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NONLINEAR PROGRAMMING WITH SEMILOCALLY B-PREINVEX AND RELATED FUNCTIONS

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A nonlinear programming problem is considered where the functions involved are η- semidifferentiable. Fritz John type and Karush-Kuhn-Tuker type necessary optimality conditions are obtained. Moreover, a result relative to sufficiency of optimality conditions is given. Wolfe type and Mond-Weir type duality results are formulated in terms of η- semidifferentials. The duality results are given using the concepts of generalized semilocally b-preinvex functions. Our results generalize the results obtained by Preda, Stancu-Minasian and Batatorescu [2], Suneja and Gupta [5], Suneja *et al.* [6].

1. PRELIMINARIES

In this section, we introduce the notation and definitions which are used throughout the paper.

Let $\mathbf{R}^n$ be the n-dimensional Euclidean space and $\mathbf{R}^n_+$ its positive orthant, i.e. $\mathbf{R}^n_+ = \{x\in \mathbf{R}^n,\ x = (x_j),\ x_j \geqq 0, j=1,\dots, n\}$.

For $x,y \in \mathbf{R}^n$, by $x \leqq y$ we mean $x_i \leqq y_i$ for all i, $x \leq y$ means $x_i \leqq y_i$ for all i and $x_j < y_j$ for at least one j, $1 \leqq j \leqq n$. By $x < y$ we mean $x_i < y_i$ for all i, and by $x \nleq y$ we mean the negation of $x \leq y$.

Throughout the paper all definitions, theorems, lemmas, corollaries, remarks are numbered consecutively in a single numeration system in each section.

Let $X^0 \subseteq \mathbf{R}^n$ be a set and $\eta : X^0 \times X^0 \to \mathbf{R}^n$ a vector function.

Definition 1.1. We say that X^0 is η-*vex* at $\bar{x} \in X^0$ if $\bar{x} + \lambda\eta(x, \bar{x}) \in X^0$ for all $x \in X^0$ and $\lambda \in [0, 1]$.

We say that X^0 is η-*vex* if X^0 is η-*vex* at any $x \in X^0$.

We remark that if $\eta(x, \bar{x}) = x - \bar{x}$ for any $x \in X^0$, then X^0 is η-vex at $\bar{x} \in X^0$ iff X^0 is a convex set at $\bar{x}$.

Definition 1.2. [7] Let $X^0 \subseteq \mathbf{R}^n$ be a nonempty set. A function $f : X^0 \to \mathbf{R}$ is said to be *preinvex* on X^0 (with respect to η) (f is η-vex, for short) if there exists an n-dimensional vector function $\eta : X^0 \times X^0 \to \mathbf{R}^n$ such that for all $x, u \in X^0$ and $\lambda \in [0, 1]$ we have

$$f(u + \lambda\eta(x, u)) \leqq \lambda f(x) + (1 - \lambda) f(u).$$

Definition 1.3. We say that $X^0 \subseteq \mathbf{R}^n$ is an η-*locally starshaped set* at $\bar{x}$ ($\bar{x} \in X^0$) if for any $x \in X^0$ there exists $0 < a_\eta(x, \bar{x}) \leqq 1$ such that $\bar{x} + \lambda\eta(x, \bar{x}) \in X^0$ for any $\lambda \in [0, a_\eta(x, \bar{x})]$.

We say that X^0 is η-locally starshaped if X^0 is η-locally starshaped at any $\bar{x} \in X^0$.

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Definition 1.4. Let $f: X^0 \to \mathbf{R}$ be a function, where $X^0 \subseteq \mathbf{R}^n$ is an η-locally starshaped set at $\bar{x} \in X^0$, with the corresponding maximum positive number $a_\eta(x, \bar{x})$ satisfying the required conditions. We say that f is:

(i_1) *semilocally b-preinvex* (*slb-preinvex*) at $\bar{x}$ if for any $x \in X^0$, there exist a positive number $d_\eta(x, \bar{x}) \leqq a_\eta(x, \bar{x})$ and a function $b: X^0 x X^0 x[0,1] \to \mathbf{R}_+$ such that $f(\bar{x} + \lambda\eta(x,\bar{x})) \leqq \lambda b(x,\bar{x},\lambda)f(x) + (1-\lambda b(x,\bar{x},\lambda))f(\bar{x})$ for $0<\lambda< d_\eta(x,\bar{x})$, $\lambda b(x,\bar{x},\lambda) \leqq 1$.

If f is semilocally b-preinvex at each $\bar{x} \in X^0$ for the same b, then f is said to be semilocally b-preinvex on X^0.

(i_2) *semilocally quasi b-preinvex (slqb-preinvex)* at $\bar{x}$ if for any $x \in X^0$, there exist a positive number $d_\eta(x, \bar{x}) \leqq a_\eta(x, \bar{x})$ and a function $b: X^0 x\ X^0 x\,[0,1] \to \mathbf{R}_+$ such that

$$\left.\begin{array}{l} f(x) \leqq f(\bar{x}) \\ 0 < \lambda < d_\eta(x,\bar{x}) \\ \lambda b(x,\bar{x},\lambda) \leqq 1 \end{array}\right\} \Rightarrow b(x,\bar{x},\lambda)\, f[\bar{x} + \lambda\eta(x,\bar{x})] \leqq b(x,\bar{x},\lambda)\, f(\bar{x})$$

If f is semilocally quasi b-preinvex at each $\bar{x} \in X^0$ for the same b, then f is said to be semilocally quasi b-preinvex on X^0.

Definition 1.5. [2],[3] Let $f: X^0 \to \mathbf{R}$ be a function, where $X^0 \subseteq \mathbf{R}^n$ is an η-locally starshaped set at $\bar{x} \in X^0$. We say that f is *η-semidifferentiable* at $\bar{x}$ if $(df)^+(\bar{x},\eta(x,\bar{x}))$ exists for each $x \in X^0$, where

$$(df)^+(\bar{x},\eta(x,\bar{x})) = \lim_{\lambda\to 0^+} \frac{1}{\lambda}[f(\bar{x}+\lambda\eta(x,\bar{x})) - f(\bar{x})]$$

(the right derivative at $\bar{x}$ along the direction $\eta(x,\bar{x})$).

If f is η-semidifferentiable at any $\bar{x} \in X^0$, then f is said to be η-semidifferentiable on X^0.

Note that semidifferentiable functions correspond to $\eta(x,\bar{x}) = x - \bar{x}$.

Some properties possesed by the semidifferentiable functions are given by Kaul and Lyall [1].

Definition 1.6. Let $f: X^0 \to \mathbf{R}$ be an η-semidifferentiable function on $X^0 \subseteq \mathbf{R}^n$. We say that f is *semilocally pseudo b-preinvex* (*slpb-preinvex*) at $\bar{x} \in X^0$ if

$$(df)^+(\bar{x},\eta(x,\bar{x})) \geqq 0 \Rightarrow b(x,\bar{x},\lambda)\, f(x) \geqq b(x,\bar{x},\lambda)\, f(\bar{x}).$$

If f is semilocally pseudo b-preinvex at each $\bar{x} \in X^0$ for the same b, then f is said to be semilocally pseudo b-preinvex on X^0.

Definition 1.7. Let $f: X^0 \to \mathbf{R}$ be an η-semidifferentiable function on $X^0 \subseteq \mathbf{R}^n$. We say that f is *semilocally explicitly b-preinvex* (*sleb-preinvex*) at $\bar{x} \in X^0$ if for each $x \in X^0$, $x \neq \bar{x}$, we have

$$\bar{b}(x,\bar{x})[f(x) - f(\bar{x})] > (df)^+(\bar{x},\eta(x,\bar{x}))$$

where

$$\bar{b}(x,\bar{x}) = \lim_{\lambda\to 0^+} b(x,\bar{x},\lambda)\,. \tag{1.1}$$

Definition 1.8. Let $f: X^0 \to \mathbf{R}$ be an η-semidifferentiable function on $X^0 \subseteq \mathbf{R}^n$. We say that f is *semilocally strongly pseudo b-preinvex* (*slspb-preinvex*) at $\bar{x} \in X^0$ if

$$\bar{b}(x,\bar{x})(\mathrm{d}f)^{+}(\bar{x},\eta(x,\bar{x})) \geqq 0 \Rightarrow f(x) \geqq f(\bar{x})$$

where $\bar{b}(x,\bar{x})$ is defined by (1.1).

If f is slspb-preinvex at each $\bar{x} \in X^0$ for the same b, then f is said to be slspb-preinvex on X^0.

For $b(x,\bar{x},\lambda) = 1$ these definitions reduce to those of semilocally preinvex, semilocally quasi-preinvex, semilocally pseudo-preinvex considered by Preda, Stancu-Minasian and Batatorescu [2].

Theorem 1.9. *Let $f : X^0 \to \mathbf{R}$ be an η-semidifferentiable function on an η-locally starshaped set X^0.*

a) *The function f is slb-preinvex at $\bar{x} \in X^0$ if and only if $(\mathrm{d}f)^{+}(\bar{x},\eta(x,\bar{x}))$ exists and*

$$\bar{b}(x,\bar{x})[f(x) - f(\bar{x})] \geqq (\mathrm{d}f)^{+}(\bar{x},\eta(x,\bar{x}))$$

b) *If f is slqb-preinvex, then*

$$f(x) \leqq f(\bar{x}) \Rightarrow \bar{b}(x,\bar{x})(\mathrm{d}f)^{+}(\bar{x},\eta(x,\bar{x})) \leqq 0,$$

where

$$\bar{b}(x,\bar{x}) = \lim_{\lambda\to 0^{+}} b(x,\bar{x},\lambda) \ \textit{and}\ \lambda b(x,\bar{x},\lambda) \leqq 1.$$

2. SUFFICIENT OPTIMALITY CRITERIA

Consider the nonlinear programming problem

$$(\mathrm{NP})\begin{cases}\text{Minimize } f(x)\\ \text{subject to}: g(x) \leqq 0,\ x \in X^0\end{cases}$$

where $X^0 \subseteq \mathbf{R}^n$ is a nonempty η-locally starshaped set and $f : X^0 \to \mathbf{R}$, $g : X^0 \to \mathbf{R}^m$ are η-semidifferentiable functions.

Let $X = \{x \in X^0 \mid g(x) \leqq 0\}$ be the set of all feasible solutions to (NP).

Let

$$N_\varepsilon(\bar{x}) = \{x \in \mathbf{R}^n \mid \| x - \bar{x} \| < \varepsilon\}$$

Definition 2.1. (a) $\bar{x}$ is said to be a local minimum solution to problem (NP) if $\bar{x} \in X$ and there exists $\varepsilon > 0$ such that

$$x \in N_\varepsilon(\bar{x}) \cap X \Rightarrow f(x) \geqq f(\bar{x}).$$

(b) $\bar{x}$ is said to be the minimum solution to problem (NP) if $\bar{x} \in X$ and $f(\bar{x}) = \min_{x \in X} f(x)$.

The next theorem gives a sufficient optimality criterion.

Theorem 2.2. *Let $\bar{x} \in X^0$ and let f be slb_1-preinvex at $\bar{x}$ and g be slb_2-preinvex at $\bar{x}$. If there exists $\bar{u} \in \mathbf{R}^m$ such that $(\bar{x},\bar{u})$ satisfies the conditions*

$$(\mathrm{d}f)^{+}(\bar{x},\eta(x,\bar{x})) + \bar{u}^T(\mathrm{d}g)^{+}(\bar{x},\eta(x,\bar{x})) \geqq 0,\ \forall x \in X, \tag{2.1}$$

$$\bar{u}^T g(\bar{x}) = 0, \tag{2.2}$$

$$g(\bar{x}) \leqq 0, \tag{2.3}$$

$$\bar{u} \geqq 0, \tag{2.4}$$

with $\bar{b}_1(x\ \bar{x}) = \lim_{\lambda\to 0^{+}} b_1(x,\bar{x},\lambda) > 0$, then $\bar{x}$ is an optimal solution to problem (NP).

Corollary 2.3. *Let $\bar{x} \in X^0$ and let f be slb_1-preinvex at $\bar{x}$ and g be slb_2-preinvex at $\bar{x}$. If there exists $\bar{u}_0 \in \boldsymbol{R}$ and $\bar{u} \in \mathbf{R}^m$ such that $(\bar{x},\bar{u}_0,\bar{u})$ satisfy (2.2) and (2.3) of Theorem 2.2., and the conditions*

$$\bar{u}_0\,(\mathrm{d}f)^+(\bar{x},\eta(x,\bar{x})) + \bar{u}^T(\mathrm{d}g)^+(\bar{x},\eta(x,\bar{x})) \geqq 0,\ \forall x \in X$$
$$(\bar{u}_0,\bar{u}) \geqq 0,\ (\bar{u}_0,\bar{u}) \neq 0$$
$$\bar{u}_0 > 0$$

with $\bar{b}_1(x,\bar{x}) = \lim_{\lambda\to 0^+} b_1(x,\bar{x},\lambda)$, then $\bar{x}$ is an optimal solution to problem (NP).

Remark 2.4. In the statement of Corollary 2.3, it suffices to assume only the slb$_2$-preinvexity of $g_I\,(I = \{i \mid g_i(\bar{x}) = 0\})$, instead of $g_i\,(i = 1, ..., m)$ at $\bar{x}$.

Theorem 2.5. *Let $\bar{x}\in X^0$, f be slspb-preinvex and g_I be η-semidifferentiable and slqb-preinvex at $\bar{x}$. If there exists $\bar{u}\in\mathbf{R}^m$ such that $(\bar{x},\bar{u})$ satisfy conditions (2.1) - (2.4) of Theorem 2.2, then $\bar{x}$ is an optimal solution to Problem (NP).*

Theorem 2.6. *Let $\bar{x}\in X^0$. We assume that there exists $\bar{u}\in\mathbf{R}^m$ such that at $\bar{x}$, f is slspb-preinvex, the numerical function $\bar{u}_I\, g_I$ is η-semidifferentiable and slqb-preinvex and such that $(\bar{x},\bar{u})$ satisfies conditions (2.1) - (2.4) of Theorem 2.2. Then $\bar{x}$ is an optimal solution to Problem (NP).*

Theorem 2.7. *Let $\bar{x}\in X^0$. We assume that there exists $\bar{u}\in\mathbf{R}^m$ such that $(\bar{x},\bar{u})$ satisfies conditions (2.1) - (2.4) of Theorem 2.2 and the numerical function $f + \bar{u}_I\, g_I$ is slspb-preinvex at $\bar{x}$. Then $\bar{x}$ is an optimal solution to Problem (NP).*

3. NECESSARY OPTIMALITY CRITERIA

Definition 3.1. We say that g satisfies the generalized Slater's constraint qualification (GSQ) at $\bar{x}\in X$, if g_I is slpb-preinvex at $\bar{x}$ and there exists $\hat{x}\in X$ such that $g_I(\hat{x}) < 0$.

Lemma 3.2. *Let $\bar{x}\in X$ be a local minimum solution to (NP). We assume that g_i is continuous at $\bar{x}$ for any $i\in J$, and that f, g_I are η-semidifferentiable at $\bar{x}$. Then the system*

$$\begin{cases}(\mathrm{d}f)^+(\bar{x},\eta(x,\bar{x})) < 0\\ (\mathrm{d}g_I)^+(\bar{x},\eta(x,\bar{x})) < 0\end{cases}$$

has no solution $x\in X^0$.

Theorem 3.3. (Fritz John type necessary optimality criteria) *Let us suppose that g_i is continuous at $\bar{x}$ for $i\in J$. Assume also that $(\mathrm{d}f)^+(\bar{x},\eta(x,\bar{x}))$ and $(\mathrm{d}g_I)^+(\bar{x},\eta(x,\bar{x}))$ are preinvex functions of x on X^0, which is an η-locally starshaped set at $\bar{x}$. If $\bar{x}$ is a local minimum solution to Problem (NP), then there exist $\bar{u}_0\in\boldsymbol{R}$, $\bar{u}\in\mathbf{R}^m$ such that*

$$\bar{u}_0\,(\mathrm{d}f)^+(\bar{x},\eta(x,\bar{x})) + \bar{u}^T(\mathrm{d}g)^+(\bar{x},\eta(x,\bar{x})) \geqq 0 \textit{ for all } x\in X^0,$$
$$\bar{u}^T g(\bar{x}) = 0,$$
$$(\bar{u}_0,\bar{u}) \neq 0,\ (\bar{u}_0,\bar{u}) \geqq 0.$$

Theorem 3.4. (Kuhn-Tucker type necessary optimality criteria) *Let $\bar{x}\in X$ be a local minimum solution to Problem (NP) and let g_i be continuous at $\bar{x}$ for $i\in J$. Assume also that $(\mathrm{d}f)^+(\bar{x},\eta(x,\bar{x}))$ and $(\mathrm{d}g_I)^+(\bar{x},\eta(x,\bar{x}))$ be preinvex functions of x on X^0 - an η-locally starshaped set at $\bar{x}$. If g satisfies GSQ at $\bar{x}$, then there exists $\bar{u}\in\mathbf{R}^m$ such that*

$$(\mathrm{d}f)^+(\bar{x},\eta(x,\bar{x})) + \bar{u}^T(\mathrm{d}g)^+(\bar{x},\eta(x,\bar{x})) \geqq 0, \textit{ for all } x\in X^0,$$
$$\bar{u}^T g(\bar{x}) = 0,\ g(\bar{x}) \leqq 0,\ \bar{u} \geqq 0.$$

4. WOLFE DUALITY

Relative to the Problem (NP) we consider the Wolfe dual

(WD) Maximize $\Psi(u, y) = f(u) + y^{\mathrm{T}} g(u)$

subject to $(\mathrm{d}f)^{+}(u,\eta(x\, u)) + y^{\mathrm{T}}(\mathrm{d}g)^{+}(u,\eta(x, u)) \geqq 0$, for all $x \in X$,

$y \geqq 0$, $u \in X^{0}$, $y \in \mathbf{R}^{m}$,

where X^{0} is a nonempty η-locally starshaped set at any $x \in X^{0}$.

Let W denote the set of all feasible solutions to Problem (WD).

Theorem 4.1. (Weak Duality) *Let $\bar{x} \in X$ and $(\bar{u}, \bar{y}) \in W$. If f and g are slb-preinvex on X^{0}, with $\bar{b}(\bar{x}, \bar{u}) = \lim_{\lambda \to 0^{+}} b(\bar{x}, \bar{u}, \lambda) > 0$, then $f(\bar{x}) \geqq \Psi(\bar{u}, \bar{y})$.*

Corollary 4.2. *Let $\bar{x} \in X$ and $(\bar{u}, \bar{y}) \in W$ such that $f(\bar{x}) = \Psi(\bar{u}, \bar{y})$. If the hypotheses of Theorem 4.1 are satisfied, then $\bar{x}$ and $(\bar{u}, \bar{y})$ are the optimal solutions to (NP) and (WD) respectively.*

Theorem 4.3. (Direct Duality) *Let $\bar{x} \in X$ be an optimal solution to (NP), f and g be η-semidifferentiable at $\bar{x}$ and*

i_1) $(\mathrm{d}f)^{+}(\bar{x}, \eta(x, \bar{x}))$ and $y^{\mathrm{T}}(\mathrm{d}g)^{+}(\bar{x}, \eta(x, \bar{x}))$ are preinvex functions of x on X^{0}, an η-locally starshaped set at $\bar{x}$;

i_2) g_i $(i \in J)$ are continuous at $\bar{x}$;

i_3) g satisfies the generalized Slater's constraint qualification at $\bar{x}$.

Then there exists $\bar{y} \in \mathbf{R}^{m}$ such that $(\bar{x}, \bar{y}) \in W$ and $f(\bar{x}) = \Psi(\bar{x}, \bar{y})$.

Moreover, if the functions f and g are slb-preinvex on X^{0} and $\bar{b}(x,u) > 0$ for all $(u, y) \in W$, then $\bar{x}$ is an optimal solution to (NP) and $(\bar{x}, \bar{y})$ is an optimal solution to (WD).

Theorem 4.4. (Strict Converse Duality) *Let $\bar{x} \in X$ be an optimal solution to (NP), f and g be η-semidifferentiable at $\bar{x}$ and:*

i_1) $(\mathrm{d}f)^{+}(\bar{x}, \eta(x, \bar{x}))$ *and* $y^{\mathrm{T}}(\mathrm{d}g)^{+}(\bar{x}, \eta(x, \bar{x}))$ *are preinvex functions of x on X^{0}, an η-locally starshaped set at $\bar{x}$;*

i_2) *g_i $(i \in J)$ are continuous at $\bar{x}$;*

i_3) *g satisfies the generalized Slater's constraint qualification at $\bar{x}$;*

i_4) *g is slb-preinvex on X^{0}.*

If (x^{*}, y^{*}) is an optimal solution of (WD), f is sleb-preinvex on X^{0} and $\bar{b}(\bar{x}, x^{*}) > 0$, then $x^{*} = \bar{x}$, i.e. x^{*} is an optimal solution to (NP) and $f(\bar{x}) = \Psi(x^{*}, y^{*})$.

Remark 4.5. If $\eta(x, u) = x - u$ we obtain the Wolfe dual considered by Suneja and Gupta [5].

5. MOND-WEIR DUALITY

For problem (NP) we consider a general Mond-Weir dual problem

(MWD) Maximize $f(u)$

subject to: $(\mathrm{d}f)^{+}(u,\eta(x,u)) + y^{\mathrm{T}}(\mathrm{d}g)^{+}(u,\eta(x, u)) \geqq 0$, $\forall x \in X$,

$$y^{\mathrm{T}} g(u) \geqq 0,$$
$$y \geqq 0,\ u \in X^0,\ y \in \mathbf{R}^m.$$

Let W_1 denote the set of all feasible solutions to Problem (MWD). We assume that X^0 is a nonempty η-locally starshaped set .

Theorem 5.1. (Weak Duality) *If $x \in X$, $(u, y) \in W_1$, f is slspb-preinvex and $y^{\mathrm{T}} g$ is slqb-preinvex on X^0, then $f(x) \geqq f(u)$.*

Corollary 5.2. *Let $\bar{x} \in X$ and $(\bar{u}, \bar{y}) \in W_1$ such that $f(\bar{x}) = f(\bar{u})$. If the hypotheses of Theorem 5.1 hold, then $\bar{x}$ and $(\bar{u}, \bar{y})$ are the optimal solutions to (NP) and (MWD) respectively.*

Theorem 5.3. (Direct Duality) *Let $\bar{x} \in X$ be an optimal solution to (NP), let f and g be η-semidifferentiable at $\bar{x}$, and assume that*

i_1) $(df)^+(\bar{x}, \eta(x, \bar{x}))$ *and* $y^{\mathrm{T}} (dg)^+(\bar{x}, \eta(x, \bar{x}))$ *are preinvex functions of x on X^0, an η-locally starshaped set at $\bar{x}$;*

i_2) *g_i ($i \in J$) are continuous at $\bar{x}$;*

i_3) *g satisfies the generalized Slater's constraint qualification at $\bar{x}$.*

Then there exists $\bar{y} \in \mathbf{R}^m$ such that $(\bar{x}, \bar{y}) \in W_1$ and $f(\bar{x}) = \Psi(\bar{x}, \bar{y})$.

Moreover, if the hypotheses of Theorem 5.1 are satisfied, then $\bar{x}$ and $(\bar{x}, \bar{y})$ are respectively optimal solutions to (NP) and (MWD).

Theorem 5.4. (Converse Duality) *Let $(\bar{u}, \bar{y}) \in W_1$. If f is slspb-preinvex and $\bar{y}^{\mathrm{T}} g$ is slqb-preinvex and there exists $\bar{x} \in X$ such that $f(\bar{x}) = f(\bar{u})$, then $\bar{x}$ solves the primal problem.*

Remark 5.5. If $\eta(x, u) = x - u$ we obtain the Mond-Weir dual considered by Suneja and Gupta [5].

Remark 5.6. Since the class of semilocally b-preinvex functions includes the class of semilocally b-vex functions, the class of semilocally preinvex functions and the class of b-preinvex functions, our results generalize those of Preda, Stancu-Minasian and Batatorescu [2], Suneja and Gupta [5] and Suneja et al. [6].

The proofs of all theorems will appear in [4].

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