PUBLISHING HOUSE
OF THE ROMANIAN ACADEMY

PROCEEDINGS OF THE ROMANIAN ACADEMY, Series A,
Volume 1, Number 2/2000,pp.000-000.

SUPERSONIC FLOW PAST A GRID OF THIN PROFILES

Adrian CARABINEANU

University of Bucharest, Faculty of Mathematics, Str. Academiei 14, Bucharest, Romania
E-mail: acara@math.math.unibuc.ro

In the framework of the small perturbation theory, we study the supersonic flow past a grid of small profiles. We calculate the lift and moment coefficients for an arbitrary profile from the grid. As a particular case we derive the aerodynamic coefficients for a profile in a wind tunnel.

Key words: Supersonic flow; small perturbations; shock waves; aerodynamic coefficients.

1. INTRODUCTION

In the present paper we use the small perturbations theory to study the supersonic flow past a grid of thin profiles. Our aim is to calculate the aerodynamic coefficients of a certain profile belonging to the grid. Unlike the case of a single profile, one has to take into account the reflections of the bow shock waves from the other profiles (in the particular geometry we have in view, the shock waves emerging from the leading edge of the profile are reflected only by the profile itself and by the profiles situated under and above the profile taken into consideration).

A similar phenomenon of reflection of the bow shock waves occurs in the case of a supersonic flow past a profile in a wind tunnel [1], [2]. It is well known that employing the method of images, one may investigate the flow past a profile in a wind tunnel by studying the flow past a particular grid of profiles. Our results generalize the results obtained in the case of the flow in a wind tunnel.

2. STATEMENT OF THE PROBLEM

We consider that a uniform supersonic flow having at infinity upstream velocity $U\mathbf{i}$, pressure p_∞ and density ρ_∞ is perturbed by the presence of a grid of thin profiles. Taking the length of the profiles L_0, the unperturbed velocity $U\mathbf{i}$, and the unperturbed density ρ_∞, respectively, as characteristic length, velocity and density, we have the following relations between the dimensional quantities x_1, y_1, U_1, V_1, p_1 and the dimensionless ones x, y, u, v, p:

$$(x_1, y_1) = L_0(x, y), U_1 = U(1+u), V_1 = Uv, p_1 = p_\infty + \rho_\infty U^2 p. \tag{2.1}$$

Obviously, (u, v) is the dimensionless perturbation velocity and p is the dimensionless perturbation pressure. From (2.1) and the Euler and continuity equations, neglecting the products of the perturbation quantities, we obtain the linearized dimensionless system

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + M^2 \frac{\partial p}{\partial x} = 0, \tag{2.2}$$

$$\frac{\partial u}{\partial x}+\frac{\partial p}{\partial x}=0, \tag{2.3}$$

$$\frac{\partial v}{\partial x}+\frac{\partial p}{\partial y}=0, \tag{2.4}$$

$$\lim_{x\to\infty}(u,v,p)=0. \tag{2.5}$$

where $M=\gamma\frac{p_\infty}{\rho_\infty}$ is the Mach number (γ is the isentropic constant).

System (2.2) – (2.4) is equivalent to the system

$$\mathbf{A}\begin{pmatrix}u\\v\\p\end{pmatrix}_x+\mathbf{B}\begin{pmatrix}u\\v\\p\end{pmatrix}_y=\begin{pmatrix}0\\0\\0\end{pmatrix},$$

with

$$\mathbf{A}=\begin{pmatrix}1&0&k^2+1\\1&0&1\\0&1&0\end{pmatrix},\mathbf{B}=\begin{pmatrix}0&1&0\\0&0&0\\0&0&1\end{pmatrix},k=\sqrt{M^2-1}.$$

The eigenvalues of the matrix

$$\mathbf{A}^{-1}\mathbf{B}=\begin{pmatrix}0&-\frac{1}{k^2}&0\\0&0&1\\0&\frac{1}{k^2}&0\end{pmatrix}$$

are $\lambda_I=0,\lambda_{II}=\frac{1}{k},\lambda_{III}=-\frac{1}{k}.$

Hence the characteristics of system (2.2) – (2.4) are

$$\frac{\mathrm{d}y}{\mathrm{d}x}=0\Rightarrow y=C_0,(I)$$
$$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{k}\Rightarrow x-ky=C_1,(II)$$
$$\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{1}{k}\Rightarrow x+ky=C_2.(III)$$

The Riemann invariants (i.e. the functions which are constant along the characteristics),

$$f_I(x,y)=f_I(y),f_{II}(x,y)=f_{II}(x-ky),f_{III}(x,y)=f_{III}(x+ky),$$

are solutions of the systems of the first order partial differential equations

$$\left(\left(\mathbf{A}^{-1}\mathbf{B}\right)^{T}-\lambda_{i}\mathbf{I}\right)\begin{pmatrix}\dfrac{\partial f_{i}}{\partial u}\\ \dfrac{\partial f_{i}}{\partial v}\\ \dfrac{\partial f_{i}}{\partial p}\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix}, i=I,II,III.$$

For $\lambda_I = 0$, we have

$$\begin{pmatrix}0 & 0 & 0\\ -\dfrac{1}{k^{2}} & 0 & \dfrac{1}{k^{2}}\\ 0 & 1 & 0\end{pmatrix}\begin{pmatrix}\dfrac{\partial f_{I}}{\partial u}\\ \dfrac{\partial f_{I}}{\partial v}\\ \dfrac{\partial f_{I}}{\partial p}\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix},$$

whence

$$\frac{\partial f_I}{\partial v}=0, \frac{\partial f_I}{\partial u}-\frac{\partial f_I}{\partial p}=0. \tag{2.6}$$

Since a solution of system (2.6) is $f_I = u + p$, we have

$$u(x,y)+p(x,y)=f_I(y). \tag{2.7}$$

For $\lambda_{II}=\dfrac{1}{k}$ we have

$$\begin{pmatrix}-\dfrac{1}{k} & 0 & 0\\ -\dfrac{1}{k^{2}} & -\dfrac{1}{k} & \dfrac{1}{k^{2}}\\ 0 & 1 & -\dfrac{1}{k}\end{pmatrix}\begin{pmatrix}\dfrac{\partial f_{II}}{\partial u}\\ \dfrac{\partial f_{II}}{\partial v}\\ \dfrac{\partial f_{II}}{\partial p}\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix},$$

whence

$$\frac{\partial f_{II}}{\partial u}=0, \frac{\partial f_{II}}{\partial p}-k\frac{\partial f_{II}}{\partial p}=0. \tag{2.8}$$

Since a solution of system (2.8) is $f_{II}=\dfrac{1}{2}\left(v+kp\right)$, we have

$$v(x,y)+kp(x,y)=2f_{II}(x-ky) \tag{2.9}$$

For $\lambda_{III}=-\dfrac{1}{k}$ we have

$$\begin{pmatrix} \frac{1}{k} & 0 & 0 \\ -\frac{1}{k^2} & \frac{1}{k} & \frac{1}{k^2} \\ 0 & 1 & \frac{1}{k} \end{pmatrix} \begin{pmatrix} \frac{\partial f_{III}}{\partial u} \\ \frac{\partial f_{III}}{\partial v} \\ \frac{\partial f_{III}}{\partial p} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix},$$

whence

$$\frac{\partial f_{III}}{\partial u} = 0, \frac{\partial f_{III}}{\partial p} + k\frac{\partial f_{III}}{\partial p} = 0. \tag{2.10}$$

Since a solution of system (2.10) is $f_{III} = \frac{1}{2}(v - kp)$, we have

$$v(x,y) - kp(x,y) = 2f_{III}(x+ky). \tag{2.11}$$

It follows from (2.9) and (2.11) that

$$v(x,y) = f_{II}(x-ky) + f_{III}(x+ky), \tag{2.12}$$

$$p(x,y) = \frac{1}{k}\left[f_{II}(x-ky) - f_{III}(x+ky)\right]. \tag{2.13}$$

Since the boundary of the flow domain is not smooth, the discontinuities of the pressure and velocity (appearing from the slipping condition in the points, including the edges, where the direction of the tangent at the boundary is not continuous) will propagate along the characteristics which will be shock waves. These shock waves can also be reflected by the profiles. The jump conditions across the shock waves are

$$[u]n_x + [v]n_y + \left(k^2+1\right)[p]n_x = 0, \tag{2.14}$$

$$\left([u]+[p]\right)n_x = 0, \tag{2.15}$$

$$[v]n_x + [p]n_y = 0. \tag{2.16}$$

We obtain these jump conditions from the Rankine – Hugoniot jump conditions

$$[\mathbf{u}]n_x + [\mathbf{f}(\mathbf{u})]n_y = 0,$$

which we impose on the weak solutions of the system

$$\frac{\partial \mathbf{u}}{\partial x} + \frac{\partial}{\partial y}\mathbf{f}(\mathbf{u}) = \mathbf{0} \tag{2.17}$$

across the lines of discontinuity ([3], page 16). In equations (2.14) – (2.16), $[u],[v]$ and $[p]$ represent the jump of u, v and p across the shock wave and $\mathbf{n} = (n_x, n_y)$ is the unit normal at the shock waves. We notice that the jump conditions (2.14) – (2.16) may be also obtained by linearizing the Rankine – Hugoniot jump conditions for the nonlinear Euler equations ([1], page 289).

In the sequel we shall demonstrate that the Riemann invariants corresponding to a characteristic from a certain family do not change when crossing a characteristic (from another family) which is a shock wave. For the characteristics $y = C_0$ from family *(I),* the unit normal is

$$\mathbf{n} = (n_x, n_y) = (1,0). \tag{2.18}$$

For the characteristics $x - ky = C_1$ from family *(II)*, the unit normal is

$$\mathbf{n} = (n_x, n_y) = \left(\frac{1}{\sqrt{k^2+1}}, \frac{-k}{\sqrt{k^2+1}} \right). \tag{2.19}$$

For the characteristics $x + ky = C_2$ from family *(III)* the unit normal is

$$\mathbf{n} = (n_x, n_y) = \left(\frac{1}{\sqrt{k^2+1}}, \frac{k}{\sqrt{k^2+1}} \right). \tag{2.20}$$

It follows from (2.7) and (2.15) that $[f_I]n_x = 0$ across the characteristics from families *(II)* and *(III)*. It follows then from (2.19) and (2.20) that

$$[f_I] = 0 \tag{2.21}$$

across the characteristics from these families. Let us prove that $[f_{II}] = 0$ on the characteristics from family *(III)*. It follows from (2.7), (2.14) and (2.21) that

$$[v]n_y + k^2[p]n_x = 0 \tag{2.22}$$

From (2.20) and (2.22) we obtain

$$[v] + k[p] = 0, \tag{2.23}$$

whence, taking into account (2.9),

$$[f_{II}] = 0 \text{ across } x + ky - C_2 = 0. \tag{2.24}$$

Similarly we prove that

$$[f_{III}] = 0 \text{ across } x - ky - C_1 = 0. \tag{2.25}$$

From (2.14), (2.15), (2.16) and (2.18) we deduce that $[v] = [p] = 0$ across the characteristics from family *(I)*, whence

$$[f_{II}] = [f_{III}] = 0 \text{ across } y - C_0 = 0.$$

Hence along the characteristics, the corresponding Riemann invariants may change only at the points where the characteristics intersect the profiles from the grid.

3. CALCULUS OF THE PRESSURE FIELD

The equations of the upper, respectively lower surface of an arbitrary airfoil belonging to the grid are

$$y = a_n + h_n^+(x), x \in [0,1], \tag{3.1}$$

$$y = a_n + h_n^-(x), x \in [0,1], \tag{3.2}$$

respectivey, where $\left|h_n^{\pm}(x)\right| << 1, \left|h_n^{\pm}{}'(x)\right| = \left|\dfrac{dh_n^{\pm}}{dx}\right| << 1$.

We suppose that $a_n > a_{n+1}$. In the sequel we are going to prove that in the domain $a_{n+1} \le y \le a_n$ we have

$$f_{III}(x+ky) = \sum_{j=0}^{\left[\frac{m}{2}\right]} h_n^{-}{}'\left(\xi - 2jk\left(a_n - a_{n+1}\right)\right) - \sum_{j=1}^{\left[\frac{m+1}{2}\right]} h_{n+1}^{+}{}'\left(\xi - (2j-1)k\left(a_n - a_{n+1}\right)\right) \tag{3.3}$$

for $mk(a_n - a_{n+1}) < \xi < (m+1)k(a_n - a_{n+1})$, $x + ky = \xi + ka_n$, $\xi \in (0,1)$.

$$f_{II}(x-ky) = \sum_{j=1}^{\left[\frac{m+1}{2}\right]} h_{n+1}^{+}{}'\left(\xi - (2j-1)k\left(a_n - a_{n+1}\right)\right) - \sum_{j=1}^{\left[\frac{m}{2}\right]} h_n^{-}{}'\left(\xi - 2jk\left(a_n - a_{n+1}\right)\right), \tag{3.4}$$

for $mk(a_n - a_{n+1}) < \xi < (m+1)k(a_n - a_{n+1})$, $x - ky = \xi - ka_n, \xi \in (0,1)$.

$$f_{III}(x+ky) = \sum_{j=1}^{\left[\frac{m+1}{2}\right]} h_n^{-}{}'\left(\xi - (2j-1)k\left(a_n - a_{n+1}\right)\right) - \sum_{j=1}^{\left[\frac{m}{2}\right]} h_{n+1}^{+}{}'\left(\xi - 2jk\left(a_n - a_{n+1}\right)\right), \tag{3.5}$$

for $mk(a_n - a_{n+1}) < \xi < (m+1)k(a_n - a_{n+1})$, $x + ky = \xi + ka_{n+1}, \xi \in (0,1)$.

$$f_{II}(x-ky) = \sum_{j=0}^{\left[\frac{m}{2}\right]} h_{n+1}^{+}{}'\left(\xi - 2jk\left(a_n - a_{n+1}\right)\right) - \sum_{j=1}^{\left[\frac{m+1}{2}\right]} h_n^{-}{}'\left(\xi - (2j-1)k\left(a_n - a_{n+1}\right)\right), \tag{3.6}$$

for $mk(a_n - a_{n+1}) < \xi < (m+1)k(a_n - a_{n+1})$, $x - ky = \xi - ka_{n+1}, \xi \in (0,1)$.

We shall first prove (3.3) – (3.6) for $m = 0$. For $0 < \xi < k\left(a_n - a_{n+1}\right)$ the characteristics $x - ky = \xi - ka_n$ and $x + ky = \xi + ka_{n+1}$ may be prolonged to infinity upstream without intersecting the grid of profiles. Hence from condition (2.5), follows

$$f_{II}\left(x-ky\right) = \lim_{x\to-\infty} f_{II}\left(x-ky\right) \quad \text{for} \quad x - ky = \xi - ka_n, 0 < \xi < k\left(a_n - a_{n+1}\right), \tag{3.7}$$

$$f_{III}\left(x+ky\right) = \lim_{x\to-\infty} f_{III}\left(x+ky\right) \quad \text{for} \quad x + ky = \xi + ka_n, 0 < \xi < k\left(a_n - a_{n+1}\right). \tag{3.8}$$

From the linearized slipping condition

$$v\left(\xi, a_n \pm 0\right) = h_n^{\pm}{}'\left(\xi\right) \tag{3.9}$$

and from relations (2.12), (3.7) and (3.18) we obtain

$$f_{III}\left(x+ky\right) = h_n^{-}{}'(\xi) \quad \text{for} \quad x + ky = \xi + ka_n, 0 < \xi < k\left(a_n - a_{n+1}\right), \tag{3.10}$$

$$f_{II}\left(x-ky\right) = h_{n+1}^{+}{}'(\xi) \quad \text{for} \quad x - ky = \xi - ka_n, 0 < \xi < k\left(a_n - a_{n+1}\right). \tag{3.11}$$

Hence relations (3.3) – (3.6) are checked for m=0. Next we shall prove that if relations (3.3) – (3.6) are valid for $mk(a_n - a_{n+1}) < \xi < (m+1)k(a_n - a_{n+1})$, $\xi \in (0,1)$, then they also remain valid for $(m+1)k(a_n - a_{n+1}) < \xi < (m+2)k(a_n - a_{n+1}), \xi \in (0,1)$. We have $mk(a_n - a_{n+1}) < \xi - k(a_n - a_{n+1}) < (m+1)k(a_n - a_{n+1})$ and taking into account the induction hypothesis and relations (3.6), (3.3) we obtain

$$f_{II}(\xi-ka_n)=f_{II}(\xi-k\left(a_n-a_{n+1}\right)-ka_{n+1})=$$
$$=\sum_{j=0}^{\left[\frac{m}{2}\right]}h_{n+1}^{+}{}'\left(\xi-(2j+1)k(a_n-a_{n+1})\right)-\sum_{j=1}^{\left[\frac{m+1}{2}\right]}h_{n+1}^{-}{}'\left(\xi-2jk(a_n-a_{n+1})\right)=$$
$$=\sum_{j=1}^{\left[\frac{m+2}{2}\right]}h_{n+1}^{+}{}'\left(\xi-(2j-1)k(a_n-a_{n+1})\right)-\sum_{j=1}^{\left[\frac{m+1}{2}\right]}h_{n}^{-}{}'\left(\xi-2jk(a_n-a_{n+1})\right), \tag{3.12}$$

$$f_{III}\left(\xi+ka_{n+1}\right)=f_{III}\left(\xi-k\left(a_n-a_{n+1}\right)+ka_n\right)=$$
$$=\sum_{j=0}^{\left[\frac{m}{2}\right]}h_{n+1}^{-}{}'\left(\xi-(2j+1)k(a_n-a_{n+1})\right)-\sum_{j=1}^{\left[\frac{m+1}{2}\right]}h_{n}^{+}{}'\left(\xi-2jk(a_n-a_{n+1})\right)=$$
$$=\sum_{j=0}^{\left[\frac{m+2}{2}\right]}h_{n+1}^{-}{}'\left(\xi-(2j-1)k(a_n-a_{n+1})\right)-\sum_{j=1}^{\left[\frac{m+1}{2}\right]}h_{n}^{+}{}'\left(\xi-2jk(a_n-a_{n+1})\right) \tag{3.13}$$

From the slipping condition (3.9) and from (2.12) we easily obtain

$$v\left(\xi,a_n-0\right)=h_n^{-}{}'(\xi)=f_{III}(\xi+ka_n)+f_{II}(\xi-ka_n)=$$
$$=f_{III}(\xi+ka_n)+f_{II}(\xi-k\left(a_n-a_{n+1}\right)-ka_{n+1}). \tag{3.14}$$

$$v\left(\xi,a_{n+1}+0\right)=h_{n+1}^{+}{}'(\xi)=f_{III}(\xi+ka_{n+1})+f_{II}(\xi-ka_{n+1})=$$
$$=f_{II}(\xi-ka_{n+1})+f_{III}\left(\xi-k\left(a_n-a_{n+1}\right)+ka_n\right).. \tag{3.15}$$

From (3.12), (3.13), (3.14) and (3.15) we get

$$f_{III}(\xi+ka_n)=h_n^{-}{}'(\xi)-f_{II}(\xi-ka_n)=$$
$$=\sum_{j=0}^{\left[\frac{m+1}{2}\right]}h_n^{-}{}'(\xi-2jk(a_n-a_{n+1}))-\sum_{j=1}^{\left[\frac{m+2}{2}\right]}h_{n+1}^{+}{}'(\xi-(2j-1)k(a_n-a_{n+1})). \tag{3.16}$$

$$f_{II}(\xi-ka_{n+1})=h_{n+1}^{+}{}'(\xi)-f_{III}(\xi+ka_{n+1})=$$
$$=\sum_{j=0}^{\left[\frac{m+1}{2}\right]}h_n^{+}{}'(\xi-2jk(a_n-a_{n+1}))-\sum_{j=1}^{\left[\frac{m+2}{2}\right]}h_{n+1}^{-}{}'(\xi-(2j-1)k(a_n-a_{n+1})). \tag{3.17}$$

So, we proved by induction the validity of relations (3.3) – (3.6) for every *m*, provided $0<\xi<1$. From (2.13), (3.3) and (3.4) we deduce that on the lower surface of the *n*th profile we have

$$p_{-}\left(\xi,a_n\right)=p\left(\xi,a_n-0\right)=$$
$$=\frac{2}{k}\sum_{j=1}^{\left[\frac{m+1}{2}\right]}h_{n+1}^{+}{}'(\xi-(2j-1)k(a_n-a_{n+1}))-\frac{2}{k}\sum_{j=1}^{\left[\frac{m}{2}\right]}h_n^{-}{}'(\xi-2jk(a_n-a_{n+1}))-\frac{1}{k}h_n^{-}{}'\left(\xi\right), \tag{3.18}$$

for $mk(a_n - a_{n+1}) < \xi < (m+1)k(a_n - a_{n+1})$, $\xi \in (0,1)$.

Putting in relations (3.5) and (3.6) $n-1$ instead of n, we deduce that on the upper surface of the nth profile we have

$$p_+(\xi, a_n) = p(\xi, a_n + 0) =$$

$$= \frac{1}{k} h_n^+{}'(\xi) - \frac{2}{k} \sum_{j=1}^{\left[\frac{m+1}{2}\right]} h_{n-1}^-{}'(\xi - (2j-1)k(a_{n-1} - a_n)) + \frac{2}{k} \sum_{j=1}^{\left[\frac{m}{2}\right]} h_n^+{}'(\xi - 2jk(a_{n-1} - a_n)). \tag{3.19}$$

4. CALCULUS OF THE AERODYNAMIC COEFFICIENTS

The aerodynamic coefficients of the n-th airfoil are the lift and the moment coefficients

$$C_L^{(n)} = \int_0^1 \left[p_-(\xi, a_n) - p_+(\xi, a_n) \right] \mathrm{d}\xi \tag{4.1}$$

$$C_M^{(n)} = \int_0^1 \xi \left[p_-(\xi, a_n) - p_+(\xi, a_n) \right] \mathrm{d}\xi \tag{4.2}$$

Taking for example $1 \geq k(a_n - a_{n+1}) \geq \frac{1}{2}$, $1 \geq k(a_{n-1} - a_n) \geq \frac{1}{2}$ we have

$$C_L^{(n)} = -\frac{1}{k} \int_0^1 \left[h_n^+{}'(\xi) + h_n^-{}'(\xi) \right] \mathrm{d}\xi + \frac{2}{k} \int_0^{1-k(a_n - a_{n+1})} h_{n+1}^+{}'(\xi) \mathrm{d}\xi + \frac{2}{k} \int_0^{1-k(a_{n-1} - a_n)} h_{n+1}^+{}'(\xi) \mathrm{d}\xi \tag{4.3}$$

$$C_M^{(n)} = -\frac{1}{k} \int_0^1 \xi \left[h_n^+{}'(\xi) + h_n^-{}'(\xi) \right] \mathrm{d}\xi + \frac{2}{k} \int_0^{1-k(a_n - a_{n+1})} (\xi + k(a_n - a_{n+1})) h_{n+1}^+{}'(\xi) \mathrm{d}\xi +$$
$$+ \frac{2}{k} \int_0^{1-k(a_{n-1} - a_n)} (\xi + k(a_{n-1} - a_n)) h_{n-1}^-{}'(\xi) \mathrm{d}\xi \tag{4.4}$$

In the case of the grid of flat plates we have

$$h_n^+(\xi) = h_n^-(\xi) = -\varepsilon\xi, \xi \in [0,1], \tag{4.5}$$

and the corresponding coefficients are

$$C_L^{(n)} = \frac{2\varepsilon}{k}(k(a_{n-1} - a_{n+1}) - 1), \tag{4.6}$$

$$C_M^{(n)} = \frac{\varepsilon}{k}\left(k^2(a_n - a_{n+1})^2 + k^2(a_{n-1} - a_n) - 1\right) \tag{4.7}$$

5. WIND TUNNEL AND GROUND EFFECTS FOR A SINGLE PROFILE

We consider the profile whose equations are

$$y = h_+(x), x \in [0,1] \tag{5.1}$$

for the upper surface and

$$y = h_{-}(x), x \in [0,1]. \tag{5.2}$$

for the lower surface.

The profile is situated inside a wind tunnel which has two straight walls

$$y = -\frac{a}{2}, y = \frac{b}{2}, x \in \mathbf{R}, a > 0, b > 0 \tag{5.3}$$

It is well known that using the method of images one can obtain the velocity and pressure fields in the domain $-\frac{a}{2} < y < \frac{b}{2}$ if one investigates the flow past a grid of profiles adequately chosen instead of the flow past the profile in the wind tunnel. The ”-*1*” profile will be the symmetric of the “*0*” profile with respect to the straight line $y = \frac{b}{2}$ while the “*1*” profile will be the symmetric of the “*0*” profile with respect to the straight line $y = -\frac{a}{2}$. Hence we have

$$a_{-1} = b, h_{-1}^{+}(x) = -h_{-}(x), h_{-1}^{-}(x) = -h_{+}(x), \tag{5.4}$$

$$a_0 = 0, h_0^{+}(x) = h_{+}(x), h_0^{-}(x) = h_{-}(x), \tag{5.5}$$

$$a_1 = -a, h_1^{+}(x) = -h_{-}(x), h_1^{-}(x) = -h_{+}(x). \tag{5.6}$$

From (3.18), (3,19), (5.4), (5.5) and (5.6) we get

$$p(\xi,-0) = -\frac{1}{k}\left[h_{-}'(\xi) + 2h_{-}'(\xi - ka) + ... + 2h_{-}'(\xi - mka\right], x \in (mka,(m+1)ka) \cap [0,1], \tag{5.7}$$

$$p(\xi,+0) = \frac{1}{k}\left[h_{+}'(\xi) + 2h_{+}'(\xi - ka) + ... + 2h_{+}'(\xi - mka\right], \; x \in (mka,(m+1)ka) \cap [0,1]. \tag{5.8}$$

The lift and moment coefficients are

$$C_L = \int_0^1 \left[p(\xi,-0) - p(\xi,+0)\right] d\xi, \tag{5.9}$$

$$C_M = \int_0^1 \xi \left[p(\xi,-0) - p(\xi,+0)\right] d\xi. \tag{5.10}$$

Let us consider some examples. For the thin profile in a free stream $(a = \infty, b = \infty)$ we have

$$C_L = -\frac{1}{k}\int_0^1 \left[h_{+}'(\xi) + h_{-}'(\xi)\right] d\xi, \tag{5.11}$$

$$C_M = -\frac{1}{k}\int_0^1 \xi \left[h_{+}'(\xi) + h_{-}'(\xi)\right] d\xi. \tag{5.12}$$

For the thin profile in ground effects $(b = \infty)$, taking for example $1 \geq ka \geq \frac{1}{2}$, we have

$$C_L = -\frac{1}{k}\int_0^1 \left[h_{+}'(\xi) + h_{-}'(\xi)\right] d\xi - \frac{2}{k}\int_0^{1-ka} h_{-}'(\xi) d\xi, \tag{5.13}$$

$$C_M = -\frac{1}{k}\int_0^1 \xi\left[h_+'(\xi) + h_-'(\xi)\right]\mathrm{d}\xi - \frac{2}{k}\int_0^{1-ka}(\xi + ka)h_-'(\xi)\mathrm{d}\xi\,. \tag{5.14}$$

For the thin profile in a wind tunnel, taking for example $1 \ge ka \ge \frac{1}{2}$, $1 \ge kb \ge \frac{1}{2}$, we have

$$C_L = -\frac{1}{k}\int_0^1\left[h_+'(\xi) + h_-'(\xi)\right]\mathrm{d}\xi - \frac{2}{k}\int_0^{1-ka} h_-'(\xi)\mathrm{d}\xi - \frac{2}{k}\int_0^{1-kb} h_+'(\xi)\mathrm{d}\xi\,,$$

$$C_M = -\frac{1}{k}\int_0^1 \xi\left[h_+'(\xi) + h_-'(\xi)\right]\mathrm{d}\xi - \frac{2}{k}\int_0^{1-ka}(\xi + ka)h_-'(\xi)\mathrm{d}\xi - \frac{2}{k}\int_0^{1-kb}(\xi + kb)h_+'(\xi)\mathrm{d}\xi. \tag{5.15}$$

For the flat plate profile the equations are

$$h_+(x) = h_-(x) = -\varepsilon x, x \in [0,1], \tag{5.16}$$

and the corresponding coefficients are

$$C_L = \frac{2\varepsilon}{k}, C_M = \frac{\varepsilon}{k}, \tag{5.17}$$

for the profile in a free stream,

$$C_L = \frac{2\varepsilon}{k}(2 - ka), C_M = \frac{\varepsilon}{k}(2 - k^2a^2)\,, \tag{5.18}$$

for the ground effects $\left(1 \ge ka \ge \frac{1}{2}\right)$, and

$$C_L = \frac{2\varepsilon}{k}(3 - ka - kb),\,,\; C_M = \frac{\varepsilon}{k}(3 - k^2a^2 - k^2b^2), \tag{5.19}$$

$\left(1 \ge ka \ge \frac{1}{2}, 1 \ge kb \ge \frac{1}{2}\right)$, for the profile in a tunnel. Formulas (5.10) – (5.20) are those obtained in [1], page 302 and [2].

ACKNOWLEDGEMENT

This work has been supported by the Ministry of Education and Research, Romania, under CNCSIS Grant 35262/2001.

REFERENCES

1. DRAGOŞ, L., *Metode Matematice în Aerodinamică*, Bucureşti, Editura Academiei, 2000.
2. DRAGOŞ, L., CARABINEANU, A., *The supersonic flow past a thin profile including ground and tunnel effects*, Zeit. Angew. Math. Mech., **82**, nr. 9, pp. 649 -652, 2001.
3. GODLEWSKI, E., RAVIART P.A., *Numerical Approximation of Hyperbolic Systems of Conservation Laws,* New York, Berlin, Heidelberg, Springer Verlag, 1996.

Received January 10, 2003.