PUBLISHING HOUSE
OF THE ROMANIAN ACADEMY

PROCEEDINGS OF THE ROMANIAN ACADEMY, Series A,
Volume 5, Number 1/2004, pp.000 - 000

CONTROL SYNTHESIS METHODOLOGY RELATED TO AN ADVANCED NONLINEAR ELECTROHYDRAULIC SERVO SYSTEM

Ioan URSU, Florica POPESCU, Felicia URSU

"Elie Carafoli" National Institute for Aerospace Research-Bucharest
e-mail: iursu@aero.incas.ro

The obtaining of control law for an advanced four-dimensional, nonlinear electrohydraulic servo mathematical model is the main contribution of the paper. This control law ensures the asymptotic stability of references tracking by constructing a Control Lyapunov Function (CLF) on the errors concerning the state variables and theirs desired values. An approach based on partitioning the state system into two subsystems – a first one internal stable, and a second one taken as framework of control synthesis – was developed; this is another main contribution of the present paper. The paper's interest derives also from illustrating the key idea of CLFs synthesis by using of celebrated Barbalat's Lemma. Numerical simulations were reported from viewpoint of assessing the servo time constant performance.

Key words: nonlinear control synthesis, backstepping, Control Lyapunov Function, Barbalat's Lemma, electrohydraulic servo.

1. INTRODUCTION

This paper develops control strategies for the electrohydraulic servo (EHS) control synthesis using the concept of *Control Lyapunov Function* (CLF), concept introduced by Artstein [1] and Sontag [2], and based on the *backstepping* approach [3] of building these functions. In the beginning, Lyapunov's stability theory deals with dynamic systems without inputs. For this reason, it has traditionally been applied only to closed loop control systems, that is, to systems for which the input has been eliminated through the substitution of a predetermined feedback control. Recently, some authors started using Lyapunov function candidates in feedback design itself by making the Lyapunov function derivative negative when choosing the control. Such idea has been made precise with the introduction of the CLF concept. The building of a CLF is not a unique matter, and the JurdjeviæQuinn's approach [4] and Sontag's approach [5] are only two such examples. Another approach is just backstepping methodology [3], a recursive type control design brought out in 1990's. Its initial limitation to a class of *strict feedback systems* [3] stimulated the development of various recursive procedures, applicable to more general nonlinear systems, as feedforward systems, adaptive systems and systems with structured uncertainty [3], [6] and [7].

In the following, the backstepping [3], [8], [11] is applied to position control synthesis for an EHS four-dimensional mathematical model, whose equation belongs to a general class of nonlinear systems ($\dot{f}$ means the derivative of the function $f(t)$ with respect to the time t)

$$\dot{x} = \boldsymbol{F}(\boldsymbol{x}) + \boldsymbol{G}(\boldsymbol{x})\boldsymbol{u} \tag{1}$$

where $\boldsymbol{x} \in \mathbb{R}^n$ is the state vector, $\boldsymbol{u} \in \mathbb{R}^m$ is the control vector and $\boldsymbol{F}$ and $\boldsymbol{G}$ are smooth vector fields of appropriate dimensions. The key idea in certifying the CLF building is using of the Barbalat's Lemma.

Recommended by Radu P.VOINEA, Member of the Romanian Academy

2. CONTROL SYNTHESIS FOR ELECTROHYDRAULIC SERVO FOUR-DIMENSIONAL MATHEMATICAL MODEL

In a previous work [8], a simplified three-dimensional mathematical model for EHS was considered:

$$\dot{x}_1 = x_2, \ \dot{x}_2 = \frac{1}{m}\left(-kx_1 - fx_2 + Sx_3\right), \ \dot{x}_3 = \frac{1}{k_c}\left(-Sx_2 - k_\ell x_3 + c\sqrt{\frac{p_a - x_3}{2}}k_v u\right), \ c := c_d w\sqrt{\frac{2}{\rho}}. \qquad (2)$$

Now, the following extended EHS mathematical model [10], [12], more realistic, is considered:

$$\begin{gathered}\dot{x}_1 = x_2, \ \dot{x}_2 = \frac{1}{m}\left[-kx_1 - fx_2 + S(x_3 - x_4)\right]\\ \dot{x}_3 = \frac{B}{V + Sx_1}\left[cx_5\sqrt{p_a - x_3} - Sx_2 - k_\ell(x_3 - x_4)\right], \ \dot{x}_4 = \frac{B}{V - Sx_1}\left[-cx_5\sqrt{x_4} + Sx_2 + k_\ell(x_3 - x_4)\right] \qquad (3)\\ \dot{x}_5 = \frac{-x_5 + k_v u}{\tau}.\end{gathered}$$

This time, the variables x_3 and x_4 are the pressures p_1, p_2 in the cylinder chambers, and x_5 stands for the valve position. These differential equations, governing the dynamics of the EHS, are those given in [10] and are reported having as a reference point the hydromechanical servomechanism SMHR (HM SMHR) included in the aileron control chain of Romanian military jet IAR 99. The state variables are denoted by: x_1 [cm] – EHS load displacement; x_2 [cm/s] – EHS load velocity; x_3 [daN/cm^2] – load pressure differential; u [V] – control variable. The valve dynamics is evaded in the mathematical model (2); a proportionality coefficient k_v between the control (input voltage to servovalve) and valve displacement was considered.

The proposed in [8] control synthesis by backstepping technique cannot be identically applied in the case of advanced nonlinear model (3), because this system is not a strict feedback system

$$\dot{x}_1 = x_2 + \varphi_1(x_1), \ \dot{x}_2 = x_3 + \varphi_2(x_1, x_2), \ \ldots, \ \dot{x}_{n-1} = x_n + \varphi_{n-1}(x_1, \ldots, x_{n-1}), \ \dot{x}_n = \beta(\boldsymbol{x})u + \varphi_n(\boldsymbol{x}).$$

Thus, a idea of partitioning the state system (3) into two subsystems – a first one internal stable, and a second one taken as framework of control synthesis – will be introduced. The internal stable system is the subsystem of the first two equations, with "perturbation term" $S(x_3 - x_4)/m$. The control will be constructed on the last three equations of the system (3) and then will be certified as ensuring the stability and tracking performance of the whole system. Summarising, the obtained result is given in the following

Proposition. *Consider the system* (3). *Let* $k_1 > 0$, $k_2 > 0$ *be tuning parameters. Under the* (*rather physical*) *assumptions* $0 < x_3 < p_a$, $0 < x_4 < p_a$, $|x_1| < V/S$, *the control* u *given by*

$$u = \frac{1}{k_v}\left[x_{5d} + \tau\left(\dot{x}_{5d} - k_2 g_2 e_p\right)\right] \qquad (4)$$

$$x_{5d} = -\frac{1}{g_2}\left(k_1 e_p - \dot{p}_d + g_1\right) \qquad (5)$$

$$g_1 = g_1(x_1, x_2) := \frac{-2BV\left(Sx_2 + k_\ell p\right)}{V^2 - S^2 x_1^2}, \ g_2 = g_2(x_1, x_2, x_3) := Bc\left(\frac{\sqrt{p_a - x_3}}{V + Sx_1} + \frac{\sqrt{x_4}}{V - Sx_1}\right) \qquad (6)$$

$$e_p := p - p_d, \quad p := x_3 - x_4, \ e_5 := x_5 - x_{5d} \qquad (7)$$

$$p_d(t) := \frac{k}{S}x_{1d}(t) := \frac{k}{S}x_{1s}\left(1 - e^{-t/t_{1r}}\right) \qquad (8)$$

when applied to system (3), *guarantees asymptotic stability for the position tracking error* $e_1 := x_1 - x_{1d}$; *more precisely,* $\lim_{t\to\infty} e_1(t) = 0$.

The notations x_{1s}, t_{1r} define the expression of the time response x_{1d} of a first order stable system to step input x_{1s}; t_{1r} is the time constant and $x_{1d}(0)$ is taken zero (see (21)).

Proof: Consider Lyapunov like function

$$V_1 = \frac{1}{2} e_p^2 . \tag{9}$$

Then, its derivative along the system (a.s.) (3), by using (5), (6) and (7), is

$$\dot{V}_1 = -k_1 e_p^2 + g_2 e_p e_5 . \tag{10}$$

Now, extend V_1 as

$$V_2 = V_1 + \frac{1}{2k_2} e_5^2 \tag{11}$$

and, taking into account (4), the derivative a.s. (3) is

$$\dot{V}_2 = -k_1 e_p^2 - \frac{1}{k_2 \tau} e_5^2 . \tag{12}$$

The equations for the errors e_p, e_5 can be written as

$$\dot{e}_p = -k_1 e_p + g_2 e_5 ,\ \dot{e}_5 = -\frac{e_5}{\tau} - k_2 g_2 e_p . \tag{13}$$

A tedious way to continue the proof is that of checking the asymptotic stability of the errors e_p, e_5 by using the second method of Lyapunov for systems with variable coefficients (see [13], Theorem 1). *An alternative and very efficient procedure will be shown in the following: that of using Barbalat's Lemma* [14]. The reasoning is as follows. Making use of the definitions (5), (6), (7) and (8) for e_p and e_5, it is easily to see that when $t \to 0$, we have $V_2(0) > 0$. Since $\dot{V}_2 \le 0$, it follows that $0 \le V_2(t) \le V_2(0)$, $(\forall)t > 0$, hence the positive function $V_2(t)$ is bounded and it follows that e_p and e_5 are bounded; so, $p = p(t)$ is also bounded on $\mathbb{R}_+ = [0, \infty)$. Now, taking the derivative of (12) yields

$$\ddot{V}_2 = 2\left(k_1^2 e_p^2 + \frac{1}{k_2 \tau^2} e_5^2 \right) - 2\left(k_1 - \frac{1}{\tau} \right) g_2 e_p e_5 .$$

Furthermore, $\ddot{V}_2$ is bounded, provided that g_2 remains bounded during the dynamical process; this condition holds, having in view the assumptions involving the variables x_1, x_3, x_4. So, $\dot{V}_2$ is uniformly continuous (as having a bounded derivative). Let us now consider Barbalat's Lemma [14]:

If the function $f(t)$ is differentiable and has a finite limit $\lim_{t\to\infty} f(t)$, and if $\dot{f}$ is uniformly continuous, then $\lim_{t\to\infty} \dot{f}(t) = 0$.

Thus, Barbalat's Lemma will be applied to show that the errors e_p and e_5 tend to zero as time tends to infinity. Indeed, applying Barbalat's Lemma, $\dot{V}_2 \to 0$. Consequently, e_p and e_5 tend to zero.

Now, let's look at the first two equations in (3), which can be rewritten as follows

$$\ddot{x}_1 + 2h\dot{x}_1 + r^2 x_1 = p_1,\ h := \frac{f}{2m},\ r := \sqrt{\frac{k}{m}},\ p_1 = \frac{S}{m} p \tag{14}$$

and p is now seen as a bounded function of t, $p := p(t) = x_3(t) - x_4(t)$. With initial conditions $x_1(0) = \dot{x}_1(0) = 0$, the solution of (14) is

$$x_1(t) = \frac{1}{\omega} e^{-ht} \int_0^t e^{hu} p_1(u) \sin \omega (t-u) \mathrm{d}u \tag{15}$$

where the condition $\omega^2 := r^2 - h^2 > 0$ is inherent to hydraulic servo systems owing to small viscous friction in cylinder. Introducing

$$p_{1d}(t) := \frac{S}{m} p_d(t) \tag{16}$$

let us also consider

$$\tilde{x}_{1d}(t) := \frac{1}{\omega} e^{-ht} \int_0^t e^{hu} p_{1d}(u) \sin \omega (t-u) du\,. \tag{17}$$

Since $e_p \to 0$ when $t \to \infty$, it is clear that $p_1(t) \to p_{1d}(t)$, as $t \to \infty$; this means: $(\forall)\, \varepsilon > 0$, $(\exists)\, \delta(\varepsilon)$ such that for $t > \delta(\varepsilon)$ we have $\left| p_1(t) - p_{1d}(t) \right| < \varepsilon$. Then, if $t > \delta(\varepsilon)$

$$\left| x_1(t) - \tilde{x}_{1d}(t) \right| \le \frac{1}{\omega} e^{-ht} \int_0^t e^{hu} \left| p_1(t) - p_{1d}(t) \right| \left| \sin \omega (t-u) \right| \mathrm{d}u \le \frac{\varepsilon}{\omega} \int_0^t e^{-h(t-u)} \mathrm{d}u = \frac{\varepsilon}{\omega} \frac{1 - e^{-ht}}{h}.$$

It results

$$x_1(t) \to \tilde{x}_{1d}(t) \text{ as } t \to \infty\,. \tag{18}$$

Then, successive calculations give:

$$\tilde{x}_{1d}(t) = \frac{1}{\omega} e^{-ht} \int_0^t e^{hu} \frac{S}{m} \frac{k}{S} x_{1s} \left(1 - e^{-u/t_r} \right) \sin \omega (t-u) \mathrm{d}u = \frac{x_{1s} k e^{-ht}}{\omega m} \int_0^t e^{hu} \left(1 - e^{-u/t_r} \right) \sin \omega (t-u) \mathrm{d}u$$

$$\int_0^t e^{hu} \left(1 - e^{-u/t_r} \right) \sin \omega (t-u) \mathrm{d}u =$$

$$\frac{(h - 1/t_r) \sin \omega t - \omega \left(e^{(h-1/t_r)t} - \cos \omega t \right)}{\omega^2 + (h - 1/t_r)^2} + \frac{-h \sin \omega t + \omega \left(e^{ht} - \cos \omega t \right)}{\omega^2 + h^2}$$

therefore

$$\tilde{x}_1(t) \to x_{1s} \text{ as } t \to \infty\,. \tag{19}$$

Thus, from (18) and (19), a standard proceeding gives

$$x_1(t) \to x_{1s} \text{ as } t \to \infty \tag{20}$$

and so ends the proof.

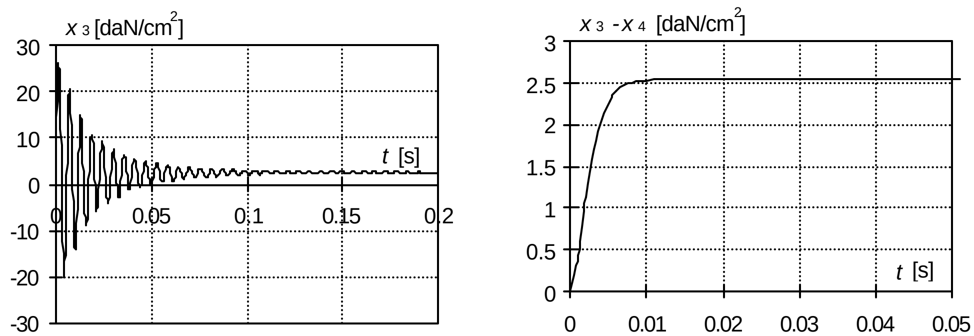
3. SIMULATION RESULTS

As it was pointed out, the goal of the backstepping synthesised control is to have the EHS tracking the specified x_{1d} position reference. Such references can be expressed as a response of a first order system to step inputs x_{is}

$$x_{1d}(t) = x_{1s}\left(1 - e^{-t/t_{1r}}\right) \tag{21}$$

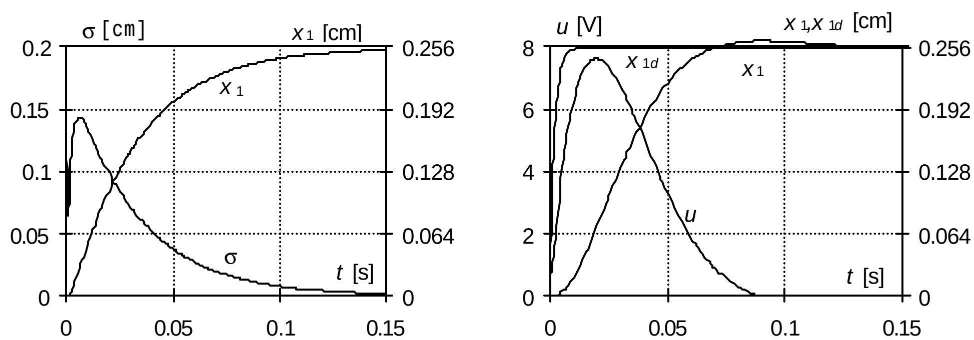
where x_{1s} stands for stationary value of the state x_1 and t_{1r} stands for associated desired time constant.

The nominal values of the parameters appearing in equations (2), (3) were as follows: $m = 0.033$ daNs^2/cm – equivalent inertial load of primary control surface reduced at the EHS's rod; $S = 10$ cm^2 – EHS's piston area; $f = 1$ daNs/cm – equivalent viscous friction force coefficient; $k = 100$ daN/cm – equivalent aerodynamic elastic force coefficient; $w = 0.05$ cm – valve-port width; $p_a = 210$ daN/cm^2 – supply pressure; $c_d = 0.63$ – volumetric flow coefficient of the valve port; $k_\ell = 5/210$ cm^5/(daN×s) – internal leakage cylinder's coefficient; $\rho = 85/(981\times10^5)$ daNs^2/cm^5 – volumetric density of oil; $k_c = 30/12\,000$ cm^5/daN $\left(:= V/(2B)\right)$– coefficient involving the bulk modulus B of the oil used and the EHS's cylinder semivolume V; $k_v = 0.0085/(0.05\times10)$ cm/V – valve displacement/voltage coefficient.



a) b)

Figure 1 – Comparison of the load pressure evolution:
a) the three-dimensional model (2) (with load pressure state variable);
b) EHS system (3), (4)-(7), with two pressure state variables.



a) b)

Figure 2 – Comparison of the position step input tracking systems with two pressure state variables:
a) SMHR case (22): $\tau_s \cong 0.037$ s;
b) EHS case (3), (4)-(7), with backstepping position control: $\tau_s \cong 0.037$ s.

As reference points of the numerical simulations were taken both the experimental data [9] and a four-dimensional mathematical model of the HM SMHR [12]:

$$\dot{x}_1 = x_2, \dot{x}_2 = \frac{1}{m}\left[-kx_1 - fx_2 + S(x_3 - x_4)\right]$$
$$\dot{x}_3 = \frac{B}{V + Sx_1}\left[c\sigma\sqrt{p_a - x_3} - Sx_2\right], \dot{x}_4 = \frac{B}{V - Sx_1}\left[-c\sigma\sqrt{x_4} + Sx_2\right], \sigma := \lambda(r - x_1). \tag{22}$$

where $\lambda = 2/3$ is the coefficient of the rigid feedback kinematics, r is the reference input at servo rigid feedback kinematics linkage point [cm] and σ is the valve error signal. Zero equilibrium point for the systems (22) corresponds to closed valve – $\sigma = 0$ – in the case of zero reference input, $r = 0$.

Numerical simulations have been performed to investigate the performance of the proposed nonlinear control (4)-(7). Zero initial conditions – corresponding to the zero equilibrium point – were chosen for a fourth order Runge-Kutta integration, with integration step 0.001 s, of the systems (2), with nonlinear control given in [8], and (3), with nonlinear control (4)-(7); in the last case, initial conditions on the variables x_3 and x_4 were chosen 1 daN/cm^2.

Consider the systems (3) at equilibrium (0, 0, $x_3(0)$, $x_4(0)$, 0). The calculation of starting control $u(0)$, derived from (4)-(8), involves the system parameters, the turning parameters and certain derivatives, as follows:

$$u(0) = \frac{1}{k_v}\left[x_{5d}(0) + \tau\left(\dot{x}_{5d}(0) - k_2 g_2(0) e_p(0)\right)\right]$$

$$x_{5d}(0) = -\frac{k_1 e_p(0) - \dot{p}_d(0) + g_1(0)}{g_2(0)},\ e_p(0) = x_3(0) - x_4(0),\ \dot{p}_d(0) := \frac{k x_{1s}}{S t_{1r}}$$

$$g_1(0) = \frac{-2BVk_l e_p(0)}{V^2},\ g_2(0) = \frac{Bc}{V}\left(\sqrt{p_a - x_3(0)} + \sqrt{x_4(0)}\right)$$

$$\dot{x}_5(0) = -\frac{\left(k_1 \dot{e}_p(0) - \ddot{p}_d(0) + \dot{g}_1(0)\right) g_2(0) - \left(k_1 e_p(0) - \dot{p}_d(0) + g_1(0)\right)\dot{g}_2(0)}{g_2^2(0)}$$

$$\dot{e}_p(0) = \dot{x}_3(0) - \dot{x}_4(0) - \dot{p}_d(0),\ \dot{x}_3(0) = -\frac{Bk_l e_p(0)}{V},\ \dot{x}_4(0) = \frac{Bk_l e_p(0)}{V}$$

$$\ddot{p}_d(0) = -\frac{k x_{1s}}{S t_{1r}^2},\ \dot{g}_1(0) = -\frac{2B}{V}\left(S\dot{x}_2(0) + k_l \dot{p}(0)\right),\ \dot{x}_2(0) = \frac{S}{m} e_p(0),\ \dot{p}(0) = \dot{x}_3(0) - \dot{x}_4(0)$$

$$\dot{g}_2(0) = \frac{Bc}{V}\left(\frac{-\dot{x}_3(0)}{2\sqrt{p_a - x_3(0)}} + \frac{\dot{x}_4(0)}{2\sqrt{x_4(0)}}\right).$$

Comparing the simulation results, an interesting feature is that the mathematical model (3) promotes a somewhat “stiffer” object than the three-dimensional mathematical model (2). Indeed, the system with two pressures as state variables are dynamically slower than the system having the load pressure as state variable. The time histories of the load pressures, depicted in Fig. 1, offer an *a posteriori* explanation of this dynamic behaviour: the faster system is by all means that actuated by greater values of the external driving force. This force is derived from load pressure Sx_3, in the case of the first two equations in (2), and, respectively, $S(x_3 - x_4)$, in the case of the first two equations in (3).

Figure 2 shows fit results obtained choosing r expressed in the form (21), with $x_{1s} = 0.255$ cm and $t_{1r} = 0.002$ s. Thus, HM SMHR mathematical model produces the *servo time constant* $\tau_s = 0.037$ s (Fig. 2a). With the fit tuning parameters $k_1 = 1000\ \mathrm{s}^{-1}$ and $k_2 = 10^{-5}\ \mathrm{daN}^{-2} \times \mathrm{cm}^6$, the corresponding (3), (4)-(7) system denotes the same servo time constant $\tau_s = 0.037$ s. Worthy noting, these values are in a close neighbourhood of the measured SMHR time constant $\tau_s = 0.02$ s, given in [9] for step reference amplitude $r = 0.12$ cm (roughly speaking, τ_s decreases with decreasing r or x_{1s}).

4. CONCLUSIONS

In this paper the problem of the control law synthesis which provides asymptotic stability and tracking performance for an EHS four-dimensional mathematical model [10], [12] has been addressed; this model is more realistic than the three-dimensional one considered as usual in the literature [8]. The full state information was considered available. In practice, a state estimator will be needed and this problem will be considered in a further research.

The main contributions of the paper are: 1) obtaining of control law for an advanced four-dimensional, nonlinear EHS mathematical model; 2) illustrating how the main theory can be brought or adapted to control design practice as defined by a given mathematical model and showing that the backstepping controllers are able to work with a complex plant such as EHS; 3) CLFs synthesis by using of Barbalat's Lemma; 4) developing the idea of partitioning the state system into two subsystems: a first one internal stable, and a second one taken as framework of control synthesis.

The simulation studies attest good tracking performance of the EHS in the presence of step signals. Based on sampled signals paradigm, a good tracking performance of arbitrary references can be also stipulated, having in view the decomposition of an arbitrary signal into a succession of step signals. Furthermore, a close correspondence between the theoretical predictions and the experimental result [9] has been found.

The backstepping technique has not yet been applied to control a position tracking EHS, used in aviation, to the best of the author's knowledge.

REFERENCES

[1] ARTSTEIN, Z., *Stabilisation with relaxed controls*, Nonlinear Analysis, 7, 1163–1173, 1983.

[2] SONTAG, E. D., *A Lyapunov-like characterisation of asymptotic controllability,* SIAM Journal of Control and Optimization, **21**, 462–471, 1983.

[3] KRSTIÆ, M., KANELLAKOPOULOS, I., KOKOTOVIÆ, P. V., *Nonlinear and adaptive control design,* Wiley and Sons, New York, 1995.

[4] JURDJEVIÆ, V., QUINN, J. P., *Controllability and stability*, Journal of Differential Equations, **28**, 381 389, 1978.

[5] SONTAG, E. D., *An universal construction of Artstein's theorem on nonlinear stabilisation*, Systems and Control Letters, **13**, 2, 117–123, 1989.

[6] SEPULCHRE, R., JANKOVIÆ, M., KOKOTOVIÆ, P. V., *Constructive nonlinear control,* Springer-Verlag, London, 1997.

[7] FREEMAN, R. A., KOKOTOVIÆ, P. V., *Robust nonlinear control design. State-space and Lyapunov techniques*, Birkhäuser, Boston, 1996.

[8] URSU, I., POPESCU, F., *Nonlinear control synthesis for position and force electrohydraulic servos*, Proceedings of the Romanian Academy, Series A, 2, 2003.

[9] URSU, I., *Theoretical and experimental data concerning qualification testing and flight clearance for aircraft servomechanism SMHR*, National Institute for Scientific and Technical Creation – INCREST Report, N-5303, Bucharest, 1984.

[10] URSU, I., URSU, F., *Active and semiactive control* (in Romanian), Romanian Academy Publishing House, 2002.

[11] URSU, I., URSU, F., POPESCU, F., *Backstepping design for controlling electrohydraulic servos* (submitted).

[12] HALANAY, A., SAFTA, C. A., URSU, I., URSU, F., *Stability of equilibria in a four-dimensional nonlinear model of a hydraulic servomechanism* (submitted).

[13] KALMAN, R. E, BERTRAM, J. E., *Continuous system analysis and design via the second method of Lyapunov. I. Continuous-time systems*, Transactions of the ASME, Journal of Basic Engineers, **82**, Series D, 2, 371–393, June 1960.

[14] POPOV, V. M., *Hyperstability of automatic control systems*, Springer-Verlag, Berlin, 1973.

Received November 5, 2003