THE PUBLISHING HOUSE
OF THE ROMANIAN ACADEMY

PROCEEDINGS OF THE ROMANIAN ACADEMY, Series A,
Volume 9, Number 1/2008, pp. 000–000

ON THE CHARACTERIZATION OF AUXETIC COMPOSITES

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The effective Young's modulus of an auxetic composite is the aim of this paper. The composite is made up of alternating layers of auxetic material (with a negative Poisson's ratio) and aluminum. The problem is solved in the light of Cosserat elasticity which admits degrees of freedom not present in classical elasticity, i.e. rotation of points in the material and couple stresses. The enhancement in the Young's modulus for auxetic over nonauxetic materials is reported.

Key words: Auxetic material, Negative Poisson'ratio, Bécus homogenization method.

1. INTRODUCTION

The auxetic material (with negative Poisson's ratio ν) is not a continuous medium. It has unique properties, instead of getting thinner like an elongated elastic band, it grows fatter, expanding laterally when stretched (Rosakis, Ruina and Lakes [1], Lakes [2]$-$[4]). Love [5] presents an example of cubic single crystal pyrite as having a Poisson's ratio of -0.14, and he suggests the effect may result from a twinned crystal. Typically mechanical properties (for example indentation resistance and shear modulus) are inversely proportional to $(1-\nu^2)$ or $(1+\nu)$. The negative limit of ν for isotropic materials is -1, and $(1-\nu^2)$ or $(1+\nu)$ tend to zero, leading to enhancements in the material properties for auxetic over nonauxetic materials. The idea is to transform a non-auxetic material into auxetic form as foams or cellular materials, or to employ new techniques for architecture new auxetic materials (honeycombs, fiber-reinforced and reentrant polymer foams and composites).

The classical mechanics fails if it is extended to describe the behavior of auxetic materials, because these materials develop couple moment stresses that bend the internal connecting ligaments (Burns [6]). The cell ribs transmit bending moments, tensile and compressive forces. These bending moments can be incorporated as hidden variables in a continuum description. That is the couple stresses from Cosserat elasticity (Cosserat [7], Kröner [8], Berglund [9]). The auxetic materials imply the chiral effects. That is the properties are described by a fifth rank modulus tensor, which changes under an inversion. In this paper we specialize in Sec.2 the theory of the auxetic material described as a chiral Cosserat medium. The formalism of determining the effective Young' s modulus for an auxetic composite is presented in Sec.3 by using the Bécus homogenization technique. The enhancements in Young' s modulus for auxetics are reported in this section.

2. THEORY

Consider a chiral Cosserat medium in a Cartesian coordinates system (x,y,z). The equations of motion for the case without body forces and body couples are (Eringen [10], [11], Mindlin [12], [13], Gauthier [14])

$$\sigma_{kl,k}-\rho\ddot{u}_l=0\,,\quad m_{rk,r}+\varepsilon_{klr}\sigma_{lr}-\rho j\ddot{\varphi}_k=0\,. \tag{1}$$

Here σ_{kl} is the stress tensor, m_{kl} is the couple stress tensor, u is the displacement vector, φ_k is the microrotation vector which in Cosserat elasticity is cinematically distinct from the macrorotation vector $r_k = 1/2\varepsilon_{klm}u_{m,l}$, and ε_{klm} is the permutation symbol. We remember that φ_k refers to the rotation of points themselves, while r_k refers to the rotation associated with movement of nearby points. In (1) ρ is the mass density and j the microinertia. The constitutive equations are

$$\begin{aligned}\sigma_{kl} &= \lambda e_{rr}\delta_{kl} + (2\mu+\kappa)e_{kl} + \kappa\varepsilon_{klm}(r_m - \varphi_m) + C_1\varphi_{r,r}\delta_{kl} + C_2\varphi_{k,l} + C_3\varphi_{l,k},\\ m_{kl} &= \alpha\varphi_{r,r}\delta_{kl} + \beta\varphi_{k,l} + \gamma\varphi_{l,k} + C_1 e_{rr}\delta_{kl} + (C_2+C_3)e_{kl} + (C_3 - C_2)\varepsilon_{klm}(r_m - \varphi_m),\end{aligned} \tag{2}$$

where $e_{kl} = 1/2(u_{k.l} + u_{l,k})$ is the macrostrain vector. λ, and μ are Lamé elastic constants,. κ is the Cosseraţ rotation modulus, α,β,γ, the Cosseraţ rotation gradient moduli, and C_i, $i = 1,2,3$ are the chiral elastic constants associated with noncentrosymmetry. For $C_i = 0$ the equations of isotropic micropolar elasticity are recovered. For $\alpha = \beta = \gamma = \kappa = 0$, (1) reduces to the constitutive equations of classical isotropic linear elasticity theory.

In this paper we do not introduce the requirement that the internal energy must be nonnegative (the material is stable) in order to obtain restrictions on the micropolar elastic constants. The condition of a positive Young's modulus and $-1 < \nu < 0.5$ corresponds to the usual range of properties for stability of an unconstrained material. The existence of negative material constants (shear modulus, bulk modulus, stiffness) is also permitted (experimentally reported in Teodorescu, Badea, Munteanu and Onisoru [15]). For most materials, the shear modulus is two times to three times greater than Young's modulus, most commonly though $\nu = 0.3$ .The initial conditions are

$$\begin{gathered} u_i(x,y,z,0) = u_i^0(x,y,z),\ \varphi_i(x,y,z,0) = 0,\ i = 1,2,3,\\ m_{ij}(x,y,z,0) = 0,\ \ \sigma_{ij}(x,y,z,0) = 0,\ \ \ i = j \neq 3. \end{gathered} \tag{3}$$

Consider the case of the laminated plates made up of a periodic layering of sheets normal to the direction x of wave propagation, each of elastic material with constant properties. For simplicity, without loss of generality, the particular 2D case in which all quantities depend only on x and z is considered.

Let $\mathbb{F} = \{\sigma_{kl}, m_{kl}, u_k, \varphi_k, k,l = 1,2,3\}$ be a set composed of the asymmetric tensors σ_{kl}, m_{kl}, e_{kl}, $k,l = 1,2,3$, and the vectors u_k, φ_k. We call $\mathbb{F}$ an elastodynamic state on the bounded medium, if it satisfies (1)−(3). The theory is based on the following theorem:

THEOREM 1.*The one-by-one transformation*

$$\hat{u}_1 = K_{10}^2(u_1 + u_2 - u_3),\ \ \hat{u}_2 = K_{11}^2(u_2 + u_3 - u_1),\ \hat{u}_3 = K_{12}^2(u_3 + u_1 - u_2),$$

$$\hat{\varphi}_1 = K_{10}^2(\varphi_1 + \varphi_2 - \varphi_3),\ \ \hat{\varphi}_2 = K_{11}^2(\varphi_2 + \varphi_3 - \varphi_1),\ \hat{\varphi}_3 = K_{12}^2(\varphi_3 + \varphi_1 - \varphi_2),$$

with

$$K_{10}^2 = \frac{(C_2 + C_3)^2}{4(2\mu+\kappa)(\beta+\gamma)},\ K_{11}^2 = \frac{(C_2 - C_3)^2}{4(2\mu+\kappa)(\gamma-\beta)},\ K_{12}^2 = \frac{(3C_1 + C_2 + C_3)^2}{4(3\lambda+2\mu+\kappa)(3\alpha+\beta+\gamma)},$$

transforms the elastodynamic state $\mathbb{F}$ *into another elastodynamic state* $\hat{\mathbb{F}} = \{\hat{\sigma}_{kl}, \hat{m}_{kl}, \hat{u}_k, \hat{\varphi}_k, k,l = 1,2,3\}$, *composed by the symmetric tensors* $\hat{\sigma}_{kl}$, $\hat{m}_{kl}$, $\hat{e}_{kl}$, $k,l = 1,2,3$, *and the vectors* $\hat{u}_k$, $\hat{\varphi}_k$, *that satisfies* (1)−(3). *The state* $\hat{\mathbb{F}}$ *can be decomposited in the form* $\hat{\mathbb{F}} = \hat{\mathbb{F}}_1 + \hat{\mathbb{F}}_2$, *where* $\hat{\mathbb{F}}_1 = \{\hat{\sigma}_{11}, \hat{\sigma}_{13}, \hat{\sigma}_{33}, \hat{m}_{22}, \hat{u}_1, \hat{u}_3, \hat{\varphi}_2\}$, $\hat{\mathbb{F}}_2 = \{\hat{\sigma}_{22}, \hat{m}_{11}, \hat{m}_{13}, \hat{m}_{33}, \hat{u}_2, \hat{\varphi}_1, \hat{\varphi}_3\}$.

Proof. The proof is immediately. We have $\hat{e}_{kl} = 1/2(\hat{u}_{k.l} + \hat{u}_{l,k}) = \hat{e}_{lk}$, and from (2) we obtain $\hat{\sigma}_{kl} = \hat{\sigma}_{lk}$, $\hat{m}_{kl} = \hat{m}_{lk}$. The state $\hat{\mathbb{F}}$ verifies equations (1)−(3). Then, we introduce (2) into (1). After a proper combination of equations, the following equations in $\hat{u} = (\hat{u}_1, \hat{u}_2, \hat{u}_3)$ and $\hat{\varphi} = (\hat{\varphi}_1, \hat{\varphi}_2, \hat{\varphi}_3)$ are found

$$\begin{gathered}(\lambda+2\mu+\kappa)\nabla\nabla\hat{u}-(\mu+\kappa)K_0^2\nabla\times\nabla\times\hat{u}+\kappa(1-K_0^2)\nabla\times\hat{\varphi}=\rho\ddot{\hat{u}},\\(\alpha+\beta+\gamma)\nabla\nabla\hat{\varphi}-\gamma K_0^2\nabla\times\nabla\times\hat{\varphi}+\kappa(1-K_0^2)\nabla\times\hat{u}-2\kappa(1-K_0^2)\hat{\varphi}=\rho j\ddot{\hat{\varphi}},\end{gathered} \tag{4}$$

with a coupling coefficient K_0 defined as

$$K_0^2=1+\frac{(C_1+C_2+C_3)^2}{(\lambda+2\mu+\kappa)(\alpha+\beta+\gamma)} \tag{5}$$

We see that (4) are decoupled into two sets of equations in $\hat{\mathbb{F}}_1$, and $\hat{\mathbb{F}}_2$.

Each set of equations corresponding to $\hat{\mathbb{F}}_1$ and $\hat{\mathbb{F}}_2$ is solved in a similar way (Teodorescu, Munteanu and Chiroiu [15], Teodorescu, Munteanu, Badea and Onisoru [16], Chiroiu, Munteanus and Dumitriu.[17]).

3. A LAMINATED COMPOSITE PLATE

As a study case, let us have a laminated 2D composite plate which occupies the region $x\in[0,L]$, $z\in[-c,c]$, and made up of alternating the N aluminum and auxetic material layers, normal to the direction x of wave propagation (fig.1). The layers are parallel, planar, periodically, across which the displacements are continuous. The length of each layer is l. The interfaces between layers are located at nl, $n=1,2,...,N$, and each joint having two faces identified by $+$ and $-$. We choose coordinates so that the waves lie in the (x,z) plane. The plate is assumed to be in plane strain and to support waves running in the x-direction.

The motion equations are given by (4) with K_0^2 given by (5). Let us to suppose that all material constants are functions of x. The equations (4) are decoupled into two sets of equations in $\hat{\mathbb{F}}_1$, and $\hat{\mathbb{F}}_2$. So, we will concentrate only to the set of equations corresponding to $\hat{\mathbb{F}}_1$. The continuity of solutions v_1, v_2 and ϕ_2 at the interface nl are given by

$$v_i(\frac{\omega}{c_1}nl^-,\frac{\omega}{c_1}z,\omega t)=v_i(\frac{\omega}{c_1}nl^+,\frac{\omega}{c_1}z,\omega t),\ i=1,3,\ \phi_2(\frac{\omega}{c_1}nl^-,\frac{\omega}{c_1}z,\omega t)=\phi_2(\frac{\omega}{c_1}nl^+,\frac{\omega}{c_1}z,\omega t),$$

for $n=1,2,...,N$. To predict the Young' modulus from the Lamé elastic constants λ, μ, we have the formula

$$E_0=\frac{(3\lambda+2\mu)\mu}{\lambda+\mu}.$$

We are interested in knowing the influence of the Cosserat rotation modulus κ, the Cosserat rotation gradient moduli α,β,γ, and the chiral elastic constants C_i, $i=1,2,3$, on the effective Young' modulus value of the laminated plate. The material constants $\tilde{C}=\{\lambda,\mu,\kappa,\alpha,\beta,\gamma,C_1,C_2,C_3\}$ for this laminated composite are periodic functions of x

$$\tilde{C}(x+P)=\tilde{C}(x)$$

where P is the period equals to the length of the basic cell (for the composite $P=2l$ with l the length of the basic cell for the composite).

Let us to introduce a new length scale

$$\eta=\frac{x}{\varepsilon}, \tag{6}$$

where $\varepsilon>0$ is a parameter, so that (4) can be written as

$$[(1-a^2(\eta))v_{3,z}]_{,x}+v_{1,xx}+a^2(\eta)v_{1,zz}-s_4^*(\eta)\phi_{2,z}=\frac{1}{s_1(\eta)+s_2(\eta)}\ddot{v}_1\,,$$

$$[a^2(\eta)v_{3,x}+s_4^*(\eta)\phi_2]_{,x}+v_{3,zz}+(1-a^2(\eta))v_{1,xz}=\frac{1}{s_1(\eta)+s_2(\eta)}\ddot{v}_3\,, \tag{7}$$

$$-[\frac{c_1^2(\eta)\mu(\eta)}{\omega^2(\eta)\gamma(\eta)}v_3]_{,x}+\phi_{2,xx}+\phi_{2,zz}+\frac{c_1^2(\eta)\mu(\eta)}{\omega^2(\eta)\gamma(\eta)}v_{1,z}+\frac{2c_1^2(\eta)\kappa(\eta)(1-K_0^2(\eta))}{\omega^2(\eta)\gamma(\eta)K_0^2(\eta)}\phi_2=\frac{1}{s_4(\eta)}\ddot{\phi}_2\,.$$

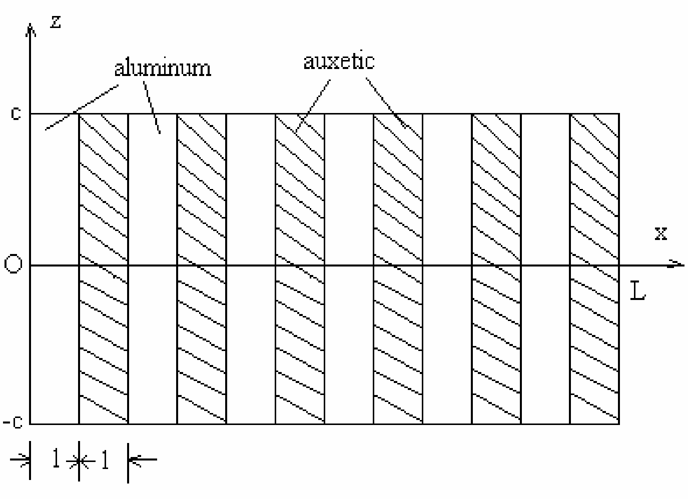

Fig. 1. The laminated composite plate.

The Bécus homogenization method via multiple scale expansion (Bécus [18]) consists in studying equations (7) as $\varepsilon\to 0$. The periodic variations of $\tilde{C}$ in (7) become frequent, so that the study of (7) will provide us some information on solutions for $P\to 0$. In view of (6), we have

$$\frac{\partial}{\partial x}=\frac{\partial}{\partial x}+\frac{1}{\varepsilon}\frac{\partial}{\partial\eta}\,.$$

The equations (7) can bee rewritten in terms of $w_i(x,z,t)=\{v_1,v_3,\varphi_2\}$, $i=1,2,3$,

$$\begin{aligned}F_i(\eta)\frac{\partial^2}{\partial t^2}w_i&=G_i(\eta)\frac{\partial^2}{\partial x^2}w_i+H_i(\eta)\frac{\partial^2}{\partial z^2}w_i+\varepsilon^{-1}G_i(\eta)\frac{\partial^2}{\partial x\partial\eta}w_i\\&+\varepsilon^{-1}\frac{\partial}{\partial\eta}\left(G_i(\eta)\frac{\partial}{\partial x}w_i\right)+\varepsilon^{-1}\frac{\partial}{\partial\eta}\left(H_i(\eta)\frac{\partial}{\partial z}w_i\right)+\varepsilon^{-2}\frac{\partial}{\partial\eta}\left(G_i(\eta)\frac{\partial^3}{\partial\eta\partial x\partial z}w_i\right)+H_i,\end{aligned} \tag{8}$$

with F, G and H are identified from (7). Upon expanding solutions $w_i(x,z,t)=\{v_1,v_3,\varphi_2\}$, $i=1,2,3$, in powers of ε, $w_i(x,\eta,z,t)=\sum_{j=0}\varepsilon^j w_{ij}(x,\eta,z,t)$, where w_{ij} are periodic of period P in η, we derive from (8)

$$F_i(\eta)\frac{\partial^2}{\partial t^2}\left(\sum_{j=0}\varepsilon^j w_{ij}\right)=\left(\varepsilon^{-2}L_0+\varepsilon^{-1}L_1+L_2\right)\left(\sum_{j=0}\varepsilon^j w_{ij}\right)+H_i, \tag{9}$$

where

$$L_0 = \frac{\partial}{\partial \eta}\left(G_i(\eta) \frac{\partial^3}{\partial \eta \partial x \partial z} w_i \right),\ L_1 = G_i(\eta) \frac{\partial^2}{\partial x \partial \eta} + \frac{\partial}{\partial \eta}\left(G_i(\eta) \frac{\partial}{\partial x} \right) + \frac{\partial}{\partial \eta}\left(H_i(\eta) \frac{\partial}{\partial z} \right),$$

$$L_2 = G_i(\eta) \frac{\partial^2}{\partial x^2} + H_i(\eta) \frac{\partial^2}{\partial z^2}.$$

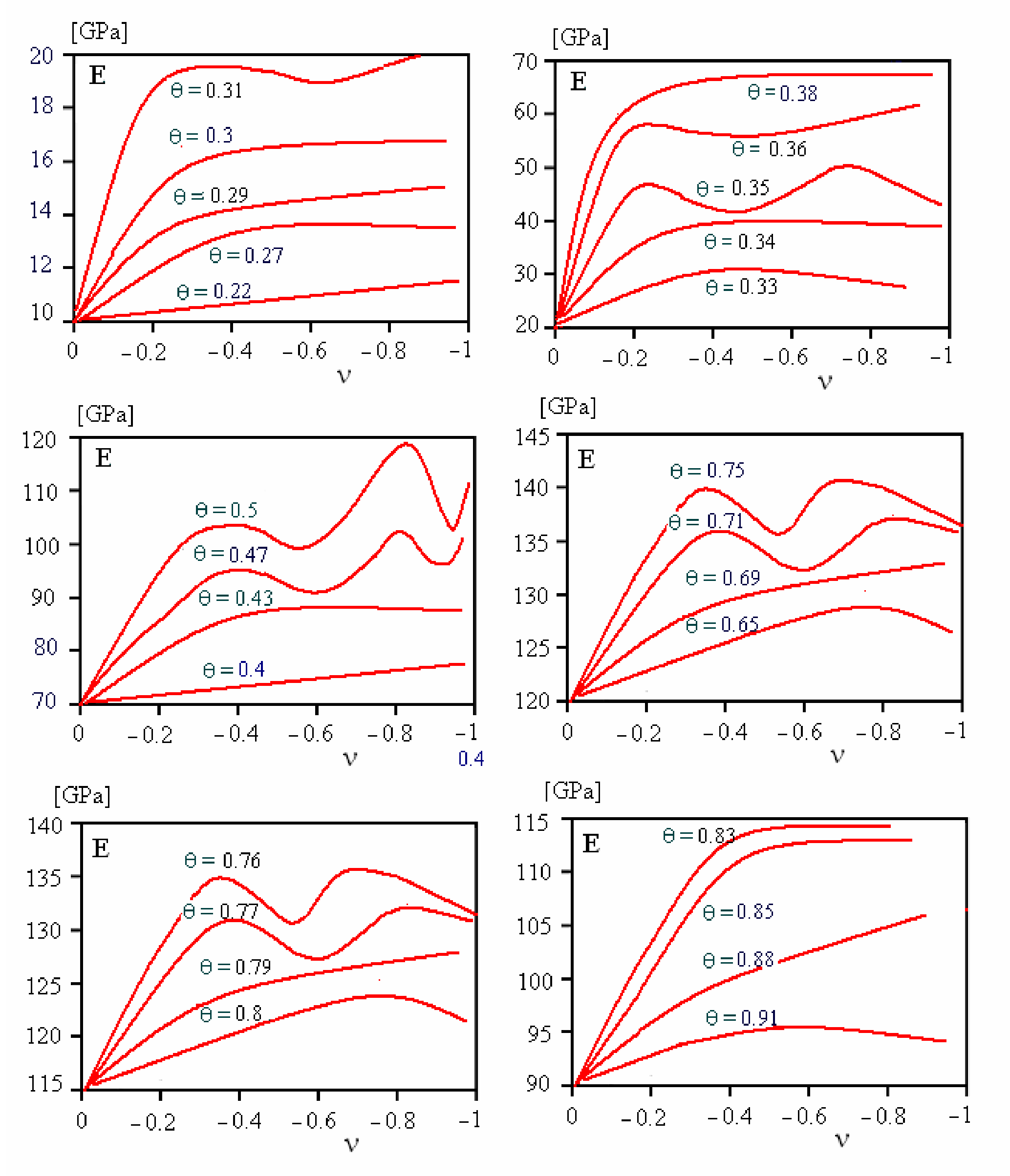

Fig. 2. The homogenized Young's modulus variation with respect to Poisson's ratio of the auxetic material.

The average of $\tilde{C}$ over one period of the composite is calculated as

$$\tilde{C}' = \frac{1}{P} \int_{\eta}^{\eta+P} \tilde{C}(\eta) \mathrm{d}\eta \tag{10}$$

From (2), (9) and (10) we find

$$E = E' + y_0\,,\ E' = \frac{(2\mu' + \kappa_{aux})(3\lambda' + 2\mu' + \kappa_{aux})}{(2\lambda' + 2\mu' + \kappa_{aux})} + \frac{1}{2}\tilde{p}^2\,,$$

$$\tilde{p}^2 = \frac{2\kappa_{aux}}{(K_0'^2 - 1)}\,,\ K_0'^2 = 1 + \frac{(C_{1aux} + C_{2aux} + C_{3aux})^2}{(\lambda' + 2\mu' + \kappa_{aux})(\alpha_{aux} + \beta_{aux} + \gamma_{aux})}\,, \tag{11}$$

where λ' and μ' are the average values given by (10) over one period of the composite, and $\tilde{C}_{aux}$ are the auxetic constants. The material constants used in this simulation are

$$\lambda = 50.01\text{GPa}\,,\ \mu = 28.21\text{GPa}\,,\ \nu = 0.32\,,$$

for aluminum, and

$$\lambda = 6.3\text{GPa}\,,\ \mu = 4.19\text{GPa}\,,\ \kappa = 0.0149\,\text{GPa}\,,$$

$$\alpha = 3.97\times10^4\,\text{N},\ \beta = 14.2\times10^4\,\text{N},\ \gamma = 2.68\times10^4\,\text{N}\,, C_1 = 13.56\times10^4\,\text{N/m}\,,$$
$$C_2 = -23.4\times10^4\,\text{N/m},\ C_3 = 19.55\times10^4\,\text{N/m}\,,$$

for the auxetics. For Poisson's ratio of the auxetic material we consider the interval $-1 \le \nu \le 0$.

Table 1. The Young's moduli for auxetic and nonauxetic virtual systems.

Young's modulus [GPa]	ν	θ	Young's modulus [GPa]	ν	θ
10.33 9.29	− 0.2 + 0.2	0.22	104.66 90.00	− 0.4 + 0.4	0.5
11.98 10.18	− 0.2 + 0.2	0.27	109.91 98.91	− 0.3 + 0.3	0.83
14.71 12.79	− 0.2 + 0.2	0.3	118.05 102.70	− 0.3 + 0.3	0.8
40.03 36.27	− 0.3 + 0.3	0.35	123.22 107.33	− 0.2 + 0.2	0.65
50.12 45.22	− 0.3 + 0.3	0.38	130.56 117.51	− 0.2 + 0.2	0.71
80.65 68.66	− 0.4 + 0.4	0.43	138.07 124.27	− 0.3 + 0.3	0.75
94.35 80.19	− 0.3 + 0.3	0.91	139.44 125.49	− 0.4 + 0.4	0.75

Fig.2 represents the variation of the homogenized Young's modulus with respect to the volume fraction θ of aluminum and the Poisson's ratio ν of the auxetic material ($-1 \le \nu \le 0$). The Young's moduli of aluminum and respectively, of the auxetic material are 109 GPa, and respectively, 1.55 GPa. We observe that the Young's modulus is increasing with respect to θ from 2 GPa, to about 140 GPa, and have a maximum value for $\theta = 0.75$. For θ above this value, the Young's modulus is decreasing with respect to θ from 135 GPa, to about 90 GPa. As expected, the limits $\theta = 0$ and $\theta = 1$ yield exactly to the Young's moduli of the auxetic material and respectively, to aluminum.

To see how the homogenized Young's modulus is varying when the auxetic material is replaced to another material with a positive Poisson's ratio ν ($0 \le \nu \le 0.5$), we calculate the modulus for the same absolute value of the Poisson's ratio. The table 1 shows that the homogenized Young's modulus decreases with 10-15% when the Poisson's ratio is positive. Consequently, by transforming a nonauxetic material into an auxetic form as foams or cellular materials, an enhancement in the Young' modulus of the composite is obtained.

Finally, fig. 3 shows the phase speed versus scaled wave number for laminated composite plate (auxetic material has $\nu = -0.32$). The fundamental mode and first harmonics are the main branches that retain finite wave velocity as the scale wave number tends to zero. The remaining harmonics come together in groups of three with another harmonic merging at a higher wave number. The behavior of the dispersive curves differ from that characterized a laminated material with positive Poisson's ratio by very low phase speeds.

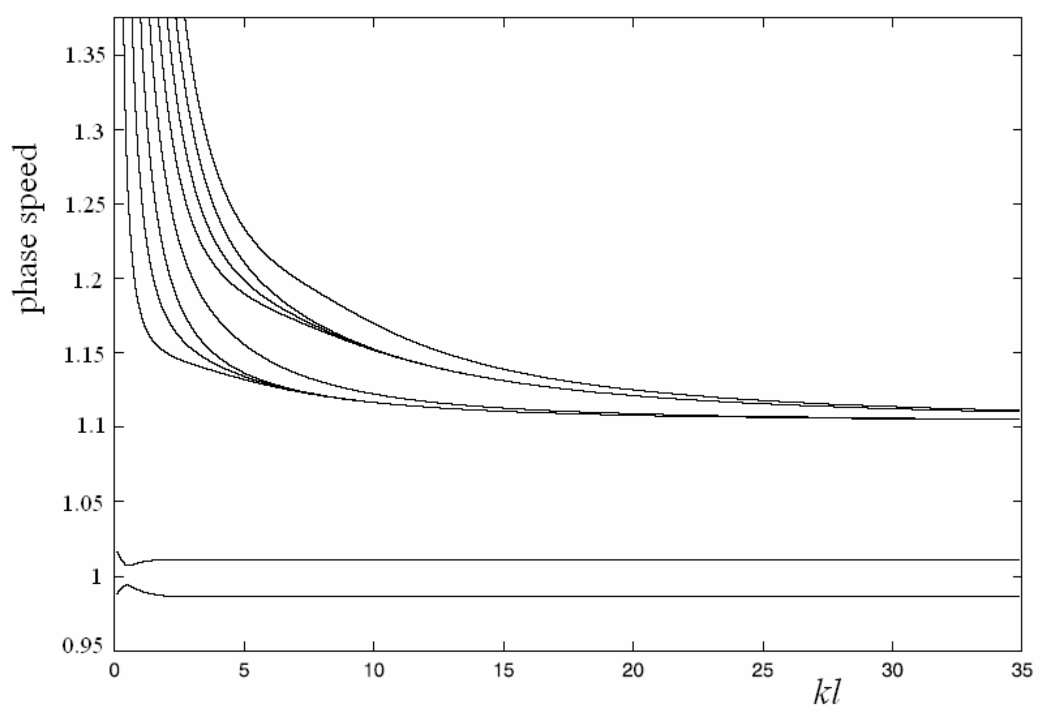

Fig. 3. Dispersion curves for the laminated composite plate.

4. CONCLUSIONS

In this paper, the auxetic material is modeled with Cosserat elasticity which admits degrees of freedom not present in classical elasticity. The Young'modulus is computed via the Bécus homogeneous technique for a laminated 2D composite plate, made up of alternating the aluminum and auxetic layers. The results confirm that the auxetic system composed from two materials with different properties can achieve various enhancements in the Young's modulus if one of the material has a negative Poisson's ratio.

The results are twofold: on the one hand, the paper provides a method to obtain the effective Young's modulus for an auxetic composites. On the other hand, they provide a means of enhancement in the material properties for auxetic over nonauxetic materials, not only for Young's modulus, but also for strength, damping, indentation resistance and shear modulus. Poisson's ratio are wider ($-\infty < \nu < \infty$) in the case of anisotropic materials. When a structure is loaded in bending, the flexural rigidity can be increased by including a core of auxetic material between these two skins to produce a sandwich structure. The key requirements for the core are normally the shear modulus and strength and compressive modulus. Light-weight, acoustic insulation and thermal insulation often result from the addition of the auxetic core.

The stiffness of one the two constituents must be at least 25 times greater than that of the other constituent to obtain a Poisson's ratio less than zero. A Poisson's ratio approaching -1 requires constituents which differ even more in stiffnesses, so that one phase is very soft, tending to empty space in its properties. The bounds on Poisson's ratio are wider ($-\infty < \nu < \infty$). Another possibility is that of creating stiffer negative Poisson's ratio materials by design on the molecular scale. A crystalline form of silica (SiO2), α-cristobalite, exhibits Poisson's ratios of $+0.08$ to -0.5, depending on direction.

The negative Poisson's ratio can often yields to a *negative stiffness* mechanism, which can be used to isolate or to cancel out vibrations in dynamical systems, better than traditional active control of vibrations solutions. The negative stiffness mechanism exerts an opposing force that cancels out the stiffness in a spring, for example (May [19], Lakes [20], [21], Lakes , Lee, T., Bersie, A., Wang [22]).

ACKNOWLEDGEMENT

The authors acknowledge the financial support of the National University Research Council (NURC-CNCSIS) Romania, Grant nr. 106/2007, code ID-247/2007.

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Received October 22, 2007