THE SOLUTION TO W. MAY’S PROBLEM FOR ISOMORPHISM OF COMMUTATIVE GROUP ALGEBRAS OF MIXED GROUPS OVER FINITE FIELDS

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Suppose $G$ is an abelian group whose $p$-component $G_p$ is totally projective (or more generally, is an $S$-group) and $F_p$ is the simple field of $p$-elements. It is shown that the $F_p$-isomorphism $F_pH \cong F_pG$ for any group $H$ implies $H_p \cong G_p$. This almost completely solves a problem of Warren May (1988) from the *Proceedings of the American Mathematical Society* posed, however, for an arbitrary field $F$ with $\text{char}(F) = p > 0$. In particular, $H \cong G$ provided $G$ is a $p$-mixed Warfield group of torsion-free rank one. The last consequence enlarges in some aspect a result of May (1988) published in the same *Proceedings* for the case when $G$ is a $p$-local Warfield group.

Key words: group algebras, isomorphisms, $\sigma$-summable and totally projective groups.

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1. INTRODUCTION

The determinative aim of this research paper is to check whether or not the total projectivity of the $p$-primary component of an arbitrary abelian group is an invariant property for the commutative modular group algebra of characteristic $p$, that is a well-known significant problem due to May [20]. In the sequel, we shall show that this question can be settled positively for group algebras over the finite modular field $F_p$ of $\text{char}(F_p) = p$.

We remember that, as usual, $G$ is an abelian group with $p$-torsion component $G_p$ and socle $G[p]$, and $F_pG$ is its group algebra over $F_p$. For such an algebra $F_pG$, the letter $S(F_pG)$ denotes the normalized Sylow $p$-group in $F_pG$ and $I(F_pG; C)$ denotes the relative augmentation ideal of $F_pG$ with respect to the subgroup $C$ of $G$. To facilitate the expressions, we designate $1 + I(F_pG; C)$ as $S(F_pG; C)$ whenever $C$ is a $p$-group. All other notations and the terminology not explicitly defined herein are standard and follow essentially the cited in the bibliography our articles and the classical books [13]. For instance, the symbol $\prod$ will always denote a restricted (= bounded) direct product, which product is termed a coproduct too.

In [2]-[7], we have studied conjectures in this branch and also partially solved the aforementioned May’s problem under some additional restrictions either on the coefficient field or on the group basis. More precisely, in [4], [9] and [12] we have used the technique of so-called *summable groups*. However, more helpful is the method by using *$\sigma$-summable groups* (see, for example, [2] and [7]) because of the fact that they are of arbitrary lengths co-final with $\omega$ while the summable groups have lengths not exceeding $\omega_1$ (cf. [13]).

2. MAIN RESULTS

The following two lemmas are well-known in the abstract group theory but, for completeness of the exposition, we include them as a folklore (for instance, see [13]).
First Lemma. Assume that $A$ is an abelian $p$-group and $N$ is its subgroup. Then, $\forall n \geq 1$, $N \cap A^n = N^p$, that is $N$ is pure in $A$, if and only if $N[p] \cap A^n = N^p[p]$. 

Remark. In the general case, when $N$ is not a neat subgroup of $A$, the intersection $N[p] \cap A^n = N^p[p]$ does not imply that $N \cap A^n = N^p$ as some simple examples show when $N^p \neq 1$.


Main Lemma. For any abelian group $A$ the equalities $A[p] = \prod_{i \in I} A_i[p]$ and $A^m[p] = \prod_{i \in I} A_i^m[p]$ hold for all natural numbers $m \geq 1$ and some subgroups $A_i \leq A \Leftrightarrow A_i \cap A = \prod_{i \in I} A_i$.

Proof. Write $A[p] = \prod_{i \in I} A_i[p] = (\prod_{i \in I} A_i)[p]$. We shall argue below that $\prod_{i \in I} A_i$ is pure in $A$. For this purpose, it is enough to see by the First Lemma that the following identities hold: $A^m[p] = A[p] \cap A^m[p] = (\prod_{i \in I} A_i)[p] \cap A^m[p] = \prod_{i \in I} A_i^m[p]$.

Third Lemma. Suppose $M$ and $N$ are subgroups of an abelian $p$-group $A$. Then $M$ is $N$-high in $A$, that is $M$ is maximal with respect to $M \cap N = 1$, if and only if $M$ is neat in $A$ and $A[p] = M[p] \times N[p]$.

Before proving our central statement that motivates the present exploration, we need one crucial proposition.

Main Proposition. Let $G_p = \prod_{i \in I} G_i$. Then $F_p H \cong F_p G$ as $F_p$-algebras for some group $H$ yields that $H_p = \prod_{i \in I} H_i$, where $length(G_i) = length(H_i)$.

Proof. Write down $G[p] = \prod_{i \in I} G_i[p]$. But, by a fundamental result of Beers-Richman-Walker ([1]), the $F_p$-isomorphism of group algebras $F_p G$ and $F_p H$ insures an isometry $\varphi$, which is a height-preserving isomorphism, between the socles $G[p]$ and $H[p]$ with heights as computed in $G$ and $H$ respectively. Hence we deduce that $H[p] = \prod_{i \in I} A_i[p]$, where $A_i$ are subgroups of $H_p$ naturally constructed in the following manner: For each $i \in I$, we set $A_i = \{h_i \in H_i \mid 1 = \varphi^i(h_i^p) \in G_i[p] \text{ for some } s \in N \cup \{0\} \text{ and } < h_i^p > \cap (h_i^p < h_i^p >) = 1 \text{ running over each other generating } h_i^p \neq h_i >$. Henceforth, $\varphi^i(h_i^p) = \varphi^j(h_j^p)$ otherwise $h_i^p = h_j^p$ implies $h_i^p = 1 = h_j^p$, whence $1 = \varphi^i(h_i^p) = \varphi^j(h_j^p)$ which is against our choice. So, in more precise words, $A_i$ is generated via such distinct elements of $H_i$ whose nonidentity images under the action of $\varphi^i$ lie in $G_i[p]$ and such that the images of $p$-degrees of arbitrary products between the different generating members are different as well. For instance, $u_{i_1}^{\alpha_1} \cdots u_{i_s}^{\alpha_s} \neq 1$ and $v_{j_1}^{\alpha_1} \cdots v_{j_t}^{\alpha_t} \neq 1$ must have different nonidentity images in $G[p]$ by the action of the isomorphism $\varphi^i$. Thereby, $u_{i_1}^{\alpha_1} \cdots u_{i_s}^{\alpha_s} = v_{j_1}^{\alpha_1} \cdots v_{j_t}^{\alpha_t}$ implies that $u_{i_1} = \cdots = u_{i_s} = v_1^{\alpha_1} = \cdots = v_{j_t}^{\alpha_t} = 1$.

Thus we observe that $A_i \subseteq H_p$ and $A_i[p] = \{a_i \in A_i \mid \varphi^i(a_i) \in G_i[p]\}$. In fact, if $a_i \in A_i[p]$ it follows that $a_i = h_{i_1}^{\alpha_1} \cdots h_{i_k}^{\alpha_k}$ where $0 < \epsilon_m < order(h_m)$; $1 \leq m \leq k$. Since $(h_{i_1}^{\alpha_1}) \cdots (h_{i_k}^{\alpha_k}) = 1$, in accordance with our construction we find $(h_{i_1}^{\alpha_1}) = \cdots = (h_{i_k}^{\alpha_k}) = 1$. But by definition $\varphi^i(h_{i_1}^{\alpha_1}) \in G_i[p]$, $\cdots$, $\varphi^i(h_{i_k}^{\alpha_k}) \in G_i[p]$, so $\varphi^i(a_i) = \varphi^i(h_{i_1}^{\alpha_1}) \cdots \varphi^i(h_{i_k}^{\alpha_k}) = \varphi^i(e_{i_1}^{\alpha_1} \cdots e_{i_k}^{\alpha_k}) \\G_i[p]$. Moreover, $A_i[p] = \prod_{i \in I} H_i[p] | 1 \neq \varphi^i(h_{i}^{\alpha_i+1}) \in G_i[p]$ for some $s \in N \cup \{0\}$ and $< h_{i}^{\alpha_i+1} > \cap (\prod_{i \in I} H_i[p] < h_{i}^{\alpha_i+1} >) = 1 \text{ running over each other generating } h_{i}^{\alpha_i+1} \neq h_{i}^{\alpha_i} >$, hence as in the preceding situation $A_i[p] = \{c_i \in A_i[p] \mid \#^i(c_i) \\G_i[p]\}$ for every natural number $n$.

Furthermore, take $g_i \in G_i$. Then we have $g_{i}^{\alpha_i} = 1$ for some natural $t_i$, hence $g_{i}^{t_i-1} \in G_i[p]$. But $\varphi(g_{i}^{t_i-1}) = h_{i}^{t_i-1}$ for some $h_i \in H_i$ because $\varphi$ as an isometric map preserves the heights. That is why $h_{i}^{t_i-1} \in A_i[p]$. Consequently, $\varphi(G_i[p]) = A_i[p]$ and $\varphi(G^m[p]) = \varphi(G[p] \cap G^m[p]) = \varphi(G[p]) \cup \varphi(G^m[p]) = A_i[p] \cap H(p)[p] = (A \cap H(p))[p]$. By what we have shown above, by using the definition, $\varphi(G^m[p]) = A_i^m[p]$ for all $n \in N$. Thus all $A_i$ are pure in $H_p$. 

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Besides, the groups $A_i$ are independent for every subscript $i \in I$, namely $A_i \cap (\prod_{j \neq i} A_j) = 1$. To confirm this, we observe that $G_i \cap (\prod_{j \neq i} G_j) = 1 \iff G_i[p] \cap (\prod_{j \neq i} G_j[p]) = 1$ and taking in both sides the homomorphism $\varphi$, we are done.

Another approach to select certain pure subgroups is like this. We claim that $H[p] = \prod_{i \in I} H_i[p]$, where every subgroup $H_i \leq H_p$ is pure in $H$. In fact, by what we have just argued above, $H[p] = A_1[p] \times \prod_{i \in I \setminus \{1\}} A_i[p]$, where $A_1 \cap \prod_{i \in I \setminus \{1\}} A_i = 1$. But $A_1 \leq H_i$, where $H_i \leq H_p$ is maximal with respect to the last intersection property. Therefore $H[p] = H_i[p] \times \prod_{i \in I \setminus \{1\}} A_i[p]$, whence $A_i[p] = H_i[p]$. By the same token, $A_2 \cap \prod_{i \in I \setminus \{2\}} A_i = 1$ hence it is routine to verify that $A_2 \cap (\prod_{i \in I \setminus \{2\}} A_i \times H_1) = 1$. Thus $A_2 \leq H_2$ with $A_2[p] = H_2[p]$, where $H_2 \leq H_p$ is maximal with respect to this intersection, etc.

We wish to indicate as in the previous approach that $\varphi(G_i[p]) = H_i[p]$ for each $n \in N_0 = N \cup \{0\}$ such that $G_i[p] \neq G[p]$ and $H_i[p] \neq H[p]$, since otherwise the Second Lemma will assure that $G_i = G$ and $H_i = H$, a contradiction.

Now, we shall prove that $\prod_{i \in I} A_i = \prod_{i \in I} H_i$ will generate $H_p$. Indeed, $G[p] = \prod_{i \in I} G_i[p]$ and $G'[p] = \prod_{i \in I} G_i'[p]$ for each positive integer $n$. Thus, by what we have already shown above, $\varphi(G[p]) = H[p] = \prod_{i \in I} H_i[p] = \prod_{i \in I} \varphi(G_i[p]) = \varphi(\prod_{i \in I} G_i[p])$, and moreover $H'[p] = \varphi(G'[p]) = \varphi(\prod_{i \in I} G_i'[p]) = \prod_{i \in I} \varphi(G_i'[p]) = \prod_{i \in I} H_i'[p]$. We therefore have the necessary relations to arrive at the Main Lemma that $H_p = \prod_{i \in I} H_i$. The proof is complete.

Example: The conditions on the intersection cannot be removed, since otherwise for some fixed $h_i \in A_i$, of order($h_i$) > $p$ and for each $h \in H[p]$ it follows that $hh_i \in A_i$, hence $h \in A_i$ that gives $H[p] \subseteq A_i$ and which is impossible.

Moreover, \(<h_i> \cap \prod_{h \neq h_i} <h'\> = <h_i> \cap <h'_1, ..., > \neq 1$ in general, since otherwise we should have a direct sum of cyclic groups. For example, if $p = 5$, $h_i^3 = h'_3 \neq 1$ may occur.

Now we are ready to attack the main attainment.

Main Theorem. Suppose $G$ is an abelian group for which $G_n$ is an $A$-group (in particular, is an S-group or an N-group). Then $F_p H \simeq F_p G$ as $F_p$-algebras for any group $H$ implies $H_p \simeq G_p$.

Proof. Complying with the Main Proposition in [2] along with [21], we consider $G_p$ as a reduced group.

First of all, we assume that $G_p$ is totally projective. We note that it suffices to establish only that $H_p$ is totally projective because, by virtue of [21], the Ulm-Kaplansky invariants of $G_p$ and $H_p$ can be determined from the group algebra $F_p G = F_p H$. Whence, owing to [13], $G_p \simeq H_p$ as stated. In order to do this, we will further distinguish the following two basic cases:

Case 1 - length($G_p$) is limit. Thus it follows from [13] that $G_p = \prod_{i \in I} G_i$, where length($G_i$) < length($G_p$) = $\lambda$ and all $G_i$ are totally projective. Applying the Main Proposition, we infer that $H_p = \prod_{i \in I} H_i$ where length($H_i$) = length($G_i$) < length($G_p$) = length($H_p$) = $\lambda$. Next, we shall detect that all $H_i$ are totally projective by the usage of standard transfinite induction on the lengths and more especially on $\lambda$. Toward this end, we may assume that the theorem holds true for all ordinals $\alpha < \lambda$. Without loss of generality, we can presume also that all length($G_i$) are limit, consequently so are the lengths like length($H_i$). In fact, this is possible because if for some abelian $p$-group $A$ the ratio $A^{\alpha+n} = 1$ is fulfilled then $(A^{\alpha+n})^{\alpha} = 1$, and thus from [23] it follows obviously that $A$ is totally projective if and only if both is $A/A^{\alpha}$, where, of course, $\alpha$ is limit. Moreover, we may restrict our attention on cofinal with $\omega$ ordinals since if $\beta$ is an arbitrary ordinal, then $\beta + \omega$ is always cofinal with $\omega$. Besides, we assume that length($H_i$) + $\omega \leq \lambda$ (notice that if length($H_i$) = length($G_i$) are not cofinal with $\omega$, they however are limit and because $G_i$ are totally projective then $G_i = \prod_{j \in I} A_j$, where eventually length($A_j$) are cofinal with $\omega$ etc.). And so, such a component $H_i$ will be totally projective by a
criterion due to Linton-Megginson (e.g. [2] or [3]) if and only if \( H/H_p^{p'} \) is \( \sigma \)-summable for all cofinal with \( \omega \) ordinals \( \gamma \leq \lambda \) and \( H/H_p^{p''} \) is totally projective for all other \( \gamma < \lambda \). But by application of the Main Proposition in [1], \( F_pH \cong F_pG \) guarantees that \( F_p(G/G_p^{p'}) \cong F_p(H/H_p^{p''}) \). Thus since \( G_p^{p}G_p^{p''} = (G/G_p^{p''})_p \) is totally projective, by virtue of the induction hypothesis \( H_p/H_p^{p''} = (H/H_p^{p''})_p \) is totally projective as well. Therefore the same conclusion is valid by [13] even for \( H/H_p^{p''} \) as its direct factor because of the canonical isomorphism \( H_p/H_p^{p''} \cong \prod_{i \in I} H_i/H_i^{p''} \). In this way, \( G_p^{p}G_p^{p''} \) is totally projective hence \( \sigma \)-summable for each cofinal with \( \omega \) ordinal \( \gamma \leq \lambda \), the \( \sigma \)-summability for \( H_p/H_p^{p''} \) holds referring to [2]. But, for each \( i \in I \), the factor-groups \( H_i/H_i^{p''} \) may be regarded as subgroups of \( H_p/H_p^{p''} \) with equal lengths whenever \( \gamma \leq \text{length}(H_i) < \lambda = \text{length}(H_p) \), whence from [2] they are trivially \( \sigma \)-summable groups. Finally, by what we have foregoing illustrated, we conclude that all \( H_i \) are totally projective and thus their coproduct \( H_p \) must be itself totally projective too by exploiting [13]. So, the first half is completed.

**Case 2 – length\((G_p)\) is nonlimit.** Write \( \text{length}(G_p) = \lambda + n \), where \( \lambda \) is limit and \( n \) is natural. Hence \( (G_p^{p})^{p^n} = I \), and on the other hand as we have seen above \( F_pG^{p^n} \cong F_pHp^{p^n} \) and \( F_p(G/G_p^{p^n}) \cong F_p(H/H_p^{p^n}) \) where \( G_p^{p} \) and \( G_p^{p^n} \) are both totally projective. Invoking standard facts and Case 1, we infer that \( H_p^{p^n} \) and \( H_p^{p''} \) are also totally projective, hence so is \( H_p \). See [23] or [13], thus completing the second part as well.

Let us now choose \( G_p \) to be a \( \mu \) - elementary \( A \)-group for some arbitrary but fixed ordinal \( \mu \) (see, for instance, [15] or [16]) so that \( G_p \) is not totally projective. Suppose \( G_p \) is an isotype and almost nice subgroup (hence an almost balanced subgroup) of the totally projective \( p \)-group \( T \). We next consider the outer direct products \( G \times T \) and \( H \times T \). Thus \( F_p(G \times T) = (F_pG)T = (F_pH)T \), whence \( S(F_pG) = (F_pH)T \) and \( S(F_p(G \times T); T) = S(F_p(H \times T); T) \). Consulting with [D1] we have \( S(F_pG) \cap S^{\alpha}(F_p(H \times T); T) = S(F_p(G \times T); T) = S(F_pG) \cap S(F_p(G^{p^n} \times T^{p^{n+1}}); T^{p^{n+1}}) = S(F_p(G^{p^n}); G^{p^n}) = S^{p^n}(G^{p^n}; G^{p^n}) = S^{p^n}(F_pG) = S^{p^n}(F_pH) \) for each ordinal \( \alpha \).

Therefore \( S(F_p(G \times T); T) \). Moreover, \( G_p \) is almost balanced in \( T \) and thus, employing [7], we obtain that \( T \) is balanced in \( S(F_p(G \times T); T) \). Hence from [13] it follows at once that \( G_p \) is almost balanced in \( S(F_p(G \times T); T) \), and so \( S(F_pG) = S(F_pG; G_p) \) (cf. [2]) is almost balanced in \( S(F_p(G \times T); T) \). Therefore \( S(F_pH) \) is almost balanced in \( S(F_p(H \times T); T) \). But it is well-known that \( H_p \) is balanced in \( S(F_pH) \), so \( H_p \) is almost balanced in \( S(F_p(H \times T); T) \) in view of [13]. On the other hand, \( S(F_p(G \times T); T) \) is totally projective by virtue of [3]. Consequently \( (G_p; S(F_p(G \times T); T)) \) and \( (H_p; S(F_p(H \times T); T)) \) are pairs of almost balanced subgroups of totally projective \( p \)-groups. Denote \( A = S(F_p(G \times T); T) = S(F_p(H \times T); T) \). After this, we examine the outer direct products \( G \times A \) and \( H \times A \). Clearly, \( F_p(G \times A) = (F_pG)A = (F_pH)A = F_p(H \times A) \). Besides, conforming with [D1], \( I(F_p(G \times A); G_p) = F_p(G \times A).I(F_pG; G_p) = F_p(G \times A).I(F_p(G \times A); G_p) = F_p(G \times A).I(F_pG; G_p) = F_p(G \times A) \).

We conclude that \( F_p(G \times A)/A(G_p) = G_p \times G_p = A \) and \( G_p \cong A \). So, by what we have shown in the totally projective step, we derive \( A/G_p \cong A/H_p \). Finally, [15] and [16] ensure that \( G_p \cong H_p \), as desired. Thus, we extract that \( H_p \) is also a \( \mu \)-elementary \( A \)-group.

To end the proof, let \( G_p \) be an arbitrary \( A \)-group. Then, by definition (cf. [15] or [16]), \( G_p \) is a direct sum of \( \mu \)-elementary \( A \)-groups. Furthermore, the Main Proposition plus the ideas of the above steps and some well-known standard group-theoretic arguments from [15] lead us to \( H_p \) is a direct sum of \( \mu \)-elementary \( A \)-groups, hence it is an \( A \)-group. On the other hand, we wish to employ ([15] and [16]) or ([11], Theorems 4.2 and 4.3) or [14] to infer that \( G_p \) and \( H_p \) must be isomorphic, so finishing this point. The proof is concluded in all generality.

**Main Corollary.** Suppose \( G \) is a global Warfield group (in particular, is a mixed simply presented group). Then the \( F_p \)-isomorphism \( F_pH \cong F_pG \) for some another group \( H \) yields that \( H_p \cong G_p \).

**Proof.** Since by making use of the technicalities in [17] and [18] the group \( G_p \) is an \( S \)-group, the statement follows according to the main attainment above. ♦

The following is important.
Main Remark. The formulated main assertion almost settles a question due to May raised in his paper [20] appeared in Proceedings of the AMS on 1988. The restriction is only on the coefficient field to be of finite power, but it is necessary for the proof of the Main Proposition about the decomposition and cannot be ignored yet. The case for an arbitrary cardinality of the field is examined in [6]. In this way, we formulate an actual problem.

Main Question. Does there exist a simple field $F_p$ of characteristic $p \neq 0$ such that for any field $K$ with characteristic $p > 0$ and any two arbitrary abelian groups $G$ and $H$, the $K$-isomorphism $KG \cong KH$ forces the $F_p$-isomorphism $F_pG \cong F_pH$? If yes, the May’s problem will be completely resolved.

Commentary. Adapting in our variant an analogous technique to that described in [25] and its generalization due to [26]-[17], together with some other standard group-theoretic arguments, one can conclude that the result of the type of Wallace-Warfield-Hunter may be extended for the classes of $S$-groups and $N$-groups (or more generally for $A$-groups), but this is a theme of some new research investigation.

The other valuable statement, which is an immediate consequence of the Main Theorem, states as follows:

**Theorem.** Let $G$ be a $p$-mixed abelian group of torsion-free rank one whose $G_p$ is an $A$-group (in particular, is an $S$-group or an $N$-group). Then $F_pH \cong F_pG$ as $F_p$-algebras for some group $H$ if and only if $H \cong G$.

**Proof.** It follows automatically combining the Main Theorem and the method illustrated by us in [3], by utilizing the central result of [25] plus the above commentary as well. The proof is finished.

**First Corollary.** Let $G$ be a $p$-mixed Warfield group (or, in particular, a $p$-mixed simply presented group) of torsion-free rank one. Then $F_pH \cong F_pG$ are $F_p$-isomorphic algebras for some group $H$ if and only if $H \cong G$.

**Proof.** Invoking the methods in [17] and [18] we deduce that $G_p$ is an $S$-group, whence the last affirmation is applicable, and we are done.

**Second Corollary.** Let $G$ be a $p$-primary $A$-group (in particular, an $S$-group or an $N$-group). Then $F_pH \cong F_pG$ as $F_p$-algebras for any group $H$ if and only if $H \cong G$.

**Remark.** The first consequence may be improved by dropping off the conditions on the torsion-free rank of $G$ as well as on the simplicity of the field and by using a more direct approach in the proof (e.g. [10] and [11]). The second consequence strengthens a similar result of Beers-Richman-Walker ([1], Theorem 4.3) when $G$ and $H$ are both $S$-groups. Other result in this aspect for the $p$-torsion case was obtained also in [24].

**CONCLUDING DISCUSSION**

The main observations that immediately arise are that the Main Theorem will be valid for an arbitrary coefficient field $K$ of characteristic $p \neq 0$ if the Main Proposition holds for such a ring, and as well that the same will be true for the other results of this type listed above.

Besides, whether adopting the methods described in this paper we may generalize the results obtained by us for direct sums of mixed groups of the present sorts each of which has torsion-free rank one. For example, every mixed simply presented group is the direct sum of simply presented subgroups of torsion-free rank one.

A question of some importance is of whether the major statement remains true for more general classes of abelian $p$-groups such as: the class of all $IT$-groups introduced by Hill-Megibben in [HM] or the class of all totally Zippin $p$-groups explored in [Me]. Following our scheme of proof, this is probably so.
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