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# CONTROL STRATEGIES FOR OBSTACLE AVOIDANCE BY REDUNDANT MANIPULATORS

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In redundancy resolution, obstacle avoidance is considered as a performance criterion and is usually performed using specific solution offered by Gradient Projection or Extended Jacobian Methods while the end-effector follows a pre-determined path in workspace. The paper starts with an introduction section followed by the redundancy resolution methods presentations in second and third sections, then simulation results illustrating the methods are presented in fourth section and the last section contains the conclusions regarding the methods comparison.

*Key words:* redundancy resolution, obstacle avoidance, Gradient Projection, Extended Jacobian.

## 1. INTRODUCTION

The need for dexterous and versatile robotic manipulations has led to the development of kinematic redundant arms. These particular serial systems have more joints than necessary to achieve a specified task of the manipulator's end-effector. Thus, an infinite number of system's configurations correspond to the same end-effector's configuration (position and orientation) and the user can chooses the better configuration improving the robot's dexterity while end-effector accomplishes the desired task.

Redundancy allows manipulators to achieve complex tasks by taking into account additional constraints. These constraints are comprised in a constraint function, which limits, globally or specifically, the manipulator configuration using adjustable parameters whose values are determined by computer simulations. The choice of the constraint function is done according to some constraint criteria such as: obstacle avoidance, maneuverability improvement, limiting of speed/force values or minimization of joint torques. Obstacle avoidance is one of the most important domains of redundant manipulators application because of non-redundant structures incapacity for collision avoidance with workspace's obstacles, [3], [4], [6].

The direct geometric model gives the relation between end-effector configuration vector **x** and joint coordinates (angles vector $\boldsymbol{\theta}$, in rotation joint case):

$$\mathbf{x} = f(\boldsymbol{\theta});\ \mathbf{x} = [\ x_1\ x_2\ \ldots x_m\ ]^{\mathrm{T}};\ \boldsymbol{\theta} = [\ \theta_1\ \theta_2\ \ldots\ \theta_n\ ]^{\mathrm{T}}. \tag{1.1}$$

where $n$ is the number of degrees of freedom and $m$ is the workspace dimension.

If one differentiates the direct geometric model, it one obtains the direct differential model:

$$\delta\mathbf{x} = \mathbf{J}(\theta)\delta\boldsymbol{\theta}, \tag{1.2}$$

where $\mathbf{J}(\theta) = \dfrac{\delta f(\theta)}{\delta\theta}$ is the manipulator Jacobian matrix.

For non-redundant manipulator structures, $n = m$, the inverse differential model is:

$$\delta\boldsymbol{\theta} = \mathbf{J}^{-1}\delta\mathbf{x}. \tag{1.3}$$

In the case of redundancy, $n > m$, two basic methods for obstacle avoidance can be distinguished in the literature. *Gradient Projection Method* (GPM) formulates the obstacle avoidance problem as an optimization problem. While the end-effector follows a given path by means of the least norm solution, some performance criteria are carried out by means of the null space solution that yields self-motion of the links in the joint space only, without any end-effector motion effect. *Extended Jacobian Method* (EJM) defines additional constraints including obstacle avoidance for the given task until the relationship between the joint and the space becomes non-redundant.

## 2. GRADIENT PROJECTION METHOD

The Gradient Projection Method was first introduced by Liegeois [1] to utilize the redundancy to avoid mechanical joint limits. Extending the pseudoinverse solution, a general solution to the inverse kinematics problem can be expressed as:

$$\delta\boldsymbol{\theta} = \mathbf{J}^{+}\delta\mathbf{x} + (\mathbf{I}\text{-}\mathbf{J}^{+}\mathbf{J})\mathbf{z} \tag{2.1}$$

where $\delta\boldsymbol{\theta}$ is the joint angular differential variation, $\delta\mathbf{x} \in \Re^{n\times 1}$; $\mathbf{J}^{+} = \mathbf{J}^{\mathrm{T}}\left(\mathbf{J}\,\mathbf{J}^{\mathrm{T}}\right)^{-1}$ is the Moore-Penrose pseudoinverse of manipulator Jacobian matrix, $\mathbf{J}^{+} \in \Re^{n\times m}$; $\mathbf{J} \in \Re^{m\times n}$; $\delta\mathbf{x}$ is the differential variation of end-effector position (computed in closed-loop), $\delta x(k) = x(k) - x(k\text{-}1)$ where: $x(k)$ - imposed position at the moment $kT$; $x(k\text{-}1)$ - achieved position at previous moment $(k\text{-}1)T$, $\delta\mathbf{x} \in \Re^{m\times 1}$; $\mathbf{I} - \mathbf{J}^{+}\,\mathbf{J} = \mathbf{P}$ is the "projector" matrix, $\mathbf{P} \in \Re^{n\times n}$; and $\mathbf{z}$ is an arbitrary vector, $\mathbf{z} \in \Re^{n\times 1}$.

The first term on the right of equation (2.1) is the least norm solution. The second term is the homogeneous or null-space solution, which is orthogonal to the first term. The homogeneous solution is called the self-motion of the manipulator and produces no end-effector motion. For a desired end-effector trajectory, a homogeneous solution is selected such that the resulting robot configuration optimizes a performance measure. To optimize a performance criterion $h(\theta)$, $\mathbf{z}$ is chosen to be:

$$\mathbf{z} = \pm K\nabla h(\theta) \tag{2.2}$$

where $K$ is a positive real number and $\nabla h(\theta)$ is the gradient of $h(\theta)$. A positive sign in equation (2.2) indicates that the criterion is to be maximized; a negative sign indicates minimization.

For the performance function construction a *potential repulsive field* is used, [6]. Khatib originally proposed potential field method for on-line collision avoidance for a robot with proximity sensors. This method treats the robot as being under the influence of a virtual potential field U. The potential function is defined as the sum between an attractive potential, which attracts the robot toward the goal configuration, and a repulsive potential, which take off the robot of obstacles, [4].

The use of potential fields is a strong and efficient tool to solve the collision avoidance problems in the workspace. Virtual potential fields generated by the restriction surfaces are introduced. Links of the manipulator are subjected to potential field forces arising from the obstacles; these forces induce a virtual repulsion from the obstacle surfaces. The basic idea of following method is application of a repulsive potential field in *p Control Points* on the links of manipulator.

One choice for the mathematical expression of the repulsive potential field is:

$$\mathrm{U} = \begin{cases} \dfrac{1}{2}\eta\left(\dfrac{1}{\rho} - \dfrac{1}{\rho_0}\right)^2 & \text{if } \rho \le \rho_0, \\ 0 & \text{if } \rho > \rho_0. \end{cases} \tag{2.3}$$

where $\eta$ is a positive scaling factor, $\rho$ is the minimum distance from robot to obstacle, $\rho_0$ is a positive constant called *distance of influence*.

The constraint function $\mathbf{z}$ is defined as the sum of the potential repulsive forces that induce a virtual repulsion from the restriction surface:

$$\mathbf{z} = \sum_{j=1}^{p} \mathbf{F}_j \tag{2.4}$$

where $p$ is the maximum number of Control Points and the potential repulsive forces $\mathbf{F}_j$ have the following mathematical expression:

$$\mathbf{F}_j - \frac{\partial \mathrm{U}_j}{\partial \theta_i} = \left[ -\frac{\partial \mathrm{U}_j}{\partial \theta_i} \;\; . \;\; . \;\; . \;\; -\frac{\partial \mathrm{U}_j}{\partial \theta_n} \right]^{\mathrm{T}} ;\; i = 1,2,\ldots,n; j = 1,2,\ldots,p\,. \tag{2.5}$$

The computation procedure is composed of the following steps [6]:

- Determine the Control Points where the repulsive potential forces actions.
- Express, relative to the base frame, the coordinates of this configuration Control Points depending on the joint coordinates.
- Write the relations that specify the distances between the configuration control points and the restriction surface.
- Compose the potential repulsive fields and determine the potential repulsive forces (equations (2.3) and (2.5)).
- Write the equations of the direct kinematic model for the given manipulator (equation (1.1)).
- Determine the constraint functions **z** by summing the potential repulsive forces (equation (2.4)).
- Obtain the Jacobian matrix **J** and determine the pseudo-inverse matrix $\mathbf{J}^{+}$.
- Solve equation (2.1) and determine the joint angular differential variation $\delta\boldsymbol{\theta}$.

## 3. EXTENDED JACOBIAN METHOD

The *Configuration Control* method augments the manipulator forward kinematics with a set of kinematic functions in Cartesian or joint space that reflects the desired additional task, [5]. Let $\mathbf{x}_E = f_E(\boldsymbol{\theta})$ be the forward kinematic model of the robot which maps the $n \times 1$ joint displacement vector $\boldsymbol{\theta}$ to the $m \times 1$ end-effector coordinate vector $\mathbf{x}_E$. Let $\mathbf{x}_c = f_c(\boldsymbol{\theta})$ define a set of $r$ (= $n - m$) kinematic functions. The augmented kinematic model is then given by:

$$\mathbf{x} = \begin{pmatrix} \mathbf{x}_E \\ \mathbf{x}_c \end{pmatrix} = \begin{pmatrix} f_E(\theta) \\ f_c(\theta) \end{pmatrix}, \tag{3.1}$$

where $\mathbf{x}$ is the $n \times 1$ configuration vector. The user can then set up the desired additional task by imposing the constraint $\mathbf{x}_c(t) = \mathbf{x}_{cd}(t)$, where $\mathbf{x}_{cd}(t)$ is the user specified desired time variation of $\mathbf{x}_c$. The configuration control problem is then to ensure that the configuration vector $\mathbf{x}$ tracks the desired trajectory $\mathbf{x}_d(t) = \begin{pmatrix} \mathbf{x}_{Ed}(t) \\ \mathbf{x}_{cd}(t) \end{pmatrix}$ using a kinematic or dynamic control law.

The direct differential model obtained from the direct geometric model presented in (3.1) is:

$$\delta\mathbf{x} = \begin{pmatrix} \dfrac{\partial f_E(\theta)}{\partial \theta} \\ \dfrac{\partial f_c(\theta)}{\partial \theta} \end{pmatrix} \delta\boldsymbol{\theta} = \begin{pmatrix} \mathbf{J}_E(\theta) \\ \mathbf{J}_c(\theta) \end{pmatrix} \delta\boldsymbol{\theta} = \mathbf{J}\,\delta\boldsymbol{\theta}\,. \tag{3.2}$$

where $\mathbf{J}_E$ is the end-effector Jacobian matrix, $\mathbf{J}_c$ is the additional constraints Jacobian matrix and **J** is the extended Jacobian matrix.

In a special case when the desired additional task is to optimize an objective function, this method is called the *Extended Jacobian Method*, introduced by Baillieul [2]. One defines $f_c(\theta) = \mathbf{N}^{\mathrm{T}} \dfrac{\partial g}{\partial \theta}$, where $g(\theta)$ is the scalar kinematic objective function to be optimized and **N** is the $n \times r$ null space matrix of **J** that corresponds to the self-motion of the redundant manipulator:

$$\mathbf{N} = \det\left(\mathbf{J}_a\right)\begin{pmatrix} \mathbf{J}_a^{-1}\mathbf{J}_b \\ -\mathbf{I}_r \end{pmatrix} \\ \text{with } \mathbf{J} = \left(\mathbf{J}_a \mathbf{J}_b\right) \tag{3.3}$$

$\mathbf{J}_a$ is an $m$-squared matrix of the first columns of **J** and $\mathbf{J}_b$ is an $m \times r$ matrix of the remaining columns.

The necessary condition for optimality of $g(\theta)$ is $f_c = 0$. Thus, if we define the desired trajectory as $f_{cd}(t) = 0$ and use the configuration control to track $\mathbf{x}_d(t)$, then the kinematic optimization problem can be solved.

*Transpose Jacobian Matrix* gives a resolution method of inverse kinematic problem (redundancy) [3]:

$$\delta\boldsymbol{\theta} = \mathbf{J}^{\mathrm{T}}(\theta)\mathbf{K}\varepsilon\,. \tag{3.4}$$

where **K** is a positive definite matrix used for variation of the additional constraints effect on the constraints imposed to the end-effector and ε is the error, $\varepsilon = \mathbf{x}_d - \mathbf{x}$.

Because the right matrix **K** choice for the error accomplished is difficult, another method for inverse kinematic problem resolution consists in matrix **K** elimination and Transpose Jacobian application until $\varepsilon \le \varepsilon_d$. For the $k$-th sampling step of end-effector's task the control process is:

$$\begin{aligned} &\text{while } \varepsilon > \varepsilon_d \\ &\quad \mathbf{x}_E^{(i)} = f_E\left(\boldsymbol{\theta}^{(i)}\right) \\ &\quad f_c^{(i)}(\theta) = \left(\mathbf{N}^{\mathrm{T}}\right)^{(i)}\left(\frac{\partial g}{\partial \theta}\right)^{(i)} \\ &\quad \varepsilon = \mathbf{x}_d - \mathbf{x}^{(i)} = \begin{pmatrix} \mathbf{x}_{Ed} \\ 0 \end{pmatrix} - \begin{pmatrix} f_E\left(\boldsymbol{\theta}^{(i)}\right) \\ \left(\mathbf{N}^{\mathrm{T}}\right)^{(i)}\left(\dfrac{\partial g}{\partial \theta}\right)^{(i)} \end{pmatrix} \\ &\quad \delta\boldsymbol{\theta} = \mathbf{J}^{\mathrm{T}}\varepsilon \\ &\quad \boldsymbol{\theta}^{(i+1)} = \boldsymbol{\theta}^{(i)} + \delta\boldsymbol{\theta} \\ &\text{end} \end{aligned} \tag{3.5}$$

## 4. COMPUTER SIMULATION RESULTS

The propose of following application is to generate the references (positions and orientations) along the contour of a circle with radius $r$, whose surface is considered to be restrictive for all elements of a SCARA like planar redundant manipulator with revolute joints (fig. 4.1), having $n = 4$ degrees of freedom in 2D Cartesian space ($m = 2$) [6].

The initial position of the manipulator is given by:

$$\theta_0 = [30^{\circ}\ 30^{\circ}\ 60^{\circ}\ 30^{\circ}]^{\mathrm{T}} \\ l_0 = l_1 = l_2 = l_3 = l_4 = 2;\quad l_5 = 1;\quad R = 2;\quad r = 1;\quad \alpha_0 = 0 \tag{4.1}$$

where $\boldsymbol{\theta}_0$ – the vector of initial joint coordinates; $l_0$, $l_1$, $l_2$, $l_3$, $l_4$, $l_5$ – the lengths of the links, $R$ – the radius of end-effector positions generation, $r$ – the radius of the restriction circle; $\alpha_0$ – the initial angular position (at moment $t = 0$).

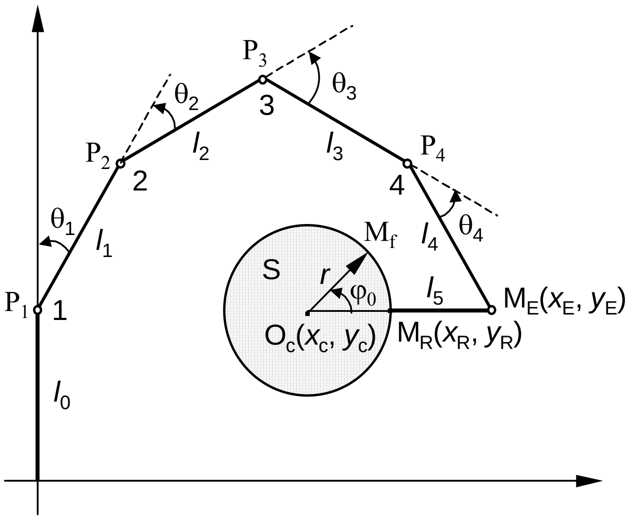


Fig. 4.1. Initial manipulator position [6].

The restriction surface is a circle with imposed values for both radius $r$ and its centre coordinates $x_c$ , $y_c$. The end-effector is the link $l_5$ and the coordinates of its extremities ($M_E$ and $M_R$ points) are generated according to the following relations:

$$
\begin{aligned}
x_E &= x_c + R\cos(\alpha_0 + k\,\Delta\alpha)\\
y_E &= y_c + R\sin(\alpha_0 + k\,\Delta\alpha)\\
x_R &= x_c + r\cos(\alpha_0 + k\,\Delta\alpha)\\
y_R &= y_c + r\sin(\alpha_0 + k\,\Delta\alpha)
\end{aligned}
\tag{4.2}
$$

where: $\Delta\alpha$ - the angular step of generation; $k$ – the sampling step of generation; $\overline{O_cM_E} = R$; $\overline{O_cM_R} = r$.

Using the Gradient Projection Method, the Control Points of the repulsive potential field are positioned in the 3-th and 4-th joint of manipulator. For the Extended Jacobian Method, the objective function is the sum of the inverses of the distances between the 3-th and 4-th joint of manipulator and the restriction surface.

The Gradient Projection and Extended Jacobian methods used to simulate the kinematic behaviour of the given redundant manipulator offers the possibility to study the influence of some parameters on manipulator behaviour, as follows: $\eta$ - the positive scaling factor and $\rho_0$ - the distance of influence of repulsive potential field and the imposed error $\varepsilon_d$, respectively.

The interpretation of computer simulation results is made considering the following qualitative aspects:

- the manipulator links should not intersect the restriction surface for a complete $360^0$ tracking of its contour;
- the joint coordinates should have small variations;
- for this case study, the direction of contour tracking is imposed by the chosen initial position.

To achieve this qualitative requirements one uses computer simulation for various values of the input parameters. The simulations were realized using Matlab R6 program and the results are illustrated in the Fig. 4.2 follow as: using gradient projection method with potential repulsive filed (first figure) and using Extended Jacobian method (second figure).

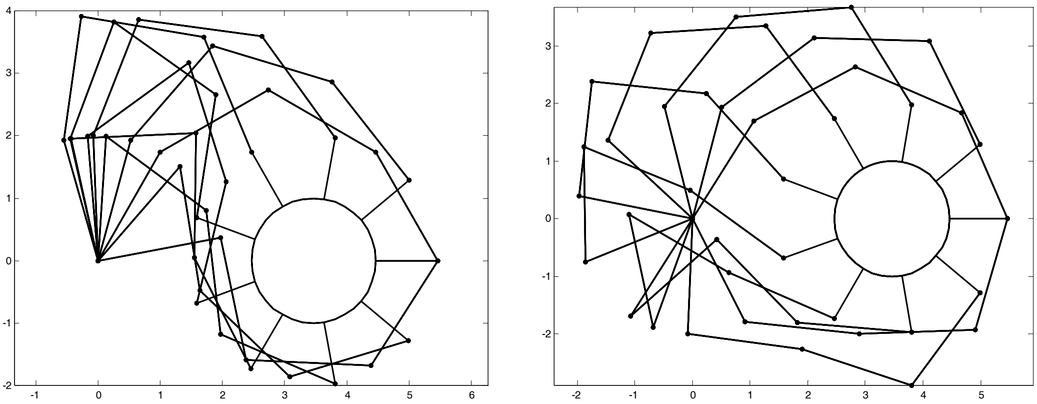

Fig. 4.2. Computer simulations

## 5. CONCLUSIONS

One advantage of GPM is that works in real time. Computational time required is sensible smaller than using the EJM that requires expensive computational resources because of increasing Jacobian dimension and of a greater number of iterations necessary for optimal solution identification. The most important advantages of the EJM are the idea having a square Jacobian matrix (thus, the use of a generalized inverse such as Moore-Penrose pseudoinverse is eliminated) and the possibility of an imposed error value selection (which can guarantee good end-effector task accuracy while obstacle avoidance is certainly accomplished). In GPM, choice of $\eta$ and $\rho_0$ parameters is difficult and don't guarantees optimal solution identification. The most common disadvantages of both methods consist in difficult constraint expressions choosing (criteria must be differentiable and have complicated expressions in symbolic forms) and in algorithmic singularities introduced by these additional constraints. These two disadvantages are eliminated by new redundancy resolution approaches, which consist in direct search techniques, neural networks or genetic algorithms. However, these methods require expensive computational resources and not deal with real-time applications.

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