THE PUBLISHING HOUSE
OF THE ROMANIAN ACADEMY

PROCEEDINGS OF THE ROMANIAN ACADEMY, Series A,
Volume 11, Number 1, pp. 63–68

ON DYNAMICS OF SYSTEMS WITH FRICTION

Valeria MOŞNEGUŢU, Veturia CHIROIU

Institute of Solid Mechanics, Romanian Academy, Ctin Mille 15, 010141 Bucharest
Corresponding author: Veturia Chiroiu, E-mail: veturiachiroiu@yahoo.com

In this paper, we analyze the stick-slip motion in systems of confined liquids. The model consists in a periodically reinforced chain of harmonic oscillators with strong buckling nonlinearity, embedded between two corrugated plates. The top plate is pulled at constant velocity by a linear spring. The time-dependent relaxation of the slip part between two successive stick events is studied on the range of high stage velocities. We see that the buckling nonlinearity may coexists with different patterns of dynamics, including chaos, and leads to the observed experimental behavior and to predictions that are amenable to experimental tests.

Key Words: Stick-slip motion; Friction; Buckling nonlinearity; Chaos.

1. INTRODUCTION

Interfacial friction is one of the oldest problems in science and certainly one of the most important from a practical point of view [1–7]. Experimentally, it has been observed that, under overdamped conditions, slip relaxation manifests more than one characteristic time scale [8]. Various studies have suggested that the details of the relaxation depend on the experimental conditions and have related the stick-slip behavior to a dynamical phase transition or to a velocity-dependent frictional force [9–11]. Historically, the exploitation of friction has a significant effect on the machine and structures development [12, 13].

An interesting model was suggested by Rozman *et al*. [14–17]. A single or a string of particles (a molecular system) are embedded between two corrugated surfaces (two-wave potential). The top plate is pulled at constant velocity by a linear spring and the plate motion is monitored. It was observed that the plate exhibits exactly the same dynamic state of frictional motion as the model with the three regimes, i.e. (1) highly regular with high amplitude stick-slip for $V \ll V_c$, (2) intermittent stochastic stick-slip for $V < V_c$, and (3) smooth low friction sliding for $V > V_c$, where V_c is the critical stick-slip velocity. The Rozman works are focused only on the range of low stage velocities, where the motion is close to periodic.

In this paper, we use the Rozman's idea of embedding a molecular system of particles between two corrugated macroscopic plates. The molecular system consists of harmonic oscillators coupled through buckling sensitive elastica. We concentrate on the stick-slip regime typical of high driving stage velocities, and analyze a time window which corresponds to slip motion between two stick events. This slip relaxation behavior depends on the conditions under which the stick-slip motion is being studied.

We consider the overdamped case, where the spring constant is weak compared to the friction constant and the characteristic slip time is much longer than the response time of the mechanical system $\approx 2\pi\sqrt{M/K}$.

2. MICROSCOPIC MODEL

We consider a system experimentally studied in [18], namely a periodically reinforced structure with strong buckling nonlinearity, consisted of N harmonic oscillators. The system is modeled by a modified Toda lattice [19–23]. The lattice model is shown in Fig.1. The masses are denoted by m, all the spring constants are k_m, and all the damping coefficients are c_m.

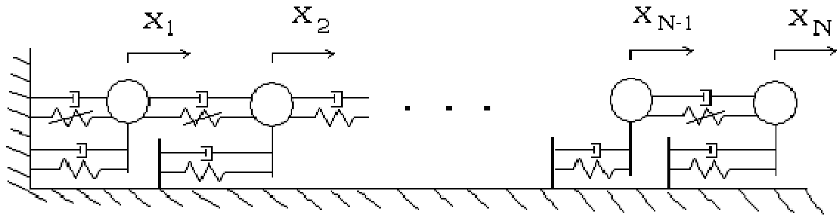

Fig. 1 – Nonlinear lattice model.

The equations of motion of the lattice are

$$L(x_i,\dot{x}_i,\ddot{x}_i)+\sum_{j\neq i}^{N}\frac{\partial U(x_i-x_j)}{\partial x_j}=0\,,\ i,j=1,2,...,N\,, \tag{1}$$

where the potential $U(x_i - x_j)$ describes the spatial interaction between buckling elements in the chain, and the operator L associated with the slow motion is defined as

$$L = m\ddot{x}_i + c_m\dot{x}_i + k_m x_i\,,$$

with boundary conditions of zero displacement of the fixed end and zero force on the free end

$$x_0 = 0\,,\ x_{N+1} = x_N\,. \tag{2}$$

The potential $U(x_i - x_j)$ is the same as that used by Toda to model the nonlinear force-displacement relationship for the buckling elements. The force F which results from this potential is given by

$$F(x_i - x_j) = \frac{k_m}{b}[\exp(b(x_i - x_j)) - 1]\,, \tag{3}$$

where b determines the strength of the nonlinearity. For the atoms interaction in a lattice, the Morse interaction force is valid in quantum mechanics of an electron motion, α_0 being a constant

$$F(x_i - x_j) = F_0[\exp(-2\alpha_0(x_i - x_j)) - 2\exp(-\alpha_0(x_i - x_j))]\,.$$

Morse force of interaction governs precisely the nearest-neighbor interaction in an anharmonic lattice of the atoms in a diatomic molecule of solids in a continuum limit. But, no exact solutions are found for elasticity problems governed by Morse force. For Toda interaction force there are some exact explicit solutions for the model. Equations (1) and (3) reduce to a Toda lattice, and the functional form of the nonlinear soliton solution is given by

$$x_i - x_j = \frac{1}{b}\ln\left[\left(\frac{\beta}{\omega_{ch}}\right)^2 \text{sech}^2(\tilde{k}(x_i - x_j) - \beta t) + 1\right], \tag{4}$$

with $\tilde{k}$ and β are constants, and $\omega_{ch} = \sqrt{k_m / m}$ is the characteristic frequency of the chain. Substitution of (4) into (1) and (3), shows that it solves these equations if we have $\beta = \omega_{ch}\sinh\tilde{k}$. If $b < 0$, then $x_{j+1}(t) - x_j(t) < 0$ for all values of j and t. This corresponds to the compressive atomic lattice waves studied by Toda. If $b > 0$, then $x_{j+1}(t) - x_j(t) > 0$, we have the tensile waves, which propagate to the speed $v = \beta a / \tilde{k}$, where a is the equilibrium spacing between adjacent masses in a free chain. The speed of this soliton is dependent upon the amplitude. If initial conditions are small, the model (1) will have approximately linear behavior. For large initial conditions, the nonlinearities may lead to chaos [18].

3. MICROSCOPIC MODEL IMBEDDED INTO A MACROSCOPIC MODEL

Next, the chain of 10 harmonic oscillators, described in previous section, is embedded into a macroscopic system of two rigid plates. The top plate of mass M is pulled by a linear spring with a spring constant k connected to a stage which moves with a velocity V (Fig. 2). Within the chain, the oscillators have the masses denoted by m_i, $i = 1,2,...,N$, and all the spring constants are k_m, and all the damping coefficients are c_m.

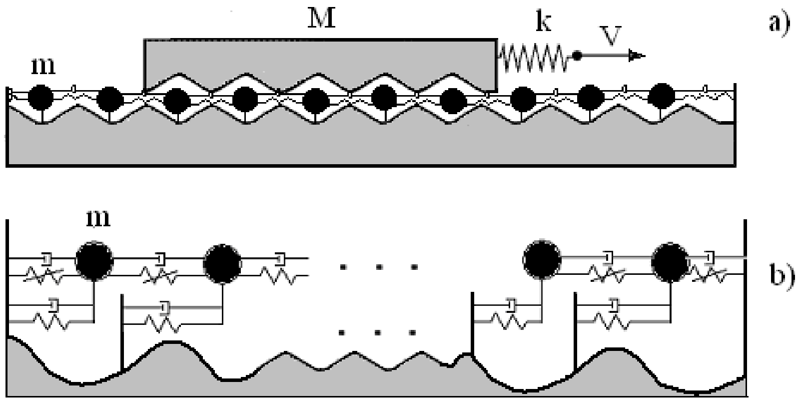

Fig. 2 – a) Sketch of model; b) chain of 10 harmonic oscillators.

The equations of motion for the top plate and chain of oscillators are [15]

$$M\ddot{X} + \sum_{i=1}^{N}\left(c_m(\dot{X} - \dot{x}_i) + \frac{\partial W(x_i - X)}{\partial X} \right) + k(X - Vt) = 0\,,$$

$$m\ddot{x}_i + c_m\dot{x}_i + c_m(\dot{x}_i - \dot{X}) + k_m x_i + \sum_{j \neq i}^{N} \frac{\partial U(x_i - x_j)}{\partial x_j} + \frac{\partial\left(W(x_i) + W(x_i - X)\right)}{\partial x_i} = 0\,, \tag{5}$$

where X is the coordinate of the top plate, x_i are the coordinates of chain masses, c_m is the damping coefficient between masses and between masses and the lower plate. The spatial interaction between the masses and the top plate is described by the potential $W(x_i - x_j)$. The dissipative forces between the oscillators and respectively, the oscillators and plates, are proportional to their relative velocities, i.e. $c_m\dot{x}_i$, and respectively $c_m(\dot{x}_i - \dot{X})$. The interaction between the plates is realized through the particles chain. For the potential $W(x_i - x_j)$ we choose [15, 17]

$$W(x_i - x_j) = -W_0\cos(r((x_i - x_j))\,, \tag{6}$$

where W_0 is the minima of the potential, $r = 2\pi/d$, and d is the spatial period of the interaction between oscillators and the plate.

Let us introduce dimensionless variables and parameters: the coordinate $y = x/d$ and the time $\tau = \omega t$, where $\omega = r\sqrt{W_0/m}$ is the frequency of the small oscillations of the particle in the minima W_0, $\gamma = c_m/m\omega$ which is a dimensionless friction constant, $\varepsilon = m/M$ the ratio of particle and plate masses, $\alpha = \Omega/\omega$ the ratio of frequencies of the free oscillations of the tope $\Omega = \sqrt{K/M}$ and ω, $\delta = (d-a)/d$, the misfit of the substrate and chain periods, and $\rho = \left(\omega_{ch}/\omega\right)^2$ the ratio of the frequencies related to interparticle and particle-plate interactions, $\tilde{V} = V/\omega d$ the top plate dimensionless stage velocity.

4. RESULTS

The overdamped regime is realized if the spring constant K is sufficiently weak, and/or if the damping constant c_m is large. In numerical simulations we use: $\varepsilon = 1/120$, $\gamma = 0.7$, $\delta = 0.1$, $\alpha = 0.15$, $\rho = 1$.

We suppose that the mass M moves with a dimensionless stage velocity $\tilde{V} < 0.8$, which is smaller then the velocity for which the chain decouples from the plates. Fig. 3 presents the time evolution of the spring force during the slip relaxation (F_s is the static force). More than one time scale in the relaxation process is observed; three slip parts: $t_0 < t < t_1$, $t_1 < t < t_2$, and $t_2 < t < t_s$ are put into evidence. Time evolution of the spring force has an additional slip part in $t_2 < t < t_s$, if we compare with the results reported in [15]. In Fig. 4 the evolution in time of the top plate velocity during relaxation, is displayed. The velocity drops abruptly in the range $t_0 < t < t_1$, and increase abruptly in the range $t_1 < t < t_2$. After that, in the range $t_2 < t < t_s$, the velocity relaxes slowly towards the stage velocity. The behavior aspects are slightly different in $t_2 < t < t_s$, than those reported in [15]. In Fig. 5 displays the variation of length of the embedded chain, during slip relaxation for the same three behaviors observed before.

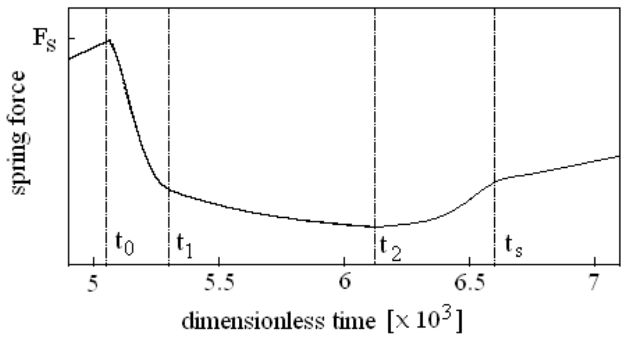

Fig. 3 – Time evolution of the spring force during slip relaxation.

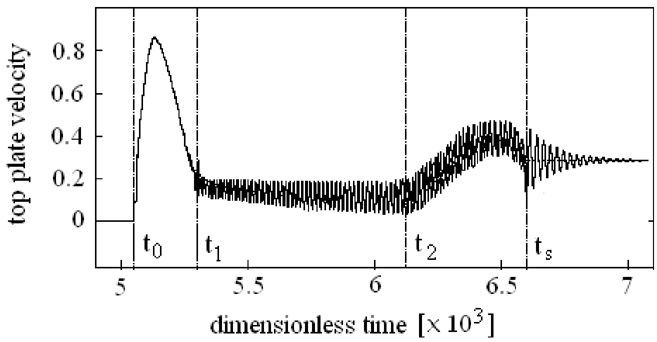

Fig. 4 – Time evolution of top plate velocity during slip relaxation.

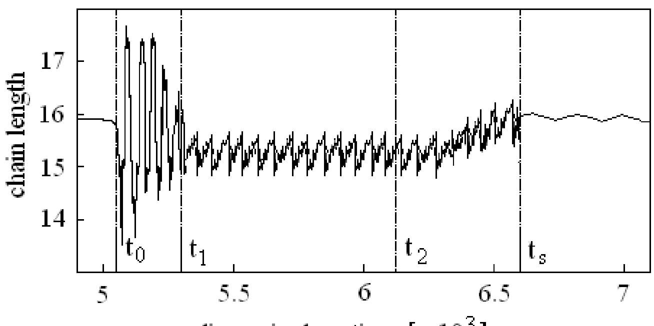

Fig. 5 – Time evolution of the embedded chain length during slip relaxation.

In the phase space of the spring force vs. the chain length, the only closed orbits are the stationary point, the stable stick slip solution, and the unstable orbit, which oscillates without coming to a complete stop. Initial conditions which lie within the unstable limit cycle, tends to go towards the stationary point, ande initial conditions, which are outside the unstable orbit, tends to go towards the periodic solution.

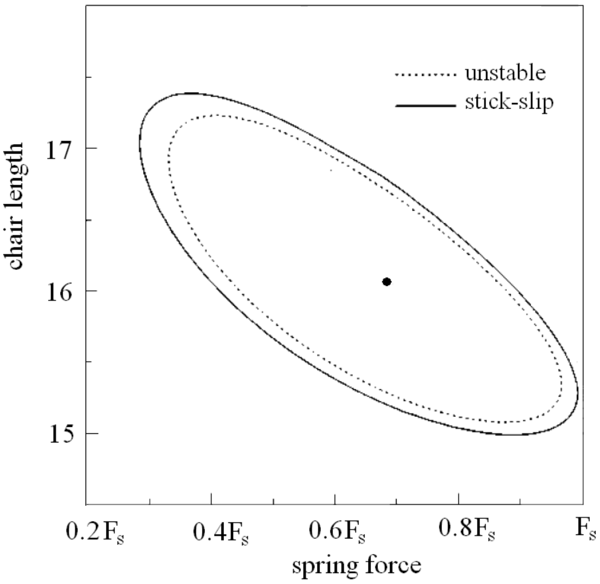

Fig. 6 – A diagram of the unstable limit cycle and the stick-slip limit cycle.

A diagram spring force-chain length of the unstable limit cycle and the stick-slip limit cycle is displayed in Fig. 6, for $\tilde{V} = 0.3$. As we increase the top plate velocity, the unstable cycle grows until it collides and eliminate the stick-clip cycle, as illustrated in terms of the maximum spring force in Fig.3. The point inside the limit cycles mark the stationary point.

5. CONCLUSIONS

In this paper, we consider a molecular system of harmonic oscillators coupled through buckling sensitive elastica, embedded between two macroscopic plates. We conclude that our results are in a good agreement with the results reported in [14–17]. We focus on the time-dependent relaxation of the slip part between two successive stick events, on the range of high stage velocities, where the motion exhibits chaos. The buckling nonlinearity of embedded chain may coexists with different patterns of dynamics, including chaos, and leads to the observed experimental behavior and to predictions that are amenable to experimental tests.

ACKNOWLEDGEMENT

The authors acknowledge the financial support of the PNII project-Idei, Grant nr. 106/2007, code ID-247/2007.

REFERENCES

1. SINGER, I. L., POLLOCK, H. M. (Eds.), *Fundamentals of Friction*, Series *E: Applied Sciences*, NATO Advanced Sciences Institutes, Kluwer Academic Publishers, Dortrecht, 1992.
2. BHUSHAN, B., Ed. *Micro/Nanotribology and Its Applications*, Series E: *Applied Sciences* 330, NATO Advanced Sciences Institutes; Kluwer Academic Publishers, Dortrecht, 1997.
3. OVERNET, R.M., TYNDALL, G., FROMMER, J., *Kinetics and Energetics in Nanolubrification*, Ch. 29 "Nanotechnology Hanndbook", B. Bhushan (Ed.), Springer-Verlag, 2004.

4. YOSHIZAWA, H., MCGUIGGAN, P., ISRAELACHVILI, J., *Identification of a second dynamic state during stick-slip motion*, Science, **259**, pp. 1305–1309, 1993.
5. REITER, G., DEMIREL, L., PEANASKY, J., CAI, L.L., GRANICK, S*., Stick to slip transition and adhesion of lubricated surfaces in moving contact*, J. Chem. Phys., **101**, pp. 2606–2615, 1994.
6. BANABIC D., KUWABARA, T., BALAN, T., COMSA, D.S., *Evaluation of an anisotropic yield criterion*, Proc. of the Romanian Academy, Series A: Mathematics, Physics, Technical Sciences, Information Science, **2**, *3*, pp. 17–21, 2001.
7. BANABIC, D., CAZACU, O., BARLAT, F., COMSA, D.S., WAGNER, S., SIEGERT, K., *Recent anisotropic yield criteria for sheet metals*, Proceedings of the Romanian Academy, Series A: Mathematics, Physics, Technical Sciences, Information Science, **3**, *3*, pp. 91–99, 2002.
8. BERMAN, A.D., DUCKER, W.A., ISRAELACHVILI, J.N., *Origin and Characterization of Different Stick-Slip Friction Mechanisms*, Langmuir, **12**, p. 4559, 1996.
9. CARLSON, J. M., BATISTA, A.A., *Constitutive relation for the friction between lubricated surfaces*, Phys. Rev., E **53**, pp. 4153–4165, 1996.
10. PERSSON, B.N.J., *Sliding Friction, Physical Properties and Applications*, Springer, Berlin, 2000.
11. YOSHIZAWA, H., ISRAELACHVILI, J. N*., Fundamental mechanisms of interfacial friction I: Relation between adhesion and friction*, J. Phys. Chem., **97**, pp. 4128–4140, 1993.
12. BURACU, V., ALECU A**.**, *On the use of a theorem by V.Vâlcovici in planar motion dynamics,* Proceedings of the Romanian Academy, Series A–Mathematics, Physics, Technical Sciences, Information Science, **6**, *1*, pp. 55–60, 2005.
13. BURACU, V., ALECU A., *Pivot friction in planar motion*, Proceedings of the Romanian Academy, Series A–Mathematics, Physics, Technical Sciences, Information Science, **5**, *3*, pp. 311–319, 2004.
14. ROZMAN, M. G., URBAKH, M., KLAFTER, J., *Stick-slip motion and force fluctuations in a driven two-wave potential*, Phys. Rev. Lett., **77**, pp. 683–686, 1996.
15. ROZMAN, M.G., URBAKH, M., KLAFTER, J., *Stick-slip dynamics as a probe of frictional forces*, Europhysics Letters, **39**, *2*, pp. 183–188, 1997
16. ROZMAN, M.G., URBAKH, M., KLAFTER**,** J*., Controling chaotic frictional forces*, Physical Review, E **57,** *6*, pp. 7340–7343, 1998.
17. ROZMAN, M. G., URBAKH, M., KLAFTER, J., *Stick-slip dynamics of interfacial friction*, Physica, A **249**, pp. 184–189, 1998.
18. DAVIES, M. A., MOON, F.C., *Solitons, chaos and modal interactions in periodic structures*, in *Dynamics with Friction: Modeling, Analysis and Experiment*, Part II, Series of Stability, Vibration and Control of Systems, World Scientific Publishing Company, 7, pp. 99–123, 2001.
19. MUNTEANU, L., DONESCU, Şt., *Introduction to Soliton Theory: Applications to Mechanics,* Book Series "Fundamental Theories of Physics", Kluwer Academic Publishers, **143**, 2004.
20. TODA, M., *Theory of Nonlinear Lattices*, Springer-Verlag, Berlin, Heidelberg, 1989.
21. TODA, M., WADATI, M., *A soliton and two solitons in an exponential lattice and related equations*, J. of the Phys. Soc. of Japan, **34**, pp. 18–25, 1973.
22. YONEYAMA, T.*, Interacting Toda equations*, J. of the Phys. Soc. of Japan., **55**, *3*, pp. 753–761, 1986.
23. CHIROIU, V., DUMITRIU, D., MUNTEANU, L., BADEA, T., *About focusing in nano elastodynamics*, Proc. of the Romanian Academy, Series A: Mathematics, Physics, Technical Sciences, Information Science, **7**, *3*, pp. 215–222, 2006.

Received April 14, 2009