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FRACTAL SPACE-TIME THEORY AND SOME APPLICATIONS IN ADVANCED MATERIALS

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Using the Scale Relativity Theory, Langmuir's double layer plasma and the heat transfer in nanofluids were analyzed by means of the "ballistic" phonons.

Key words: Fractal space-time, double layer, heat transfer, nanofluids

1. INTRODUCTION.

Since the last twenty years an increasing interest on the fractal approach on microphysics has been observed [1-9]. The Theory of fractal space-time or the Scale Relativity Theory (SRT) [1] extends Einstein's principle of relativity to scale transformation of resolution. The introduction of non-differentiable trajectories in physics dates back to the pioneering works of Feynman, who demonstrated that the typical quantum mechanical paths are non-differentiable curves of fractal dimension 2 [10].

2. THE FRACTAL STRUCTURE OF SPACE-TIME

The new geometric description of space-time is based on the explicit dependence of physical laws on the investigation scale and on the introduction of a non-differentiable space-time varying with resolution, i.e. characterized by its fractal properties.

Three main consequences arise from this approach: i) The geodesics of a non-differentiable space-time are fractal and in infinite number: this leads one to use a fluid-like description, v = v [x(t), t], and to add new terms in the differential equations of mean motion; ii) The geometry of space-time becomes fractal [2-4], i.e. explicitly resolution dependent: this allows one to describe a non-differentiable physics in terms of differential equations acting in the scale spaces. In this framework, the Planck length-time scale becomes a minimal impassable scale, invariant under dilatations, and the cosmic length-scale (related to the cosmological constant) a maximal one [5, 6]. An attempt to construct a generalized SRT includes non-linear scale transformations and scale-motion coupling. In this last framework, one can reinterpret gauge invariance as scale invariance on the internal resolutions. In such a context, each elementary displacement is described in terms of the sum, $dX = dx + d\xi$ of a mean classical displacement $dx = vdt$ and of a fractal fluctuation $d\xi$ whose behavior satisfies the principle of SRT. The fractal properties of the considered curve is characterized by $\langle d\xi \rangle = 0$ and $\langle d\xi^2 \rangle = 2Ddt$, with D the fractal dimension. The existence of this fluctuation implies the introducing of a new second order terms in the differential equation of motion; iii) Time reversibility is broken at the infinitesimal level: this can be described in terms of a two-valuedness of the velocity vector, with the complex representation, $\mathbf{v} = \left[\left(v_+ + v_-\right)/2\right] - i\left[\left(v_+ - v_-\right)/2\right]$.

These three effects can be combined to construct a complex time-derivative operator

$$d/dt = \partial_t + \mathbf{V}\cdot\nabla - iD\Delta \tag{1}$$

where the mean velocity $\mathbf{V} = dx/dt$ is now complex and D is a parameter characterizing the fractal behavior of trajectories.

Since the mean velocity is complex, the same is true for the Lagrange function, then for the generalized action S as well. Setting

$$\psi = \exp\left(\frac{i\mathbf{S}}{2m_0 D}\right), \tag{2}$$

the velocity $\mathbf{V}$ has the expression [1]

$$\mathbf{V} = -2iD\nabla(\ln\psi) \tag{3}$$

and the Newton's equation of dynamics $m_0\, d\mathbf{V}/dt = -\nabla U$ can be integrated in terms of a generalized Schrödinger equation [1]:

$$D^2\Delta\psi + iD\partial_t\psi = \left(\frac{U}{2m_0}\right)\psi \tag{4}$$

This provides us with a theory of self-organization, since the solutions of this equation yield probability densities, which are interpreted as a tendency for the system to make ordered structures.

In the present paper, using the SRT, Langmuir's double layer plasma and the heat transfer in nanofluids by means of the "ballistic" phonons are analyzed.

3. LANGMUIR'S DOUBLE LAYER PLASMA

A double layers is a local region that can sustain a potential difference. It consists of two adjacent layers with opposite net charge. Such a structure has an internal electrical field althougt as a whole arrangement it is globally neutral. The stability is determined in a self-consistent manner by particles dynamics in the electrical field set up by the net charge distribution [11]. The one dimensional (1D) potential distribution in the Langmuir's double layer (DL) plasma is given by Poisson equation [11, 12]

$$\partial_{xx}V = \frac{j_i}{\varepsilon_0[2e(V_0 - V)/m_i]^{1/2}} - \frac{j_e}{\varepsilon_0[2e(V_0 + V)/m_e]^{1/2}} \tag{5}$$

where the parameters which appear in (5) have the usual significances from [11,12]: $m_{e,i}$ – electron and ion mass, $j_{e,i}$ – electron and ion current densities through the structure, V_0 – double layer potential, e – elementary charge.

Let us consider the approximation $V/V_0 \le 1$ and thereby equation (5) with substitutions:

$$a = \frac{j_i}{\varepsilon_0}\left(\frac{m_i}{2eV_0}\right)^{1/2} \quad ; \qquad b = \frac{j_e}{\varepsilon_0}\left(\frac{m_e}{2eV_0}\right)^{1/2} \tag{6a,b}$$

can be approximate as a power series in V/V_0 :

$$\partial_{xx}V \approx (a-b) + \frac{1}{2}\left(\frac{V}{V_0}\right)(a+b) + \frac{3}{8}\left(\frac{V}{V_0}\right)^2(a-b) + \frac{5}{16}\left(\frac{V}{V_0}\right)^3(a+b) + \frac{35}{128}\left(\frac{V}{V_0}\right)^4(a-b) + O\left[\left(\frac{V}{V_0}\right)^5\right] \tag{7}$$

The restriction $a = b$ determines the Langmuir condition for the critical currents densities bellow which double layer cannot be maintained: $j_e/j_i = (m_i/m_e)^{1/2}$. This condition shows also that the electronic component has a major contribution to the electric current in double layer.

In the framework of SRT, we can build a field theory with spontaneous symmetry breaking using the following substitutions:

$$x \to ix \quad , V \to iV \tag{8a,b}$$

In this approach, space-time becomes fractal, therefore, the x coordinate has a dynamic signification (particularly by time) and V variable a probabilistic one [1-4, 9].

Using the relation between the spatial length l and the potential drop generating this arrangement, $l^2 = (V_0 / a)$ and (6b) we can obtain the Child-Langmuir relation $j_e = \varepsilon_0 (2e/m_e)^{1/2} \left(V_0^{3/2}/d^2\right)$ [11].

Let us apply the variational principle $\partial \int L dv = 0$, integrating over the whole volume, to the Lagrangean density:

$$L = \frac{1}{2}\left(\partial_\zeta f\right)^2 - \vartheta(f) \tag{9}$$

with the potential

$$\vartheta(f) = \left(\frac{f^4}{4}\right) - \left(\frac{f^2}{2}\right) \tag{10}$$

We made the substitution:

$$\left(\frac{a}{V_0}\right)^{1/2} x = \xi \quad , \qquad V = \left(\frac{8}{5}\right)^{1/2} V_0 f, \tag{11a,b}$$

and neglect all the higher order terms in *V/V_0*.

The equation (7) with the substitutions (11a-b) takes the form

$$\partial_{\zeta\zeta} f = f^3 - f \tag{12}$$

From the Lagrangean density (9) it result the energy:

$$\varepsilon(f) = \int_{-\infty}^{\infty} d\zeta [\frac{1}{2}\left(\partial_\zeta f\right)^2 + \vartheta(f)] \tag{13}$$

In order to find the explicit form of the solution of (12), we multiply it by $\partial_\zeta f$ and subsequently integrate over ζ ; this yields:

$$\frac{1}{2}\left(\partial_\zeta f\right)^2 = -\frac{f^2}{2} + \frac{f^4}{4} + \frac{1}{2} f_0 \tag{14}$$

where f_0 is an integrate constant. From this we have

$$\zeta - \zeta^0 = \int_0^f \frac{df}{\sqrt{\frac{f^4}{2} - f^2 + f_0}}, \tag{15}$$

ζ^0 is a constant of integration.

To this general solution corresponds, for an arbitrary f_0, an infinite value of the energy $\varepsilon(f)$. To obtain the solution with finite energy, we make use of the boundary conditions [13] leading us to $f_0 = 1/2$. Replacing this value into (15), the solution $f_k(\zeta)$ of the field equation (14) with a finite energy is:

$$f_k(\zeta) = f(\zeta - \zeta^0) = \tanh[\frac{1}{\sqrt{2}}\left(\zeta - \zeta^0\right)] \tag{16}$$

This is called the kink solution and was also obtained using a nonlinear diffusion equation [14].

Let us define the energy relative to an equivalent "vacuum state", considered as a space without any spatial gradients, using the formula:

$$\varepsilon(f)-\varepsilon(f_v)=\int_{-\infty}^{\infty}d\zeta[\frac{1}{2}(\partial_\zeta f)^2+\frac{1}{4}(f^2-1)^2] \tag{18}$$

Since all the terms in the right hand side in (19) are positive, due to the finiteness of this energy it results that, at $\zeta\rightarrow\pm\infty$

$$\partial_\zeta f=0 \quad , \qquad \frac{1}{4}(f^2-1)^2=0 \tag{19}$$

It follows that at $\zeta\rightarrow\pm\infty$ the function $f(\zeta)$ tends to its "vacuum" value $f_v\rightarrow\pm1$.

Considering the expression $f_v=1$ and the expression for f_k in (19) we obtain the energy of the kink relative to the vacuum:

$$\varepsilon(f_k)-\varepsilon(f_v)=\frac{2\sqrt{3}}{3} \tag{20}$$

Therefore the kink solution was obtained by a spontaneous symmetry breaking, as a sudden transition from "vacuum state" to spatial ordered state (arrangement of double layer type).

We can extend now the topological analysis and assume that the kink solution can be considered as mapping of a spatial zero-sphere S^0, taken at infinity onto the vacuum manifold of the model given by (13). The homotopy group for this model is $\Pi_0(Z_0)=Z_2$, i.e. the model gives rise to two solutions: a constant solution f_v and the kink solution. The associated topological charge is:

$$Q=\frac{1}{2}\int_{-\infty}^{\infty}j(\zeta)d\zeta=\frac{1}{2}\int_{-\infty}^{\infty}\frac{df}{d\zeta}d\zeta=\frac{1}{2}[f(+\infty)-f(-\infty)] \tag{21}$$

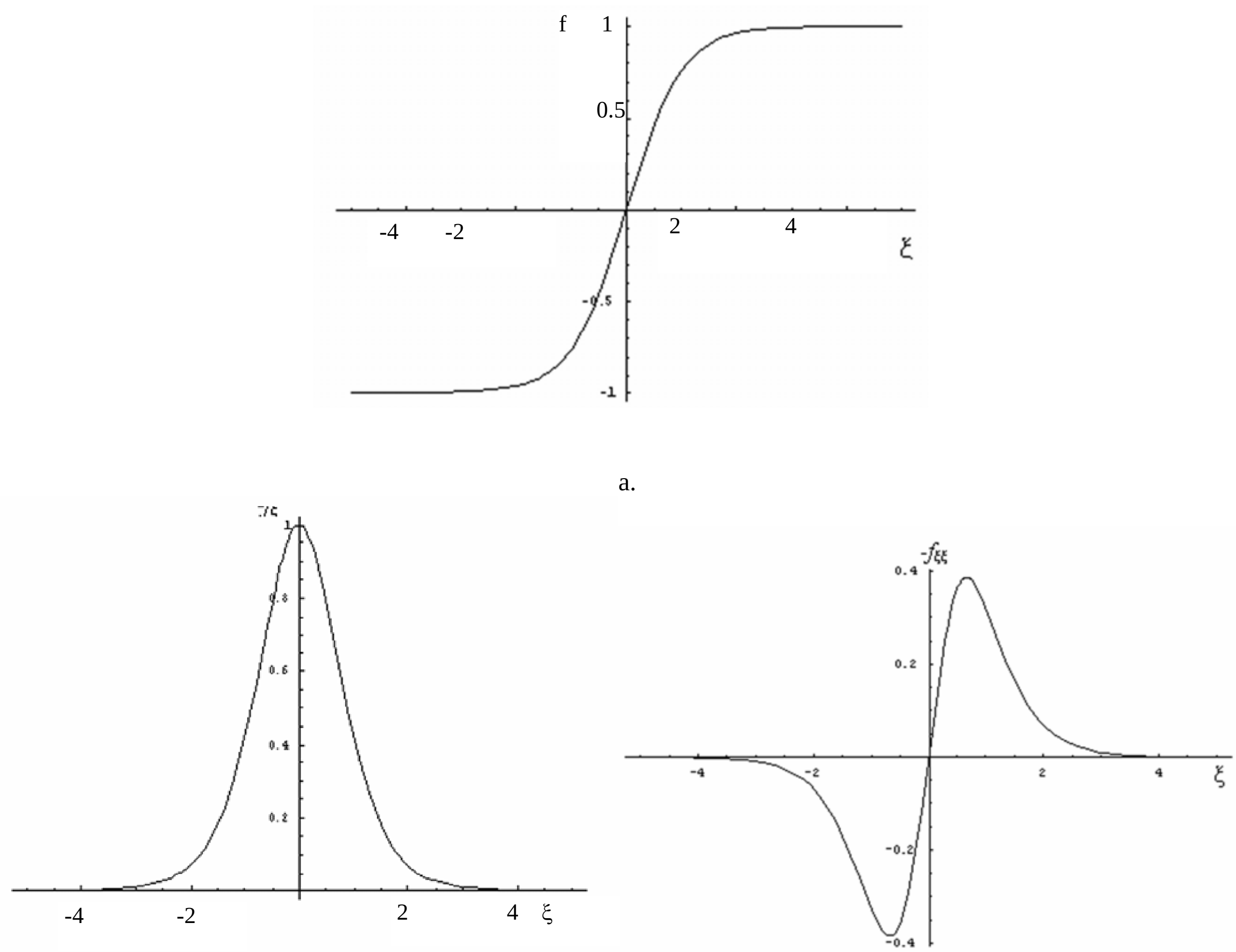
The "vacuum solution" and the kink solution can be characterized by $Q=0$ and $Q=1$, respectively (the result it obtained by an adequate normalization of *f*). Since (12) is a equation of Ginzburg-Landau type, it follows that *Q*=0 and the vacuum solution describes the behavior of the electron-ion pair in the ground state, while *Q*=1 and the kink solution describe the electron-ion pair behavior for a disturbed state. The results can be interpreted in a similar way as in [16-18]: the kink solution (18), assimilated in the field theories with an instanton [18], broke the "vacuum" of the plasma, generating electron-ions pair by a charge separating mechanism. The pair broken time is $\tau=l^2/D_a$. [14, 19]. If we continue the differentiation of the kink solution (Fig. 1a) we obtain the electric field distribution (Fig. 1b)

$$E_r(\zeta)=-f_\zeta=\frac{1}{\sqrt{2}}\frac{1}{\cosh^2[\frac{(\zeta-\zeta_0)}{\sqrt{2}}]} \tag{22}$$

and the charge density distribution (see Fig. 1c):

$$n_r(\zeta)=-f_{\zeta\zeta}=\frac{\sinh[\frac{(\zeta-\zeta_0)}{\sqrt{2}}]}{\cosh^3[\frac{(\zeta-\zeta_0)}{\sqrt{2}}]}. \tag{23}$$

All theoretical profiles are in concordance with those obtained in experiments [19,20].



a. b. c.

Figures 1a-c – Distributions of the potential, field and charge density in the Langmuir double layer.

Numerical application: Using parameters of the double layer from the experiments described in [14,15,18-20]: potential $V_0 \approx 40V$, double layer current $I \approx 10^{-3}A$, electronic temperature $T_e \approx 5eV$, ionic temperature $T_p \approx 2.5\cdot 10^{-2}eV$, electronic-ionic mass ratio $\left(m_e/m_p\right)_{Ar}^{1/2} \approx 1/270$, electronic neutral collision frequency, $\nu_{en} = 10^9 s^{-1}$, length of the discharge tube, $d = 50\cdot 10^{-2}m$, we obtain the pair breaking time of the double layer structure $\tau = l^2/D_a = 5.9\cdot 10^{-3}s$ (with $D_a \approx D_e\left(m_e/m_p\right)^{1/2}\left[1+\left(T_p/T_e\right)\right] \approx 0.57m^2/s$ and $D_e = \left(k_B T_e/m_e \nu_{en}\right) \approx 155m^2/s$), in a good agreement with the experiment $\tau_{\exp}$=4*10^{-3}*s*. With Child – Langmuir relation we obtain the width of double layer $l \approx 3.9\cdot 10^{-2}m$, value that agree with experiment $l_{\exp} \approx 4\cdot 10^{-2}m$ [20].

4. HEAT TRANSFER IN NANOFLUIDS BY MEANS OF THE "BALLISTIC" PHONONS

Thermal cnoidal oscillation modes of the nanoparticle-liquid (nP/L) interface show that this interface is ordered as a two dimensional (2D) non-linear Toda lattice vortex [21-25]. Since through the relation

$T/T_0 \equiv cn^2(u)$ minima and maxima of the thermal field overlap with zeroes $(2m+1)K + 2inK'$ and poles $2mK + i(2n+1)K'$, m, n =0, ±1, ±2, …of the elliptic function cn^2 of complex argument $\underline{u}$ [26],

$$\underline{u} = \frac{K}{a}\underline{z}, \quad \underline{z} = x + iy, \quad \frac{K'}{K} = \frac{b}{a} \tag{24 a-c}$$

$$K = \int_0^{\pi/2} \frac{d\varphi}{(1-k^2\sin^2\varphi)^{1/2}}$$

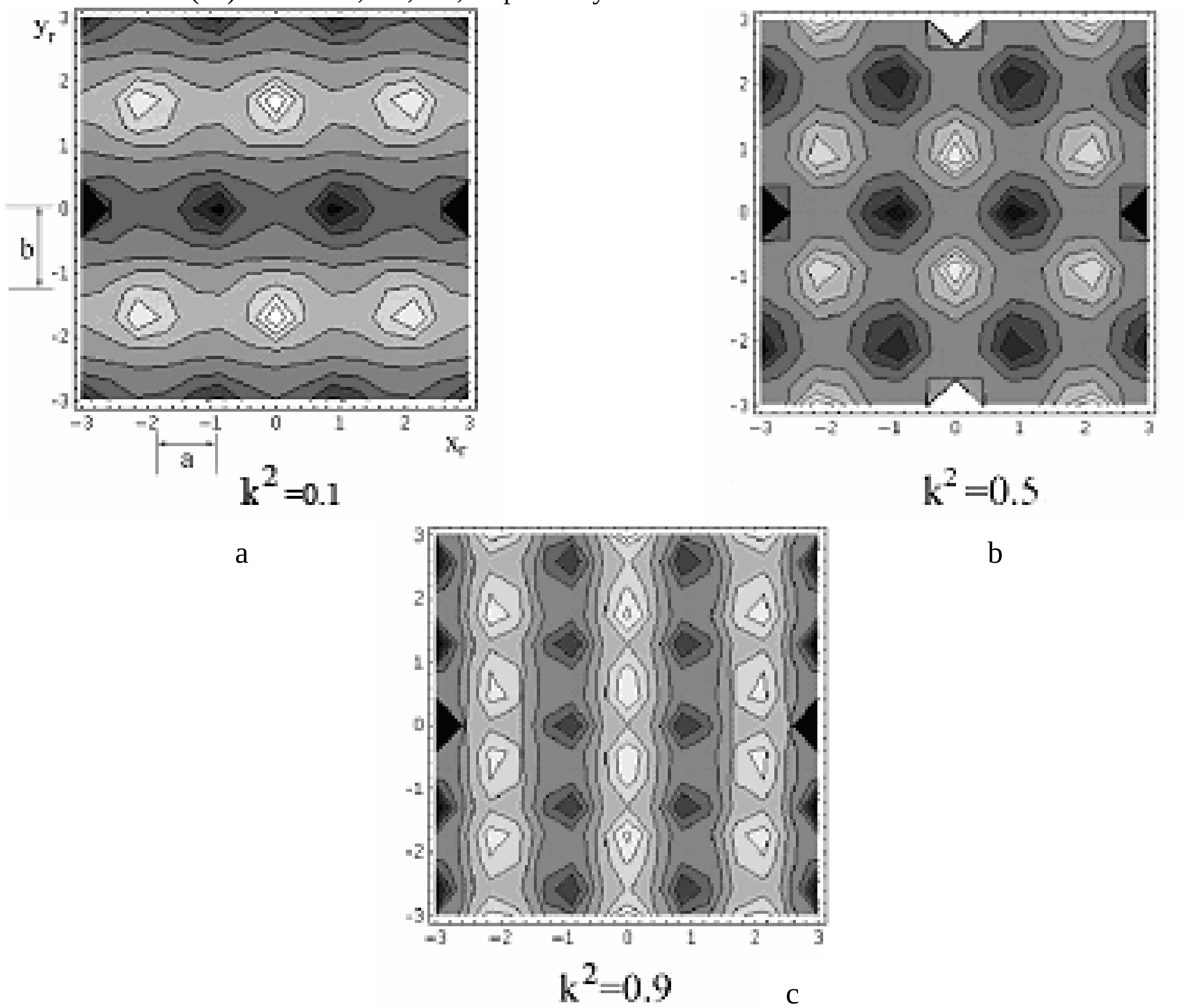
$$K' = \int_0^{\pi/2} \frac{d\varphi}{(1-k'^2\sin^2\varphi)^{1/2}}, k^2 + k'^2 = 1 \tag{25 a,b}$$

a, b are lattice constants along Ox and Oy directions.

The potential of the vortex lattice is define as the real part of the complex action

$$\underline{S} = mD\ln[cn(\underline{u};k)] \tag{26}$$

In Figs. (2a-c) it is represented the spatial dynamics (x,y) of the vortex lattice (equipotential curves), given the relation (26) for k^2=0.1; 0.5; 0.9, respectively.



Figures 2(a-c) - The dynamics of the lattice represented by equipotential curves.

It results that in the limit cases k→0 and k→1 the 'vortex streets' are generated along Ox direction, and Oy direction, respectively. Only in these limits, the vortex lattice gets coherence properties and the heat transport in nP/L interface works properly.

Let us define the "thermal implse":

$$\underline{P} = \frac{d\underline{S}}{d\underline{z}} \tag{27}$$

or explicitly

$$\underline{P} = mD\frac{K(k)}{2a}\left[\frac{sn(\underline{u};k)dn(\underline{u};k)}{cn(\underline{u};k)}\right] \tag{28}$$

where *sn* and *dn* are the elliptic functions. Since only real quantities have direct physical meaning, in the following we consider only the real part of the expression (28). Using now the relations of transformation for the elliptic function of complex argument into elliptic functions of real argument and introducing the notations [26]

$$\begin{gathered} s = sn(\alpha,k);\;\; s_1 = sn(\beta,k');\qquad c = cn(\alpha,k);\;\; c_1 = cn(\beta,k'); \\ d = dn(\alpha,k);\;\; d_1 = dn(\beta,k');\qquad \alpha = \frac{K}{a}x;\;\; \beta = \frac{K}{a}y \end{gathered} \tag{29}$$

Relation (28) becomes:

$$\underline{P} = mD\frac{K(k)}{2a}\mathrm{Re}\left[\frac{sn(\underline{u};k)dn(\underline{u};k)}{cn(\underline{u};k)}\right] = mD\frac{K(k)}{2a}\frac{scd[c_1^2(d_1^2+k^2c^2s_1^2)-s_1^2d_1^2(d^2c_1^2-k^2s^2)]}{(1-d^2s_1^2)(c^2c_1^2+s^2d^2s_1^2d_1^2)} \tag{30}$$

Then, in agreement with the previous observations, the following degenerations are imposed:

i) k=1, k'=0, K=∞, K'=π/2 for P_x, i.e.

$$P_x = \frac{\pi mD}{2b}\frac{\sinh\alpha\cosh\alpha}{\cosh^2\alpha-\sin^2\beta},\;\alpha = \frac{\pi x}{2b},\;\;\beta = \frac{\pi y}{2a} \tag{31 a-c}$$

ii) k=0, k'=1, K=π/2, K'=∞ for P_y, i.e.

$$P_y = \frac{\pi mD}{2a}\frac{\sin\gamma\cos\gamma}{\cos^2\gamma\cosh^2\delta+\sin^2\gamma\sinh^2\delta}\;\;,\;\;\gamma = \frac{\pi x}{2a},\;\;\delta = \frac{\pi y}{2b}. \tag{32 a-c}$$

The "thermal vortex field" $\mathbf{\Omega}$ will have, through $\mathbf{\Omega} = m^{-1}\nabla\times\mathbf{P}$, the non-zero component

$$\Omega_z = \frac{\pi^2 D}{4a}\left[\frac{1}{a}\frac{2\cos2\gamma(\cos^2\gamma\cosh^2\delta+\sin^2\gamma\sinh^2\delta)+\sin^2 2\gamma}{(\cos^2\gamma\cosh^2\delta+\sin^2\gamma\sinh^2\delta)^2}-\frac{1}{b}\frac{\sinh2\alpha\sin2\beta}{(\cosh^2\alpha-\sin^2\beta)^2}\right] \tag{33}$$

Averaging,

$$\langle\Omega_z\rangle = \int_0^b\int_0^a \Omega_z dS \bigg/ \int_0^b\int_0^a dS\,, \tag{34}$$

relation (34) becomes

$$\langle\Omega_z\rangle = \frac{\pi D}{2ab}\left\{\frac{b}{a}-\frac{1}{\pi}\ln\left[\frac{\cosh\left(\frac{\pi a}{b}\right)+\cos\left(\frac{\pi b}{a}\right)}{2\cos^2\left(\frac{\pi b}{2a}\right)\cosh^2\left(\frac{\pi a}{2b}\right)}\right]\right\} \tag{35}$$

In Fig. 3 it is shown the dependence of the mean "thermal vortex field" on the lattice lengths *a*,*b*.

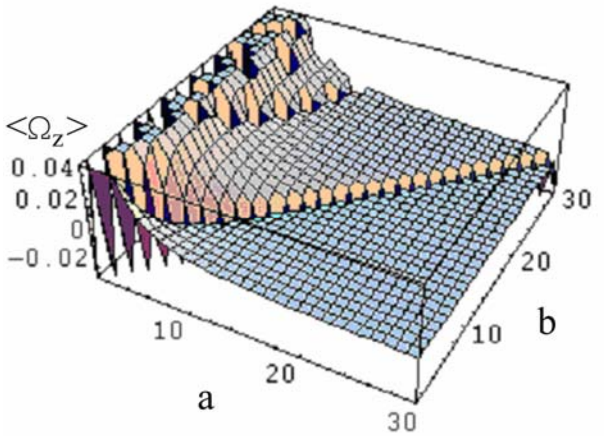


Figure 3. The dependence of the mean "thermal vortex field" on the lattice lengths a,b

This means that the nP-L interface behaves like a nonlinear vortex lattice, with acoustic and optical component respectively (the first and the second term of the relation (35)) of the phononic spectrum (for details see [22-24]). The field diverges for $a=b$, specifying an intrinsic anisotropy of nP/L interface. The existence of this anisotropy determines an anisotropy of the heat transport in nanofluides ([22-24]). For $a>>$b (35) takes the approximate form:

$$\left\langle \Omega_z \right\rangle \approx \frac{\pi D}{2a^2}. \tag{36}$$

Then the optical component of the phononic spectrum is missing. We assume that in nanofluids the heat moves in a ballistic manner (by means of the "ballistical" phonons). These results are in concordance with the experiment [22-24].

5. CONCLUSIONS

i) In the framework of a fractal space-time theory, it is possible to establish the distribution of the electrical potential, field and charge density for the Langmuir's plasma double layer. The double layer can be assimilated, as in the field theory, with a breaking of the "vacuum" state of the plasma generating pairs of electron-ions. A numerical approach gives the theoretical values for the pair breaking time of the double layer structure in agreement with the experiment;

ii) A theoretical approach of the heat transport in nanofluid using the scale relativity was developed. In such context, it was shown that the nP/L interface self-structurates as a non-linear Toda vortex lattice. The intrinsic anisotropy and the phononic spectrum components have been shown. In our opinion, the heat transfer in nanofluid is done by means of the "ballistical" phonons.

REFERENCES

1. NOTTALE L., *Fractal Space-Time and Microphysics: Towards a Theory of Scale Relativity*, World Scientific, Singapore (1992).
2. NICOLIS G., *Introduction to Nonlinear Science*, Cambridge University Press, Cambridge, 1995.
3. GOUYET J. F., *Physique et structures fractals*, Masson, Paris, 1992.
4. MANDELBROT B., *The Fractals Geometry of Nature*, Freeman, San Francisco, 1982.
5. ARGYRIS J., CIUBOTARIU C., MATTUTIS G., Chaos, Solitons and Fractals, **12**, 1 2001.
6. ARGYRIS J., MARIN C., CIUBOTARIU C., *Physics of Gravitation and the Universe*, Iasi, Spiru Haret Publishing House, (2003).
7. AGOP M., GRIGA V., CIOBANU B, et. al., Chaos, Solitons and Fractals **9,** 1143, 1998.
8. ELNASCHIE M. S., Chaos, Solitons and Fractals **11,** 1149, 2000.
9. NELSON E., Phys. Rev. **150**, 1079, 1966.

10. FEYNMAN R. P., Hibbs A. R., *Quantum Mechanics and Path Integrals*, MacGraw-Hill, NY, 1965.
11. RAIZER Y. P., *Gas Discharge Physics*, Springer Verlag, Berlin, 1991.
12. HORTON W., *Coherent Structures in Plasma:Turbulence*, Transport, Chaos and Plasma Physics 2: Advanced Science in Non-linear Dynamics, vol. **9**, World Scientific, Singapore, 1992.
13. CHAICHIAN M., NELIPA N. F. *Introduction to Gauge Field Theories*, Berlin, Heidelberg, NY, Tokyo,Springer Verlag, 1984.
14. AGOP M., STAN C., BUZEA C. Et. Al., Rom. Rep. Phys., **1-2,** 37, 1998.
15. STAN C., CRISTESCU C. P., ALEXANDROAEI D., J. Optoelectron. Adv. Mater., **7**, 2451, 2005.
16. POOLE C. P., FARACH H. A., CRESWICK R., *Superconductivity*, Academic Press, San Diego, NY, Boston, London, Sydney, Tokyo, Toronto, 1995.
17. ELNASCHIE M. S., Chaos, Solitons and Fractals, **22,** 1 (2004).
18. CRISTESCU C. P., STAN C., ALEXANDROAEI D., Phys. Rev. E, **70**, 016613, 2004.
19. AGOP M., STRAT M., STRAT G., NICA P., Chaos, Solitons and Fractals, **13,** 1541, 2002.
20. ALEXANDROAEI D., SANDULOVICIU M., Phys. Lett. A, **122**, 173, 1987.
21. TODA M., *Theory of Nonlinear Lattices*, Springer-Verlag, 1988.
22. KIM J., KANG Y. T., CHOI C. K., *Physics of Fluids*, **16**, 2395, 2004.
23. WANG B., ZHOU L. R., PENG X. F., Int. J. Heat Mass Transfer, **46**, 2665, 2003.
24. XUE Q.-Z, Phys. Lett. A **307**, 313, 2003.
25. AGOP M., IOANNOU P. D., NICA P. et al., Mat. Trans., **45**, 972, 2004.
26. BOWMAN F., *Introduction to Elliptic Functions with Applications*, English Universities Press Ltd, London, 1953.

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