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ON A MECHANICAL SYSTEM CONTAINING JOURNALS WITH CLEARANCES

Nicolae–Doru STĂNESCU

University of Pitești, Department of Applied Mechanics,
e-mail: s_doru@yahoo.com

In this paper we purpose a mechanical system consisting of an axle and two journals with clearances. The clearances are modelled each of them by an even number of identical springs. For this system we make a cinematic study, we describe the forces which appear due to the model of the clearances, we determine the equations of motion and we present the linear approximation.

Key words: clearances, motion, linear approximation

1. INTRODUCTION

In the present paper we purpose the study of a mechanical transmission consisting of an axle with two journals. The mathematical model is characterised by the existence of the clearances in both journals, such that the axle present a complex spatial motion.

From the practical point of view there exists no possibility to eliminate these clearances, and the elastic properties of these clearances can be modelled in very different ways.

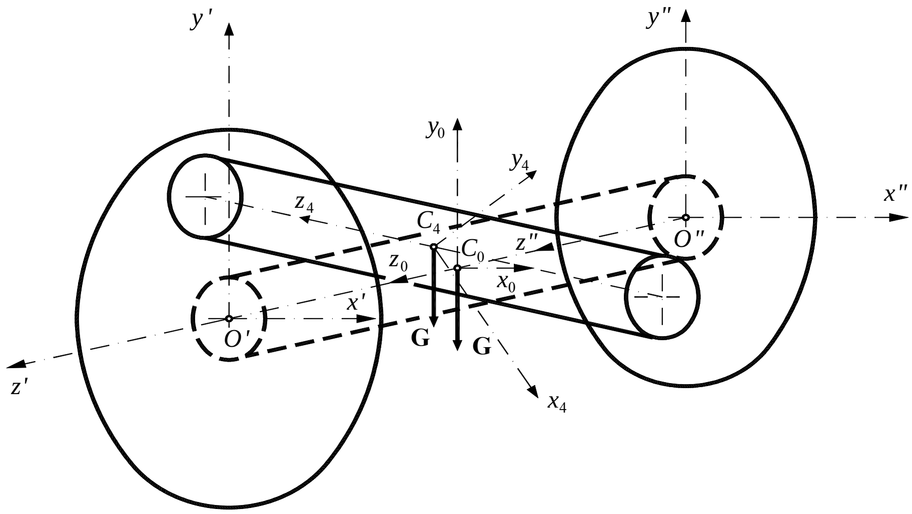

Fig. 1. The mathematical model.

The system purposed is presented in figure 1. the two journals are denoted by I and II, respectively. The radii of the shafts are R_{11} for the journal I, respectively R_{12} for the journal II, the radii of the rims being R_{21}, respectively R_{22}. The weight centre of the axle is C. The centres of the two journals are O', respectively O''. We attach to the two journals a fixed reference system $O'x'y'z'$ to the journal I, respectively a fixed reference system $O''x''y''z''$ to the journal II, with the origins in the corresponding journals, the axes $O'y'$, respectively $O''y''$ being vertical ascendant, the axes $O'x'$, respectively $O''x''$ in the horizontal direction, and the axes $O'z'$, respectively $O''z''$ in the direction of the line that connected the two centres of the journals. The axes $O'x'$ and $O'y'$ are situated in the plane of the journal I, and the axes $O''x''$ and $O''y''$ are situated in the plane of the journal II. For the horizontal position of the axle and with the symmetry axis passing through O' and O'' we denote by C_0 the position of axles' centre of weight. We attach to the axle the reference system $C_0x_0y_0z_0$ with the axes parallel to the system $O'x'y'z'$, respectively $O''x''y''z''$. Let us assume that the axes C_0x_0, C_0y_0, C_0z_0 are principal, central axes of inertia for the axle and that we know the moments of inertia J_x, J_y, and J_z with respect to these axes, respectively. We assume that are given the distances C_0O' and C_0O'', which define the position of the weight centre C_0 relative to the centres of the journals and the weight G of the axle.

2. THE CINEMATIC STUDY

Further on, we shall not consider the motion of rotation of the axle about its longitudinal axis because this motion is imposed by its acted forces.

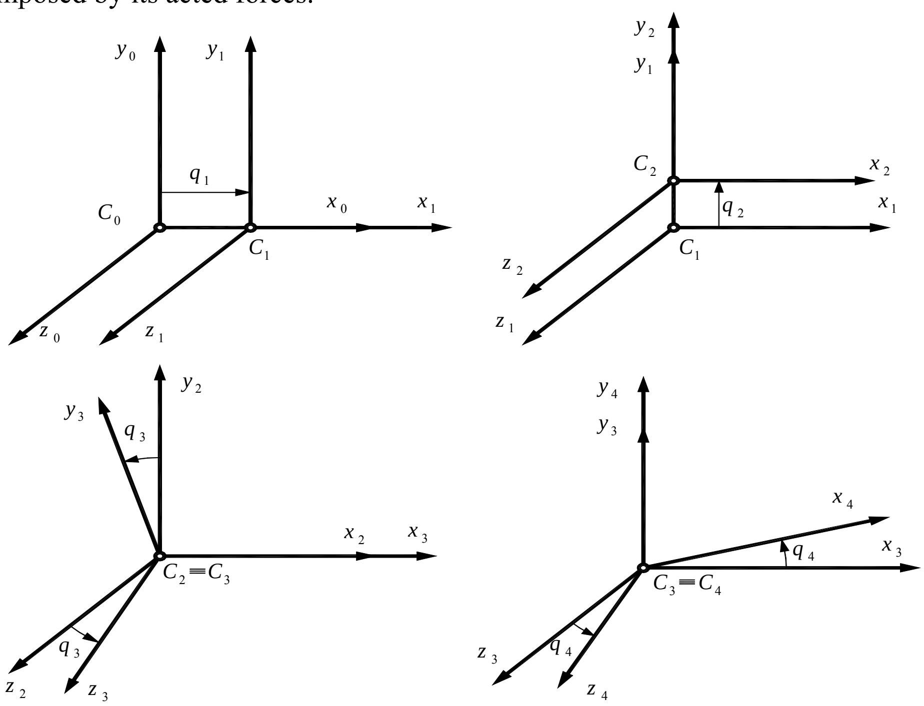

Fig. 2. The passing from the fixed reference system to the reference system linked to the axle.

Due to the clearances in the journals the system has four degrees of freedom, name them: a translation along the axis x, a translation along the axis y, a rotation about the axis x and a rotation about the axis y. After these displacements the centre of weight arrives from the position C_0 to the position C_4. The four displacements are described in figure 2, the passing relations being

$$x_1 = x_0 - q_1,\ y_1 = y_0,\ z_1 = z_0 \tag{1}$$

$$x_2 = x_1,\ y_2 = y_1 - q_2,\ z_2 = z_1 \tag{2}$$

$$x_3 = x_2,\ y_3 = y_2 \cos q_3 + z_2 \sin q_3,\ z_3 = -y_2 \sin q_3 + z_2 \cos q_3, \tag{3}$$

$$x_4 = -z_3 \sin q_4 + x_3 \cos q_4,\ y_4 = y_3,\ z_4 = z_3 \cos q_4 + x_3 \sin q_4. \tag{4}$$

The vector angular velocity $\boldsymbol{\omega}$ reads

$$\boldsymbol{\omega} = \dot{q}_3 \mathbf{i}_3 + \dot{q}_4 \mathbf{j}_4 . \tag{5}$$

From the relations (4) we obtain

$$x_3 = x_4 \cos q_4 + z_4 \sin q_4 , \tag{6}$$

the expression (5) writing now

$$\boldsymbol{\omega} = \dot{q}_3 \cos q_4 \mathbf{i}_4 + \dot{q}_4 \mathbf{j}_4 + \dot{q}_3 \sin q_4 \mathbf{k}_4 . \tag{7}$$

3. THE FORCES IN THE JOURNALS

We model the elastic behaviours of the journals using n_1, respectively n_2, linear springs, where both n_1 and n_2 are even numbers. The springs in each journal are identical, disposed at equal angles, counted started from the spring situated on the axis x, the first spring having the index 0, the last having the index $n_i - 1$, $i = 1, 2$, and their stiffness are k_1, respectively k_2.

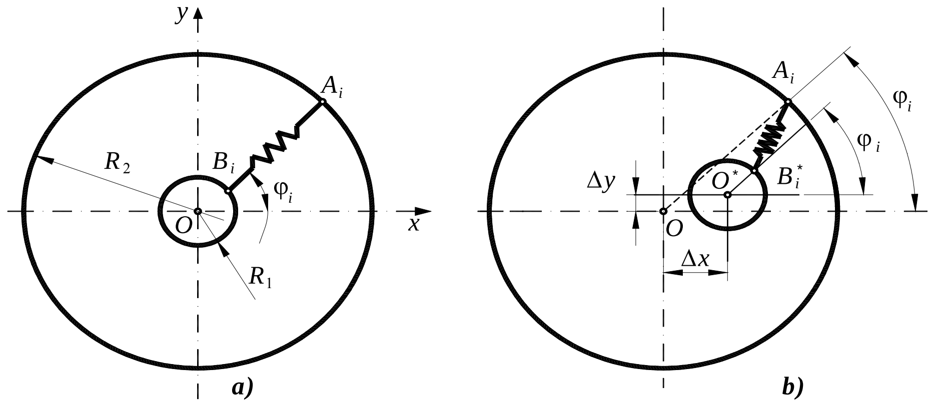

Fig. 3. The modelling of the journals.

In figure 3 is captured the created situation, where the indices 1 and 2 were eliminated, considering a generic journal. If the shaft and the rim are concentric we have the situation from figure 3, *a*, i being the index of the considered spring for which

$$\varphi_i = 2\pi \frac{i}{n} . \tag{8}$$

The points B_i and A_i mark the linking points of the spring to the shaft, respectively to the rim.

After the displacements of the axle, the centre of the shaft arrives in the point O^*, suffering the displacements Δx and Δy along the axes Ox, respectively Oy. In this way the point B_i arrives in the point B_i^*, where we made the hypothesis that the system remains plane.

Denoting by R_1 the radius of the shaft and by R_2 the radius of the rim, we have the geometrical relations

$$x_{A_i} = R_2 \cos \varphi_i , \; y_{A_i} = R_2 \sin \varphi_i , \; x_{B_i^*} = \Delta x + R_1 \cos \varphi_i , \; y_{B_i^*} = \Delta y + R_1 \sin \varphi_i , \tag{9}$$

so that one obtains

$$A_i B_i^* = \sqrt{\left[\Delta x + \left(R_1 - R_2\right) \cos \varphi_i\right]^2 + \left[\Delta y + \left(R_1 - R_2\right) \sin \varphi_i\right]^2} . \tag{10}$$

Let us also denote by l_0 the length of the non-deformed springs, resulting the force in the spring i in the form

$$\mathbf{F}_i = k\left(l_0 - A_i B_i^*\right)\frac{\overrightarrow{A_i B_i^*}}{A_i B_i^*}. \tag{11}$$

The n forces give us a resultant in O^*,

$$\mathbf{R} = \sum_{i=0}^{n-1}\mathbf{F}_i . \tag{12}$$

Obviously the forces $\mathbf{F}_i$ give also a moment relative to the point O^*, but how we do not consider the rotation about the axis z, this moment has no importance for the next calculations.

4. THE EQUATIONS OF MOTION

The theorem of the momentum offers us the relations

$$\frac{G}{g}\ddot{q}_1 = \sum F_x = R_{1x} + R_{2x}, \tag{13}$$

$$\frac{G}{g}\ddot{q}_2 = \sum F_y = R_{1y} + R_{2y} - G, \tag{14}$$

in which R_{1x}, R_{2x}, R_{1y}, R_{2y} are the components of the resultants of the forces in the journals on the axes Ox, respectively Oy, and g is the gravitational acceleration.

The theorem of the moment of momentum written relative to the weight centre gives us

$$\dot{\mathbf{K}}_{C_4} = \sum \mathbf{M}_{C_4}, \tag{15}$$

where $\sum \mathbf{M}_{C_4}$ is the sum of the all moments of forces relative to the point C_4. In fact,

$$\sum \mathbf{M}_{C_4} = L_1\mathbf{k}_4 \times \mathbf{R}_1 - L_2\mathbf{k}_4 \times \mathbf{R}_2, \tag{16}$$

in which L_1 and L_2 are the dimensions which give the position of the centre of weight of the axle relative to the two journals, and $\mathbf{R}_1$, respectively $\mathbf{R}_2$ are the reactions in the journals.

On the other hand,

$$\begin{aligned}\mathbf{K}_{C_4} &= J_x\omega_x\mathbf{i}_4 + J_y\omega_y\mathbf{j}_4 + J_z\omega_z\mathbf{k}_4 \\ &= J_x\dot{q}_3\cos q_4\mathbf{i}_4 + J_y\dot{q}_4\mathbf{j}_4 + J_z\dot{q}_3\sin q_4\mathbf{k}_4,\end{aligned} \tag{17}$$

where we kept into account the formula (7).

From the Poisson formulas we have

$$\begin{aligned}\dot{\mathbf{i}}_4 &= \boldsymbol{\omega}\times\mathbf{i}_4 = \begin{vmatrix}\mathbf{i}_4 & \mathbf{j}_4 & \mathbf{k}_4\\ \dot{q}_3\cos q_4 & \dot{q}_4 & \dot{q}_3\sin q_4\\ 1 & 0 & 0\end{vmatrix} = \dot{q}_3\sin q_4\mathbf{j}_4 - \dot{q}_4\mathbf{k}_4,\\ \dot{\mathbf{j}}_4 &= \boldsymbol{\omega}\times\mathbf{j}_4 = \begin{vmatrix}\mathbf{i}_4 & \mathbf{j}_4 & \mathbf{k}_4\\ \dot{q}_3\cos q_4 & \dot{q}_4 & \dot{q}_3\sin q_4\\ 0 & 1 & 0\end{vmatrix} = \dot{q}_3\sin q_4\mathbf{i}_4 - \dot{q}_3\cos q_4\mathbf{k}_4,\end{aligned} \tag{18}$$

$$\dot{\mathbf{k}}_4 = \boldsymbol{\omega} \times \mathbf{k}_4 = \begin{vmatrix} \mathbf{i}_4 & \mathbf{j}_4 & \mathbf{k}_4 \\ \dot{q}_3 \cos q_4 & \dot{q}_4 & \dot{q}_3 \sin q_4 \\ 0 & 0 & 1 \end{vmatrix} = \dot{q}_4 \mathbf{i}_4 - \dot{q}_3 \sin q_4 \mathbf{j}_4 .$$

By derivation, the relation (17) leads us to

$$\begin{aligned} \dot{\mathbf{K}}_{C_4} &= \left(J_x \ddot{q}_3 \cos q_4 - J_x \dot{q}_3 \dot{q}_4 \sin q_4 - J_y \dot{q}_3 \dot{q}_4 \sin q_4 + J_z \dot{q}_3 \dot{q}_4 \sin q_4\right)\mathbf{i}_4 \\ &+ \left(J_x \dot{q}_3^2 \sin q_4 \cos q_4 + J_y \ddot{q}_4 - J_z \dot{q}_3^2 \sin q_4 \cos q_4\right)\mathbf{j}_4 \\ &+ \left(- -J_x \dot{q}_3 \dot{q}_4 \cos q_4 + J_y \dot{q}_3 \dot{q}_4 \cos q_4 + J_z \ddot{q}_3 \sin q_4 + J_z \dot{q}_3 \dot{q}_4 \sin q_4\right)\mathbf{k}_4 . \end{aligned} \tag{19}$$

From the expressions (1)-(4) we find

$$\begin{aligned} \begin{pmatrix} x_4 \\ y_4 \\ z_4 \end{pmatrix} &= \begin{bmatrix} \cos q_4 & 0 & -\sin q_4 \\ 0 & 1 & 0 \\ \sin q_4 & 0 & \cos q_4 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos q_3 & \sin q_3 \\ 0 & -\sin q_3 & \cos q_3 \end{bmatrix} \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix} \\ &= \begin{bmatrix} \cos q_4 & \sin q_3 \sin q_4 & -\cos q_3 \sin q_4 \\ 0 & \cos q_3 & \sin q_3 \\ \sin q_4 & -\sin q_3 \cos q_4 & \cos q_3 \cos q_4 \end{bmatrix} \left[\begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} - \begin{pmatrix} q_1 \\ q_2 \\ 0 \end{pmatrix} \right], \end{aligned} \tag{20}$$

wherefrom

$$\begin{aligned} \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} &= \begin{bmatrix} \cos q_4 & \sin q_3 \sin q_4 & -\cos q_3 \sin q_4 \\ 0 & \cos q_3 & \sin q_3 \\ \sin q_4 & -\sin q_3 \cos q_4 & \cos q_3 \cos q_4 \end{bmatrix}^{-1} \left[\begin{pmatrix} x_4 \\ y_4 \\ z_4 \end{pmatrix} + \begin{pmatrix} q_1 \\ q_2 \\ 0 \end{pmatrix} \right] \\ &= \begin{bmatrix} \cos q_4 & 0 & \sin q_4 \\ \sin q_3 \sin q_4 & \cos q_3 & -\sin q_3 \cos q_4 \\ -\cos q_3 \sin q_4 & \sin q_3 & \cos q_3 \cos q_4 \end{bmatrix} \left[\begin{pmatrix} x_4 \\ y_4 \\ z_4 \end{pmatrix} + \begin{pmatrix} q_1 \\ q_2 \\ 0 \end{pmatrix} \right]. \end{aligned} \tag{21}$$

Making now $x_4 = 0$, $y_4 = 0$, $z_4 = L_1$, respectively $x_4 = 0$, $y_4 = 0$, $z_4 = -L_2$, we find the displacements Δx_1, Δy_1, Δx_2, Δy_2 of the ends of the axle in the two journals,

$$\begin{aligned} \begin{pmatrix} \Delta x_1 \\ \Delta y_1 \\ \Delta z_1 \end{pmatrix} &= \begin{bmatrix} \cos q_4 & 0 & \sin q_4 \\ \sin q_3 \sin q_4 & \cos q_3 & -\sin q_3 \cos q_4 \\ -\cos q_3 \sin q_4 & \sin q_3 & \cos q_3 \cos q_4 \end{bmatrix} \begin{pmatrix} q_1 \\ q_2 \\ L_1 \end{pmatrix} \\ &= \begin{pmatrix} q_1 \cos q_4 + L_1 \sin q_4 \\ q_1 \sin q_3 \sin q_4 + q_2 \cos q_3 - L_1 \sin q_3 \cos q_4 \\ -q_1 \cos q_3 \sin q_4 + q_2 \sin q_3 + L_1 \cos q_3 \cos q_4 \end{pmatrix}, \end{aligned} \tag{22}$$

respectively

$$\begin{aligned} \begin{pmatrix} \Delta x_2 \\ \Delta y_2 \\ \Delta z_2 \end{pmatrix} &= \begin{bmatrix} \cos q_4 & 0 & \sin q_4 \\ \sin q_3 \sin q_4 & \cos q_3 & -\sin q_3 \cos q_4 \\ -\cos q_3 \sin q_4 & \sin q_3 & \cos q_3 \cos q_4 \end{bmatrix} \begin{pmatrix} q_1 \\ q_2 \\ -L_2 \end{pmatrix} \\ &= \begin{pmatrix} q_1 \cos q_4 - L_2 \sin q_4 \\ q_1 \sin q_3 \sin q_4 + q_2 \cos q_3 + L_2 \sin q_3 \cos q_4 \\ -q_1 \cos q_3 \sin q_4 + q_2 \sin q_3 - L_2 \cos q_3 \cos q_4 \end{pmatrix}. \end{aligned} \tag{23}$$

Knowing now the values Δx_1, Δy_1, Δx_2, Δy_2, with the aid of the formulas (10), (11) and (12), one determines the forces in the journals $\mathbf{R}_1$, r4espectively $\mathbf{R}_2$. The projections of these forces onto the directions of axes of the mobile system $C_4x_4y_4z_4$ are given by

$$\begin{pmatrix} R'_{ix} \\ R'_{iy} \\ R'_{iz} \end{pmatrix} = \begin{bmatrix} \cos q_4 & \sin q_3 \sin q_4 & -\cos q_3 \sin q_4 \\ 0 & \cos q_3 & \sin q_3 \\ \sin q_4 & -\sin q_3 \cos q_4 & \cos q_3 \cos q_4 \end{bmatrix} \begin{pmatrix} R_{ix} \\ R_{iy} \\ R_{iz} \end{pmatrix}, \quad (24)$$

in which $i = 1, 2$, and prime marks the projections onto the axes of the mobile system.

From the formula (16), it results now

$$\sum \mathbf{M}_{C_4} = \begin{vmatrix} \mathbf{i}_4 & \mathbf{j}_4 & \mathbf{k}_4 \\ 0 & 0 & L_1 \\ R'_{1x} & R'_{1y} & R'_{1z} \end{vmatrix} - \begin{vmatrix} \mathbf{i}_4 & \mathbf{j}_4 & \mathbf{k}_4 \\ 0 & 0 & L_2 \\ R'_{2x} & R'_{2y} & R'_{2z} \end{vmatrix} = \left(-L_1R'_{1y} + L_2R'_{2y}\right)\mathbf{i}_4 + \left(L_1R'_{1x} - L_2R'_{2x}\right)\mathbf{j}_4. \quad (25)$$

One deduces the last two equations of motion

$$J_x\ddot{q}_3 \cos q_4 - J_x\dot{q}_3\dot{q}_4 \sin q_4 - J_y\dot{q}_3\dot{q}_4 \sin q_4 + J_z\dot{q}_3\dot{q}_4 \sin q_4 = -L_1R'_{1y} + L_2R'_{2y}, \quad (26)$$

respectively

$$J_x\dot{q}_3^2 \sin q_4 \cos q_4 + J_y\ddot{q}_4 - J_z\dot{q}_3^2 \sin q_4 \cos q_4 = L_1R'_{1x} - L_2R'_{2x}. \quad (27)$$

The relations (13), (14), (26) and (27) form a system of four second order differential equations with the unknowns q_1, q_2, q_3 and q_4.

5. THE LINEAR APPROXIMATION

The expression (10) can be written in the form

$$\begin{aligned} A_iB_i^* &= \sqrt{(\Delta x)^2 + (\Delta y)^2 + (R_1 - R_2)^2 + 2\Delta x(R_1 - R_2)\cos\varphi_i + 2\Delta y(R_1 - R_2)\sin\varphi_i} \\ &\approx (R_2 - R_1)\sqrt{1 - \frac{2\Delta x\cos\varphi_i}{R_2 - R_1} - \frac{2\Delta y\sin\varphi_i}{R_2 - R_1}} \approx (R_2 - R_1)\left(1 - \frac{\Delta x\cos\varphi_i}{R_2 - R_1} - \frac{\Delta y\sin\varphi_i}{R_2 - R_1}\right), \end{aligned} \quad (28)$$

where $(\Delta x)^2 \approx 0$, $(\Delta y)^2 \approx 0$ and $\sqrt{1 - z} \approx 1 - \frac{z}{2}$ for z small.

From the formulas (9) we obtain

$$\overrightarrow{A_iB_i^*} = [\Delta x + (R_1 - R_2)\cos\varphi_i]\mathbf{i} + [\Delta y + (R_1 - R_2)\sin\varphi_i]\mathbf{j}. \quad (29)$$

On the other hand, we have

$$\frac{l_0}{A_iB_i^*} \approx \frac{l_0}{R_2 - R_1}\left(1 - \frac{\Delta x\cos\varphi_i}{R_2 - R_1} - \frac{\Delta y\sin\varphi_i}{R_2 - R_1}\right)^{-1} \approx \frac{l_0}{R_2 - R_1}\left(1 + \frac{\Delta x\cos\varphi_i}{R_2 - R_1} + \frac{\Delta y\sin\varphi_i}{R_2 - R_1}\right), \quad (30)$$

where $\frac{1}{1 - z} \approx 1 + z$ for z small.

The formula (11) reads now

$$\begin{aligned}\mathbf{F}_i &= k\overrightarrow{A_iB_i^*}\left(\frac{l_0}{A_iB_i^*}-1\right) \approx k\{[\Delta x+(R_1-R_2)\cos\varphi_i]\mathbf{i}+[\Delta y+(R_1-R_2)\sin\varphi_i]\mathbf{j}\}\\ &\times\left[\frac{l_0}{R_2-R_1}\left(1+\frac{\Delta x\cos\varphi_i}{R_2-R_1}+\frac{\Delta y\sin\varphi_i}{R_2-R_1}\right)-1\right]\\ &\approx k\left[\left(\Delta x\frac{l_0}{R_2-R_1}-\Delta x-l_0\cos\varphi_i\frac{\Delta x\cos\varphi_i}{R_2-R_1}-l_0\cos\varphi_i\frac{\Delta y\sin\varphi_i}{R_2-R_1}\right)\mathbf{i}\right.\\ &\left.+\left(\Delta y\frac{l_0}{R_2-R_1}-\Delta y-l_0\sin\varphi_i\frac{\Delta x\cos\varphi_i}{R_2-R_1}-l_0\sin\varphi_i\frac{\Delta y\sin\varphi_i}{R_2-R_1}\right)\mathbf{j}\right],\end{aligned} \tag{31}$$

the resultant $\mathbf{R}$ being

$$\mathbf{R} = \sum_{i=0}^{n-1} k\left[\left(\frac{\Delta x l_0\sin^2\varphi_i}{R_2-R_1}-\Delta x\right)\mathbf{i}+\left(\frac{\Delta y l_0\cos^2\varphi_i}{R_2-R_1}-\Delta y\right)\mathbf{j}\right], \tag{32}$$

where we kept into account that for n even

$$\sum_{i=0}^{n-1}\sin\varphi_i\cos\varphi_i = 0. \tag{33}$$

The expression (32) reads also in the form

$$\mathbf{R} = k\Delta x\mathbf{i}\sum_{i=0}^{n-1}\left(\frac{l_0}{R_2-R_1}\sin^2\varphi_i-1\right)+k\Delta y\mathbf{j}\sum_{i=0}^{n-1}\left(\frac{l_0}{R_2-R_1}\cos s^2\varphi_i-1\right). \tag{34}$$

For q_3 and q_4 small, we have $\sin q_3 \approx q_3$, $\sin q_4 \approx q_4$, $\cos q_3 \approx 1$, $\cos q_4 \approx 1$, $\sin q_3 \sin q_4 \approx 0$, the relations (22) and (23) writing now as

$$\begin{pmatrix}\Delta x_1\\ \Delta y_1\\ \Delta z_1\end{pmatrix} = \begin{pmatrix}q_1+L_1q_4\\ q_2-L_1q_3\\ -q_1q_4+q_2q_3+L_1\end{pmatrix}, \tag{35}$$

respectively

$$\begin{pmatrix}\Delta x_2\\ \Delta y_2\\ \Delta z_2\end{pmatrix} = \begin{pmatrix}q_1-L_2q_4\\ q_2+L_2q_3\\ -q_1q_4+q_2q_3-L_2\end{pmatrix} \tag{36}$$

From the formulas (34), (35) and (36) it follows

$$\mathbf{R}_1 = k_1(q_1+L_1q_4)\mathbf{i}\sum_{i=0}^{n_1-1}\left(\frac{l_{01}}{R_{21}-R_{11}}\sin^2\varphi_i-1\right)+k_1(q_2-L_1q_3)\mathbf{j}\sum_{i=0}^{n_1-1}\left(\frac{l_{01}}{R_{21}-R_{11}}\cos s^2\varphi_i-1\right), \tag{37}$$

respectively

$$\mathbf{R}_2 = k_2(q_1-L_2q_4)\mathbf{i}\sum_{i=0}^{n_2-1}\left(\frac{l_{02}}{R_{22}-R_{12}}\sin^2\varphi_i-1\right)+k_2(q_2+L_2q_3)\mathbf{j}\sum_{i=0}^{n_2-1}\left(\frac{l_{01}}{R_{22}-R_{12}}\cos s^2\varphi_i-1\right). \tag{38}$$

The expression (24) leads us to

$$\begin{pmatrix} R'_{ix} \\ R'_{iy} \\ R'_{iz} \end{pmatrix} = \begin{bmatrix} 1 & 0 & -q_4 \\ 0 & 1 & q_3 \\ q_4 & -q_3 & 1 \end{bmatrix} \begin{pmatrix} R_{ix} \\ R_{iy} \\ R_{iz} \end{pmatrix} = \begin{pmatrix} R_{ix} \\ R_{iy} \\ q_4 R_{ix} - q_3 R_{iy} \end{pmatrix}, \tag{39}$$

where we kept into account that $R_{iz} = 0$, $i = 1, 2$.

We obtained

$$\mathbf{R}'_1 = \mathbf{R}_1,\ \mathbf{R}'_2 = \mathbf{R}_2. \tag{40}$$

The equations of motion read

$$\frac{G}{g}\ddot{q}_1 = k_1(q_1 + L_1 q_4)\sum_{i=0}^{n_1-1}\left(\frac{l_{01}}{R_{21} - R_{11}}\sin^2\varphi_i - 1\right) + k_2(q_1 - L_2 q_4)\sum_{i=0}^{n_2-1}\left(\frac{l_{02}}{R_{22} - R_{12}}\sin^2\varphi_i - 1\right), \tag{41}$$

$$\frac{G}{g}\ddot{q}_2 = k_1(q_2 - L_1 q_3)\sum_{i=0}^{n_1-1}\left(\frac{l_{01}}{R_{21} - R_{11}}\cos^2\varphi_i - 1\right) + k_2(q_2 - L_2 q_3)\sum_{i=0}^{n_2-1}\left(\frac{l_{02}}{R_{22} - R_{12}}\cos^2\varphi_i - 1\right) - G, \tag{42}$$

$$\begin{aligned} J_x\ddot{q}_3 &= -L_1 k_1(q_2 - L_1 q_3)\sum_{i=0}^{n_1-1}\left(\frac{l_{01}}{R_{21} - R_{11}}\cos^2\varphi_i - 1\right) \\ &+ L_2 k_2(q_2 - L_2 q_3)\sum_{i=0}^{n_2-1}\left(\frac{l_{02}}{R_{22} - R_{12}}\cos^2\varphi_i - 1\right), \end{aligned} \tag{43}$$

$$\begin{aligned} J_y\ddot{q}_4 &= L_1 k_1(q_1 + L_1 q_4)\sum_{i=0}^{n_1-1}\left(\frac{l_{01}}{R_{21} - R_{11}}\sin^2\varphi_i - 1\right) \\ &- L_2 k_2(q_1 - L_2 q_4)\sum_{i=0}^{n_2-1}\left(\frac{l_{02}}{R_{22} - R_{12}}\sin^2\varphi_i - 1\right). \end{aligned} \tag{44}$$

Considering now the displacement q_2 relative to the position of static equilibrium, the term $-G$ disappears in the formula (42).

Let us denote

$$\begin{aligned} S_{11} &= \sum_{i=0}^{n_1-1}\left(\frac{l_{01}}{R_{21} - R_{11}}\cos^2\varphi_i - 1\right),\ S_{21} = \sum_{i=0}^{n_1-1}\left(\frac{l_{01}}{R_{21} - R_{11}}\sin^2\varphi_i - 1\right), \\ S_{12} &= \sum_{i=0}^{n_2-1}\left(\frac{l_{02}}{R_{22} - R_{12}}\cos^2\varphi_i - 1\right),\ S_{22} = \sum_{i=0}^{n_2-1}\left(\frac{l_{02}}{R_{22} - R_{12}}\sin^2\varphi_i - 1\right) \end{aligned} \tag{45}$$

and results the system

$$\ddot{q}_1 - \frac{k_1 S_{21} + k_2 S_{22}}{G} g q_1 + \frac{-k_1 L_1 S_{21} + k_2 L_2 S_{22}}{G} g q_4 = 0, \tag{46}$$

$$\ddot{q}_2 - \frac{k_1 S_{11} + k_2 S_{12}}{G} g q_2 + \frac{k_1 L_1 S_{11} - k_2 L_2 S_{12}}{G} g q_3 = 0, \tag{47}$$

$$\ddot{q}_3 + \frac{L_1 k_1 S_{11} - L_2 k_2 S_{12}}{J_x} - \frac{L_1^2 k_1 S_{11} + L_2^2 k_2 S_{12}}{J_x} q_3 = 0, \tag{48}$$

$$\ddot{q}_4 + \frac{-L_1 k_1 S_{21} + L_2 k_2 S_{22}}{J_y} q_1 - \frac{L_1^2 k_1 S_{21} + L_2^2 k_2 S_{22}}{J_y} q_4 = 0 \tag{49}$$

or

$$\ddot{q}_1 + a_{11}q_1 + a_{14}q_4 = 0,\ \ddot{q}_2 + a_{22}q_2 + a_{23}q_3 = 0,\ \ddot{q}_3 + a_{32}q_2 + a_{33}q_3 = 0,\ \ddot{q}_4 + a_{41}q_1 + a_{44}q_4 = 0, \quad (50)$$

the notations being evident.

A particular case very met in practice is that of the identical journals and of the centre of weight situated at the middle of the axle, that is

$$L_1 = L_2 = L,\ k_1 = k_2 = k,\ n_1 = n_2 = n. \quad (51)$$

In this situation, we have

$$S_{11} = S_{12} = S_1,\ S_{21} = S_{22} = S_2 \quad (52)$$

and the system formed by the equations (46)-(49) reduces to

$$\ddot{q}_1 - \frac{2kS_1}{G} gq_1 = 0,\ \ddot{q}_2 - \frac{2kS_1}{G} gq_2 = 0,\ \ddot{q}_3 - \frac{2L^2kS_1}{J_x} q_3 = 0,\ \ddot{q}_4 - \frac{2L^2kS_2}{J_y} q_4 = 0. \quad (53)$$

One observes that the system (53) has periodical solution if and only if

$$S_1 < 0,\ S_2 < 0. \quad (54)$$

An evident situation when the relations (54) are fulfilled is that when $l_0 < R_2 - R_1$, that is all the springs are stretched.

6. CONCLUSIONS

In our paper we present a model for a mechanical transmission containing two journals with clearances. The clearances were modelled by even number of identical springs. The general hypothesis was that the motion can be described by two translation and two rotations, all of them being independent. The motion along and about the common axis of the journals was not considered in this paper. For this model we obtained the equations of motion and the linear form of these equations. It is easy to observe that the general form of the equations of motion is highly non-linear and very difficult to handle. Due to the fact that the clearances are small, it results that the displacements q_i, $i = \overline{1,4}$ are also small and the solution of the linear system (50) offers a good approximation for the general solution. A very often case characterized by identical journals and symmetric axle was briefly discuss and was found the necessary and sufficient condition for the existence of periodical motion. In our future works we shall discuss the equilibrium positions, stability of the equilibrium and the stability of the motion. Another study will be dedicated to the situation of the damping forces and to the influence of the damping forces on the motion.

REFERENCES

1. SIMON, A., FLOWERS, G., T., *Adaptive Disturbance Rejection and Stabilisation for Rotor Systems with Internal Damping*, International Journal of Acoustics and Vibration, **13**, 2, pp. 73-81, 2008.
2. PANDREA, N., STĂNESCU, N.–D., *Mechanics*, Didactical and Pedagogical Publishing House, Bucharest, 2002.
3. PANDREA, N., POPA, D., *Mechanisms. Theory and CAD applications*, Technical Publishing House, Bucharest, 2000.
4. STĂNESCU, N.–D., MUNTEANU, L., CHIROIU, V., PANDREA, N., *Dynamical Systems. Theory and applications*, Romanian Academy Publishing House, Bucharest, vol. 1, 2007.
5. STĂNESCU, N.–D., *Numerical Methods*, Didactical and Pedagogical Publishing House, Bucharest, 2007.
6. STĂNESCU, N.–D., *A Modelling of the Joints with Clearances*, Proceedings of the third National Seminary of Mechanisms, Craiova, 2008 (in press).

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