearly two touching nuclei is obtained.

In this paper, the cluster decay was treated within fission models. The drifting potential in the overlap region was obtained in conjunction with the minimal action principle. It was shown that the cluster decay follows a well behaved valley in the potential energy landscape. This behavior is very different from the well known double humped fission barrier. The model presented in this work represents an alternative description for cluster decay. Usually, the cluster decay is treated by calculating a preformation probability, as in alpha decay [37].

ACKNOWLEDGEMENTS

Work performed in the frame of the CNCSIS IDEI 512 project of the Romanian Ministry of Education and Research.

REFERENCES

1. A. SANDULESCU, W. GREINER, *Mass asymmetry in fission, fusion and mass-transfer due to fragmentation in valleys*, Journal of Physics G, **3**, *8*, pp. L189-L193, 1977.
2. A. SANDULESCU, H.J. LUSTIG, J. HAHN, W. GREINER, High mass-*asymmetry distributions of fissioning nuclei*, Journal of Physics G, **4**, *11*, pp. L279-L285, 1978.
3. A. SANDULESCU, D.N. POENARU, W. GREINER, *New type of decay of heavy nuclei intermediate between fission and alpha decay*, Soviet Journal of Particles and Nuclei, **11**, pp. 528–541, 1980.
4. A. SANDULESCU, *A new radioactivity*, Journal of Physics G, **15**, *15*, pp. 529–554, 1989.
5. A. SANDULESCU, W. GREINER, *Cluster decays*, Reports on Progress in Physics**, 55**, *9*, pp. 1423–1481, 1992.
6. A. SANDULESCU, D.N. POENARU, W. GREINER, J.H. HAMILTON, *Exotic nuclear decay of ^{223}Ra by emission of ^{14}C nuclei*, Physical Review Letters, **54**, *5*, pp. 490–490, 1985.
7. H.J. ROSE, G.A. JONES, *A new kind of natural radioactivity*, Nature, **307**, 5948, pp. 245–247, 1984.
8. E. HOURANI, M. HUSSONOIS, L. STAB, et al., *Evidence for the radioactive decay of ^{226}Ra by ^{14}C emission*, Physics Letters B, **160**, *6*, pp. 375–379, 1985.

9. S. GALES, E. HOURANI, M. HUSSONOIS, H.P. SHAPIRA, M. VERGNES*, Exotic nuclear Decay of ^{223}Ra by emission of ^{14}C nuclei*, Physical Review Letters, **53**, *8*, pp. 759–752, 1984.
10. S.W. BARWICK, P.B. PRICE, J.D. STEVENSON, *Radioactive decay of ^{232}U by the ^{24}Ne emission*, Physical Review C, **31**, *5*, pp. 1984–1986, 1984.
11. M. MIREA, L. TASSAN-GOT, C. STEPHAN, C.O. BACRI, R.C. BOBULESCU, *Fine structure of the 0.7 MeV resonance in the ^{230}Th neutron induced fission cross section*, Europhysics Letters, **73**, *5*, pp. 705–7011, 2006.
12. M. MIREA. L. TASSAN-GOT, C. STEPHAN, C.O. BACRI, R.C. BOBULESCU, *Landau-Zener effect in fission*, Physical Review C, **76**, 064608, 2007.
13. M. MIREA, L. TASSAN-GOT, C. STEPHAN, C.O. BACRI, P. STOICA, R.C. BOBULESCU, *Dynamical single-particle effects in the threshold fission cross section*, Journal of Physics G, **31**, pp. 1165–1181, 2005.
14. M. MIREA, *New dynamical pair breaking effect*, Physics Letters B, **680**, pp. 316–320, 2009.
15. M. MIREA, *Energy partition in low energy fission*, Preprint arXiv: 1102.2733v1, 2011.
16. M. MIREA, L. TASSAN-GOT, C. STEPHAN, C.O. BACRI, *Dissipation effects in low energy fission*, Nuclear Physics A, **735**, pp. 21–39, 2004.
17. M. MIREA, L. TASSAN-GOT, *Th and U fission barriers within the Woods-Saxon two center shell model,* Central European Journal of Physics, **9**, *1*, pp. 116–122, 2011.
18. I. COMPANIS, M. MIREA, A. ISBASESCU*, Scission configurations for ^{236}Np fission*, Romanian Journal of Physics, **56**, *1–2*, pp. 63–70, 2011.
19. M. MIREA, D.S. DELION, A. SANDULESCU, *Microscopic cold fission yields of ^{252}Cf*, Physical Review C, **81**, 044317, 2010.
20. M. MIREA, D.S. DELION, A. SANDULESCU, *Microscopic description of ^{252}Cf cold fission yields*, Romanian Journal of Physics, **55**, *3–4*, pp. 309–323, 2010.
21. J.R. NIX, *Calculations of fission barriers for heavy and superheavy nuclei,* Annual Review of Nuclear Science, **20**, pp. 65–120, 1972.
22. M. BRACK, J. DAMGAARD, A.S. JENSEN, H.C. PAULI, V. M. STRUTINSKY, *Funny hills: the shell correction approach to nuclear shell effects and its application to the fission process,* Reviews of Modern Physics, **44**, *2*, pp. 320–405, 1972.
23. K.T.R. DAVIES, J.R. NIX, *Calculation of moments, potential and energies for an arbitrarily shaped diffuse-surface nuclear density distribution*, Physical Review C, **14**, *1*, pp. 1977–1994, 1976.
24. D.N. POENARU, M. IVASCU, D. MAZILU*, Folded Yukawa-plus-exponential model PES for nuclei with different charge densities*, Computer Physics Communications, **19**, pp. 205–214, 1980.
25. M. MIREA, O. BAJEAT, F. CLAPIER, F. IBRAHIM, A.C. MUELLER, N. PAUWELS, J. PROUST, *Modeling a neutron-rich nuclei source*, European Physical Journal A, **11**, pp. 59–78, 2001.
26. M. MIREA, *Time-dependent pairing equations for seniority-one nuclear systems*, Physical Review C, **78**, 044618, 2008.
27. M. MIREA, R.C. BOBULESCU, *Cranking mass parameters for fission,* Journal of Physics G, **37**, 055106, 2010.
28. M. MIREA, R.C. BOBULESCU, M. PETRE, *Inertia for two-center nuclear shapes parametrizations*, Romanian Reports in Physics, **61**, *4*, pp. 646–655, 2009.
29. M. PETRE, R.C. BOBULESCU, C. PETRE, M. MIREA, *Cranking inertia in the frame of the Woods-Saxon two center shell model*, Romanian Journal of Physics, **55**, *7–8*, pp. 741–749, 2010.
30. M. MIREA, *Realistic orbital momentum operators for the superasymmetric two-center shell model*, Nuclear Physics A**, 780**, pp. 13–33, 2006.
31. M. MIREA*, Superasymmetric two center shell model for spontaneous heavy-ion emission*, Physical Review C, **54**, *1*, pp. 302–314, 1996.
32. M. MIREA, *Fine structure of alpha decay in odd nuclei*, Physical Review C, **63**, 034603, 2001.
33. M. MIREA, *Fine structure in the ^{14}C cluster emission from ^{225}Ac*, European Physical Journal A, **4**, pp. 335–341, 1999.
34. M. MIREA, *Landau-Zener effect in cluster decay*, Physical Review C, **57**, pp. 2484–2492, 1998.
35. P. STOICA*, Cold fusion for superheavy elements synthesis*, Romanian Reports in Physics, **63**, *1*, pp. 76–85, 2011.
36. M. MIREA, D.S. DELION, A. SANDULESCU, *Cold valleys for fission/fusion superheavy elements beyond Z = 118*, EPL, **85**, 12001, 2009.
37. R.G. LOVAS, R.J. LIOTTA, A. INSOLIA, K. VARGA, D.S. DELION*, Microscopic theory of cluster radioactivity*, Physics Reports**, 294**, pp. 265–362, 1998.

Received March 23, 2011