Abstract. The objective of this study is to quantify and investigate nonlinear motion of the normal human knee joint in different walking conditions using nonlinear analysis. The kinematic data of the human knee flexion-extension angles defined in the sagittal plane were analyzed for different kinematic conditions of human walking on the treadmill and on the over-ground. A time series was constructed for each test. For each time series the angular displacements diagrams are obtained. The kinematic data series were acquired with a complex electro-goniometer system. The curves strides were normalized by interpolation with cubic Spline functions, using MATLAB environment, and reported on the abscissa at an scaled interval from 0 to 100 percentage. The medium stride for each test is determined. The main spatio-temporal parameters of the performed tests and the results obtained from statistical analyse are presented. Profiles of phase plane portraits are presented to demonstrate the utility of the approach in extracting information about the kinematics of steady state. For all time series, the human motion was characterized with the correlation dimension and the largest Lyapunov exponent as nonlinear measures from the experimental time series of the flexion-extension angle of human knee joint. The calculation of the LLE and correlation dimension was performed using the Chaos Data Analyser software.

Key words: human knee joint, experimental walking tests, treadmill, over-ground, nonlinear analysis.

1. INTRODUCTION

Gait analysis is a modern tool that offers the possibility of measuring the biomechanical response to diseases of the musculoskeletal system. Generally, the human joint movement data are collected with different acquisition systems, are extracted, analyzed and are represented as temporal diagrams representing specific joint measures during the gait cycle over-ground or on the treadmill [1–3]. In [4] and [5] a new

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version of the CaTraSys measurement system has been used to determine the trajectory of the human limb extremity during walking operation and furthermore the system is able to measure forces that are exerted by a limb. Experimental determination of articulation mobility is also presented with numerical and experimental results. The waveform analysis techniques include qualitative comparison and subjective descriptions of the overall shapes of gait waveforms. The experimental methods used for data acquisition and kinematical analysis of the gait are also used in the case of the robotic structures [6,8]. In [9] Chau presented a review of analytical techniques for gait data. Quantitative analysis methods of the entire gait cycle data include Fourier series, neural network classifiers, and pattern recognition techniques [10] and [11]. A complex dynamic system is in slight disequilibrium with the environment and maintains this disequilibrium over time. In the many physical and biological systems, the complexity was interpreted as existence of deterministic chaos. Optimal variability as a central feature of normal human movement is consistent with a nonlinear approach. The theory and methodology for nonlinear dynamics has been applied to quantitative analysis of experimental time series, and the chaotic analysis which characterize the system using the Lyapunov exponents and the correlation dimensions, is one of dynamical nonlinear analysis approaches. Gait kinematics and dynamics of polio survivors were analyzed by Hurmuzlu et al. [12] using phase plane portraits and first return maps as graphical tools to detect abnormal patterns in the sagittal kinematics of polio gait. The subjects walked down a track at a comfortable constant speed for several passes. For each subject, the phase plane portraits were obtained from averaging the data over all passes of each individual. Same methodology and analysis were used for the development of quantitative measures in order to evaluate the dynamic stability of human locomotion. In [13] average maximum finite-time Lyapunov exponents were estimated to quantify the local dynamic stability of human walking kinematics and comparisons were made between overground and motorized treadmill walking in young healthy subjects and between diabetic neuropathic patients and healthy controls (CO) during overground walking. Yang and Wu [14] showed the potential of the Lyapunov exponents send to measure the balance control of human standing, an important aspect in the development of bipedal walking machines. Nonlinear dynamic analysis tools have been used to extract characteristic features of human postural sway. The results obtained in [15,16] confirmed that the chaotic characteristics represented by the correlation dimension and the largest Lyapunov exponents were included in the rhythmic forearm movements. The results obtained in [17] suggest that chaotic swaying of the body is a dynamically stable state of the body while receiving information from many segments within the body and its external environment. Proof of this chaotic swaying was provided by reconstruction of the dynamics in phase space and calculation of the largest Lyapunov exponent. The hand trajectories for adults were compared with the trajectories of robotic systems using nonlinear dynamic tools in [18]. Nowak presents the spatial chaos as an analytical tool that can be used in biology and any idea of an
evolutionary process or mechanism should be studied with the help of mathematical equations [19].

The ability to measure the angles would provide information about the human knee motion and stability. A quantitative measure of the human knee flexion-extension angles can provide the clinical team an essential mean to diagnose movement pathologies, administer proper treatments, and monitor patient progress. The objective of this study is to quantify and investigate nonlinear motion of the human knee joint using nonlinear dynamics stability analysis. The largest Lyapunov exponent (LLE) and correlation dimension will be calculated as a chaotic measures from the experimental time series of the flexion-extension angle of human knee joint.

2. NONLINEAR DYNAMICS

2.1. STATE SPACE RECONSTRUCTION

One method to reconstruct the state space \( S \) is to generate the so-called delay coordinates vectors (Packard, 1980) [20].

\[
x_n = \{s(t_0 + nT_s), s(t_0 + nT_s + T), ..., s(t_0 + nT_s + (d_E - 1)T)\},
\]

where \( s(\cdot) \) is a measured scalar function (COP coordinates), \( T_s \) the sampling time, \( n = 1, 2, ..., d_E \), \( T = kT_s \) an appropriately chosen time delay. The space constructed by using the vector \( x_n \) is called the reconstructed space. The integer \( d_E \) is called the embedding dimension. The number of vectors \( x_n \) is chosen in order to satisfy \( n + (m - 1)T \leq N \).

It is assumed that the geometry and the dynamics of the trajectory obtained using the vectors \( x_n \) are the same as the geometry and the dynamics of the trajectory in the actual phase space of the system. This assumption is verified with a proper choice of time delay and embedding dimension. No overlap of the orbit with itself is required in the reconstructed trajectories in the embedding space. The embedding dimension \( d_E \) must be large enough so that the reconstructed orbit does not overlap with itself. When this happens, \( d_E \) is called the proper embedding dimension. The dynamics in the reconstructed state space is equivalent to the original dynamics. As a consequence of the equivalence, an attractor in the reconstructed state space has the same invariants, such as Lyapunov exponents and dimension [21].

2.2. CORRELATION DIMENSION

An attractor’s dimension is a measure of its geometric structure. There are various ways to define the dimension of an attractor, which is generally fractional. One of
them is the correlation dimension $d_F$. The correlation dimension is one of the most used measures of the fractal dimension, and one of the chaotic characteristics, which shows the existence of the self-similarity [13]. The correlation dimension, $C_2(l)$, represents the probability that the distance between two arbitrary points $x_i$ and $x_j$ of the reconstructed space will be $l$.

$$C_2(l) \approx l^{d_F},$$

(1)

where $d_F$ is called the correlation dimension. The correlation dimension is given by the saturation value of the slopes of the curves for an increasing embedding dimension. If there is no saturation of the slopes, and they keep increasing with the increasing of the embedding dimension, then the system is stochastic.

### 2.3. EMBEDDING DIMENSION

Embedding is an important part of the study of chaotic systems. Embedding is a mapping from one dimensional space to a m-dimensional space and the main idea of embedding is that all the variables of a dynamical system influence one another. The theorem of Takens [22] states that a map exists between the original state space and a reconstructed state space and the dynamical properties of the system in the true state space are preserved under the embedding transformation. One of the most used method for measuring the minimal embedding dimension is the false nearest-neighbor (FNN) method, introduced by Kennel et al. [23]. The method is based on the assumption that the phase space of a dynamical system folds and unfolds smoothly, and there are no sudden irregularities. The main idea of the method is to unfold the observed orbits from self overlap arising from the projection of an attractor of a dynamical system on a lower dimensional space [24]. If a phase space point has a neighbor that does not fulfil this criteria then that point is said to have a false neighbor [24]. The minimum dimensionality needed to properly reconstruct the chaotic flow is marked by a vanishing fraction of FNN [25]. If two nearest neighbors are nearest neighbors in the $d_i$ dimension but they are not in the $d_{i+1}$ dimension they are called false nearest neighbors. As $i$ increases, the total percentage of false neighbors (%FN) declines and $d_E$ is chosen where this percentage of false nearest neighbors approaches zero [26]. The determination of the local dimension $d_L$ of the dynamics by the local false nearest neighbor method shows how many dimensions can be used to model the dynamics. The local dimension also shows how many true Lyapunov exponents should be evaluated for the system.

### 2.4. LYAPUNOV EXPONENTS

Lyapunov exponents, $\lambda_i$ provide a measure of the sensitivity of the system to its
initial conditions. They exhibit the rate of divergence or convergence of the nearby trajectories from each other in state space and are fundamentally used to distinguish the chaotic and non-chaotic behavior \[26\]. If a 3-dimensional state space is considered there will be an exponent for each dimension: all negative exponents will indicate the presence of a fixed point; one zero and the other negative indicate a limit cycle; one positive indicate a chaotic attractor \[13\]. If the system has more than one positive Lyapunov exponent, the system is called to be hyperchaotic and the magnitude of the largest Lyapunov exponent indicates the maximum amount of instability in any direction in the attractor. Therefore, in order to characterize the behavior of a dynamical system the sign of Lyapunov exponents must be determined. The value of the largest Lyapunov exponent is expressed in bits of information/second and it is the main exponent that quantifies the exponential divergence of the neighboring trajectories in the reconstructed state space and reflects the degree of chaos in the system. The obtained information is necessary to identify the stability of the time series.

3. EXPERIMENTAL STUDY

The experimental method which allows to obtain the kinematic parameters diagrams for the human and artificial radio-carpal joint is based on the own designed electrogoniometer based data acquisition system. The electrogoniometer is made of one potentiometer and two rods; one of them is fixed on the longitudinal axis of femur bone, while the other is fixed on the longitudinal axis of the tibia. The rods are fixed with elastic straps, type Velcro. The acquisition data and control board Arduino Mega 2560 has the role of acquisition data from the potentiometer and it stocks up this data in files data which will be processed. In the command board it is upload the code written in C/C++ which returns the angular values function of time. The board and the computer are interconnected by serial interface and the acquisition data series are displayed on the monitor. The block schema of the acquisition data system based on a electrogoniometer is presented in Fig. 1.

In Fig. 2 is shown the data acquisition system mounted on the subject.

Measurements were performed on a healthy man aged 33 years, body weight 82 kg, height 174 cm, lower limb length 81 cm hip-knee distance 43 cm, knee-ankle distance 38 cm. The participant was pain-free, had no evidence or known history of motor and skeletal disorders, or record of surgery to the lower limbs and provided written informed consent. The study was approved by the University of Craiova human ethics research committee. Before starting the experiments there were collected anthropometric data. The subject becomes familiar with the tests by repeating them several times before the beginning of the final experimental tests.

The knee flexion-extension during the locomotion process was performed in a 2D space in sagittal plane. The human subject executes the following tests for which the
experimental data were acquired for continuous walking cycles: Test 1 – walking on the treadmill for 3 minutes at a slow speed of 1.8km/h; Test 2 – walking on the treadmill for 3 minutes at a normal speed of 3.6km/h; Test 3 – walking on the treadmill for 3 minutes at a fast speed of 5.4km/h; Test 4 – walking over-ground for 2 minutes at a normal speed of about 3.6km/h; Test 5 – walking over-ground for 2 minutes at a fast speed of about 5.4km/h.

The main spatio-temporal parameters of the tests performed are presented in Table 1. Cadence and the medium stride length increased as the walking speed increased. With increasing the walking speed there was a significant linear trend of
increasing cadence and stride length. For similar speed values, the stride length is higher and the cadence is less on the treadmill than over-ground.

Table 1
Main parameters of the walking tests

<table>
<thead>
<tr>
<th>Test</th>
<th>Speed [km/h]</th>
<th>Distance [m]</th>
<th>Strides number</th>
<th>Stride length</th>
<th>Cadence [str/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test_1</td>
<td>1.8</td>
<td>92</td>
<td>196</td>
<td>0.47</td>
<td>1.08</td>
</tr>
<tr>
<td>Test_2</td>
<td>3.6</td>
<td>180</td>
<td>280</td>
<td>0.64</td>
<td>1.55</td>
</tr>
<tr>
<td>Test_3</td>
<td>5.4</td>
<td>272</td>
<td>348</td>
<td>0.78</td>
<td>1.63</td>
</tr>
<tr>
<td>Test_4</td>
<td>3.8</td>
<td>132</td>
<td>246</td>
<td>0.54</td>
<td>2.05</td>
</tr>
<tr>
<td>Test_5</td>
<td>5.5</td>
<td>184</td>
<td>318</td>
<td>0.58</td>
<td>2.65</td>
</tr>
</tbody>
</table>

4. RESULTS

The angular amplitudes of human knee flexion-extension during the gait performed on the treadmill and over-ground conditions were obtained for each test from the report generated by the acquisition system as data files. In the data analysis the beginning region and the end region of the time series were cut off in order to remove the transient data and the final data series contain an average number of 17000 points. The diagram of the knee flexion-extension angles obtained from the collected data cycles corresponding to the test 4 is obtained using MATLAB environment and it is shown in Fig. 3. Similar diagrams are obtained for all the five tests.

![Fig. 3 – Diagram of the knee flexion-extension angles [degrees] in respect with time [s].](image)

The knee flexion-extension movements occur entirely within the normal range of motion according to normative data. For more accurate results and for analyse
the good repeatability of each human test, considering the natural biological variability from one individual stride to another, for each test were selected 15 consecutive strides. In Fig. 4 is presented the flexion-extension diagram of continuous 15 strides for Test 4.

The curves strides were normalized by interpolation with cubic Spline functions, using Matlab environment, and reported on the abscissa at an scaled interval from 0 to 100 percentage. The medium stride was determined as being the arithmetical mean of the data that correspond to the 15 strides. For each test were drawn the curves of flexion-extension angles corresponding to the fifteen strides, and also it has been drawn the corresponding curve of the medium stride. In Fig. 5 there are being displayed the normalised curves corresponding to the 15 strides and the medium stride recorded over-ground at the speed of 3.6 km/h for Test 4.

Comparing the fifteen maximum amplitudes of the knee flexion-extension angles for test 4, the values ranged from 66.45 to 71.29 and the mean value was 69.09, with a standard deviation equal to 1.55. The values of the medium stride were calculated as average of the fifteen strides values. The maximum amplitude of the medium stride was 68.80, very close to 68.76 with non-significant differences. The minor differences obtained by this comparison show a good repeatability of the imposed exercise for subject 1. The range of the fifteen strides maximum values, the medium value and the standard deviation of the maximum values, and, also, the maximum value of the medium stride are presented for all the tests in Table 2.
Fig. 5 – Knee flexion-extension diagrams for each of fifteen strides and for medium stride (Test 4).

Table 2

Main spatio-temporal parameters

<table>
<thead>
<tr>
<th>Test</th>
<th>Range of max.val.</th>
<th>Medium of max.val.</th>
<th>St.dev.</th>
<th>Max. val of medium stride</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test_1</td>
<td>59.09 - 63.15</td>
<td>60.26</td>
<td>1.04</td>
<td>58.58</td>
</tr>
<tr>
<td>Test_2</td>
<td>67.22 - 71.61</td>
<td>68.76</td>
<td>0.96</td>
<td>68.67</td>
</tr>
<tr>
<td>Test_3</td>
<td>71.45 - 74.19</td>
<td>72.84</td>
<td>0.85</td>
<td>72.76</td>
</tr>
<tr>
<td>Test_4</td>
<td>66.45 - 71.29</td>
<td>69.09</td>
<td>1.55</td>
<td>68.80</td>
</tr>
<tr>
<td>Test_5</td>
<td>71.32 - 74.18</td>
<td>72.33</td>
<td>1.32</td>
<td>72.22</td>
</tr>
</tbody>
</table>

Similar results were obtained for all the tests. The variation of the medium strides for all the tests are represented in Fig. 6.

Fig. 6 – Medium strides for all the five tests.

Analysing the values from Table 2, it can be seen that as the speed increases, the maximum values for knee flexion angles increase and the standard deviation decreases, so the variation decreases and the performed repeatability increases.
The minor differences obtained for all the experimental tests between the medium value of the maximum angles values and the maximum value of the medium stride could lead to the conclusion that all the tests were performed with a good repeatability.

In this study the human lower limb was considered as a nonlinear system that can achieve dynamic equilibrium. Standard plots in which the knee joint displacements are plotted against time do not provide sufficient information about the dynamics of the system. Phase plane portraits will be used to characterize the kinematics of the system when it attained this equilibrium. Using phase plane portraits one can correlate the joint rotations with the respective joint velocities. The phase plane plot is a two-dimensional plot in which the time derivative $\dot{\phi}$ is plotted versus $\phi$ at each data point. The phase plane plots, shown in Fig. 7, contain information as the graph of data for all five tests. Phase plane portraits can be utilized to compare the joint kinematics of clinical patients with normals.

Comparing the phase-plane graphics corresponding to the five tests, it can be seen that as the speed increases, the cycles curves of these tests become more compact and their spread is decreasing. The plots traced for slow speed show more divergence in their trajectories, while the trajectories obtained for faster speed are confined within a tighter space. So at normal speed and fast speed, rhythm and amplitude of consecutive steps tend to be constant, while in the case of low speed rhythm and amplitude varies. Also, it can be seen that for same speed (3.6 km/h and, respectively, 5.4 km/h) the phase plane curves are more compact for walking on the treadmill than for walking on the ground conditions.
To complete the observations, the phase plane for medium cycle of the first three tests (treadmill conditions) are presented in Fig. 8 a) and for the last two tests are presented in Fig. 8 b).

![Phase-plane for medium cycle: angles [degree] on horizontal axis and angular velocity [degree/sec] on the vertical axis a) on treadmill b) over ground.](image)

It can be seen that for both situations, treadmill conditions a) and over-ground conditions b), the phase planes are almost concentric curves, inside is the curve corresponding to the lowest speed, and outside is the curve corresponding to the highest speed. In treadmill conditions the angular velocity has the maximum value when the amplitude is about 43 degrees on the ascendant curve, ((40 deg; 2,53 deg/s) for Test_2, respectively (41 deg; 3,1 deg/s) for Test_3), the value 0 when the amplitude is maximum and has maximum negative value when the amplitude is about 20-30 degrees on the descendant curve ((29 deg; –2,81 deg/s); (27 deg; –3,4 deg/s)). In over-ground conditions the angular velocity has the maximum value when the amplitude is 46 degrees on the ascendant curve((46 deg; 2,90 deg/s), respectively (46 deg; 3,3 deg/s)), the value 0 when the amplitude is maximum and has maximum negative value when the amplitude is about 20-30 degrees on the descendant curve ((30 deg; –2,9 deg/s); (26 deg; –3,16 deg/s)).

Traditional measures of variability, which only provide estimates of the average magnitude of variations across strides, are therefore insufficient to characterize the local dynamic stability properties of locomotor behavior. For characterizing the underlying complexity during movement, the experimental data are analysed using the Lyapunov exponents and correlation dimensions of angular amplitude in the human knee joint as a measure of the stability of a dynamical system (such as the human during walking or during different limbs movements). A suitable embedding dimension was chosen by using the false nearest neighbour (FNN) method. Embedding dimension is the minimum value that trajectories of the reconstructed state vector may not cross over each other in state space. Total percentage of false neighbours for the human knee joint was computed by FNN method, and the number of dimensions was chosen where this percentage approaches zero. In this study, as we can see in Fig. 8, the value is $d_E = 3$ and it was adopted. The results were similar for all five time series and indicated an appropriate embedding dimension of $d_E = 3$. 

![Fig. 8 – Phase-plane for medium cycle: angles [degree] on horizontal axis and angular velocity [degree/sec] on the vertical axis a) on treadmill b) over ground.](image)
Fig. 9 – Embedding dimension chosen by using the false nearest neighbour (FNN) method.

With Embedding Dimension=3 and time delay=4, using Chaos Data Analyzer (CDA) software, we calculated Mean Correlation Dimension for all five time series. This values are presented in Table 3. As we can see, the correlation dimension were increasing, the higher the frequency becomes.

<table>
<thead>
<tr>
<th>Test</th>
<th>Mean Cor. Dim.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test_1</td>
<td>1.855</td>
</tr>
<tr>
<td>Test_2</td>
<td>1.960</td>
</tr>
<tr>
<td>Test_3</td>
<td>1.968</td>
</tr>
<tr>
<td>Test_4</td>
<td>1.978</td>
</tr>
<tr>
<td>Test_5</td>
<td>2.034</td>
</tr>
</tbody>
</table>

We also calculated the largest Lyapunov exponents (LLE) for all time series using a special software to analyze nonlinear time series data, Chaos Data Analyzer, (Sprott and Rowlands, 1992) based on the method by Wolf et al [28]. The value of the LLE is the main exponent that quantifies and reflects the degree of chaos in the system. If the system is known to be deterministic, a positive Lyapunov number can be taken as proof of a chaotic system. The LLE calculated for all time series were positive, then it can be concluded that the time series of amplitude of the human knee joint movement were chaos and the human lower limb is a deterministic chaotic systems. The largest Lyapunov exponents for human knee joint movement for all five tests are presented in Table 3. The values are given in units of bits per data sample. The existence of the positive Lyapunov exponent indicates that the time series of the movement amplitude has the instability. The results are similar with those obtained by other researchers in their nonlinear dynamics studies applied at human joints movements [13].

As we can observe from Table 4, the LLE values for similar speed movements are smaller on the overground conditions than on the treadmill. One of the explanations
could be that the treadmills may artificially reduce the natural variability, compared to overground walking, because walking speed is strictly enforced. Overground walking may yield slightly larger values for the measures quantified. The value of LLE is higher in the case of Test 1 (speed equal to 1.8 km/h) because the effects of the very low speed (as the effects of very high speed) on the variability are more pronounced than in the situation of normal or rapid speed [29].

5. CONCLUSIONS

A study based on the tools of nonlinear dynamics to visualize the steady state kinematics of human knee movement is presented. The kinematic data of the flexion-extension angles for five tests of human knee joint motion were analysed. The purpose of this study was to investigate the spatio-temporal characteristics, the biomechanics of chaotic characteristics of movements of the knee joint. We applied the chaotic analysis to the rhythmic flexion-extension movements of human knee joint and showed that there existed the chaotic feature of angle positions using the chaotic measure such as the largest Lyapunov exponent and the correlation dimension. The LLE obtained for each test of human knee joint were positive values. The mean LLE for human knee joint ranged from 0.040 to 0.080. As in other research results [30], our results demonstrated that walking on the treadmill and walking overground with relative faster speed are associated with significant reductions in knee flexion variability and with improvements in local dynamic stability.

The flexion-extension movement of the human knee during the locomotion presents interest for the kinematic and dynamic study of the bipedal locomotion applied to humanoid robots. The results obtained can be used as a valuable reference for the normal knee joint movement for further studies of abnormal movement. A quantitative measure of the stability of human knee can provide the clinical team an essential mean to diagnose movement pathologies, administer proper treatments, and monitor patient progress. The results can show the potential of the Lyapunov exponents used to measure the balance control of human walking, an important aspect in the development of bipedal walking machines. We anticipate that the results of this study can be used for the design of new artificial devices that could reproduce the motion of
the lower limb, in robotics for humanoid robots, in the medical field as a prosthetic device for the human lower limb or as educational robot in order to study the complex process of locomotion.

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