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THE NON-TRIVIAL EFFECT OF FAST VIBRATION ON THE SLOW DYNAMICS OF A DAMPED OSCILLATOR

Adrian TOADER[*], Veturia CHIROIU[**], Tudor SIRETEANU[**], Viorel ŞERBAN[***]

[*] National Institute for Aerospace Research "Elie Carafoli", Bd. Iuliu Maniu 220, 061126 Bucharest
[**] Institute of Solid Mechanics of the Romanian Academy, Ctin Mille 15, P.O.Box 1-863, 010141 Bucharest
[***] Center of Technology and Engineering for Nuclear Projects, 409, Atomistilor Street, Magurele, Ilfov, POB. 5204-MG-4
Corresponding author: Veturia Chiroiu, E-mail: veturiachiroiu@yahoo.com

The paper discusses the non-trivial effect of high-frequency sinusoidal excitation on the slow dynamics of the *p*th power damped oscillators. We use the method of direct partition of motion and the linear equivalence method to obtain semi analytical solutions of the problem. We show that the fast vibration has a significant influence on the damping characteristics of the damper, especially in the low-velocity range. It is concluded that the hard dampers produce highest damping force at high-velocity. For low-velocity, the damping force is small, but may be increased for fast vibrations. Contrary, for soft dampers, the fast excitation reduces the low-velocity damping.

Key words: Fast vibration; Low-amplitude high-frequency excitation; Direct partition of motion; Linear equivalence method.

1. INTRODUCTION

By fast vibration we understand the low-amplitude high-frequency excitation with a time-period shorter than the natural period of the system. If we compare with a linear system which is functioning as a band pass filter and filters out any frequency away from its natural frequency, the behaviour of the nonlinear systems is different. For a nonlinear system, the high-frequency excitation may have a significant effect on its dynamics at a time scale comparable to the natural time period of the system [1]. A such excitation can affect certain characteristics of systems such as damping of nonlinear dampers [2–5], stick-slip motion of machine tool slide [6, 7], nonlinear elastic properties of material [8, 9], material transportation [10–12], symmetry breaking [13], equilibrium stability [14] and Hopf bifurcation [15, 16].

Blekhman [17, 18] developed a technique called the *direct partition of motion* based on two different time scales, the fast time and the slow time. According to this method, the motion is spitted into slow and fast components of the motion.

The purpose of the paper is to analyze the non-trivial effect of high-frequency sinusoidal excitation on the slow dynamics of the *p*th power damped oscillators. Semi analytical treatment is based on the method of *direct partition of motion* and the *linear equivalence method* (LEM) [19–21].

2. MODEL

Consider a single-degree-of-freedom oscillator with nonlinear damping, subjected to a high frequency base harmonic excitation $X_e = Q\sin(\omega_f t)$ (Fig.1). The base excitation is a high- frequency low- amplitude vibration, with time-period $T_0 \ll 1$ and amplitude $q \ll 1$ very small $q/T_0 = O(1)$. The non-dimensional motion equation is given by

$$\ddot{x} + f_d(\dot{x}) + x = q\Omega^2 \sin(\Omega\tau) , \qquad (2.1)$$

where $x(\tau)=(X-X_e)/x_0$ is the non-dimensional relative displacement of the mass m, x_0 is an arbitrary length, the dot denotes differentiation with respect to non-dimensional time $\tau=\omega t$, $\omega=\sqrt{k/m}$, $q=Q/x_0$, $\Omega=\omega_f/\omega$, $x_e=X_e/x_0$, and k is the stiffness. The base excitation is a high- frequency low- amplitude vibration, with time-period $T_0 \ll 1$ and amplitude $q \ll 1$ very small $q/T_0=O(1)$.

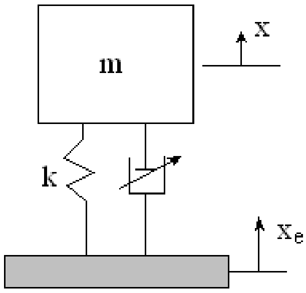

Fig.1 – Single degree-of-freedom damped oscillator with base excitation.

According to *direct partition of motion* [17, 18] we split the motion $x(\tau)$ into slow motion $z(\tau)$, where τ is the slow time, and fast motion $\phi(T)$ where $T=T_0^{-1}\tau$ is the fast time

$$x(\tau)=z(\tau)+T_0\phi(T)\,,\quad T_0^{-1}\int_0^{T_0}\phi \mathrm{d}T=0\,. \tag{2.2}$$

Here, $T_0\phi(\tau,T)$ is small compared with $z(\tau)$. By inserting (2.2) into (2.1) we obtain

$$T_0^{-1}\phi_{,TT}+2\phi_{,T}+T_0\,\phi-q\Omega^2\sin(\Omega\tau)=z_{\tau\tau}+z+f_d=\lambda\,, \tag{2.3}$$

where the indices mean the differentiation with respect to specified variable ($S_{,\tau}=\dfrac{\partial S}{\partial\tau}$, $F_{,T}=\dfrac{\partial F}{\partial T}$). The damping function f_d is defined as a power damping law [1]

$$f_d=a(\dot{x})\,|\,\dot{x}\,|^{p-1}\,,\ a=c\frac{x_0^{p-1}\omega^{p-2}}{m}\,, \tag{2.4}$$

with c a constant. For p semi-positive, the system is stable, $p=0$ means Coulomb's dry function damping. The damping function can define a soft damper for $p<1$, or a hard damper for $p>1$. The law (2.4) can be written as, according to (2.2)

$$f_d=a(z_{,\tau}+\phi_{,T}+T_0\phi)\,|\,z_{,\tau}+\phi_{,T}+T_0\phi\,|^{p-1}\,. \tag{2.5}$$

We can rewrite (2.3) as two nonlinear ODSs of the form

$$\begin{gathered} v=\phi_{,T} \\ 2T_0^{-1}v_{,T}+2v+T_0\,\phi-q\Omega^2\sin(\Omega\tau)-\lambda=0 \end{gathered} \tag{2.6}$$

and

$$\begin{gathered} u=z_{,\tau} \\ u_{,\tau}+z+f_d-\lambda=0,\quad f_d=f_d(u,v,\phi). \end{gathered} \tag{2.7}$$

The analytical solutions of (2.6) and (2.7) are difficult to obtain. In the following, we solve analytically the system (2.6) and with help of these solutions, we solve numerically the system (2.7).

3. SOLUTIONS

To understand LEM, let us consider a nonlinear ODSs of the form [19–21]

$$\dot{y}-f(y)=0\,,\quad f(y)=\left[f_j(y)\right]_{j=1,\dots,n}\,,\quad f_j(y)=\sum_{|\mu|=1}^{\infty} f_{j\mu}y^{\mu}\,, f_{j\mu}\in\Re,\ j=1,2,...,n,\ |\mu|\in \mathrm{N}\,. \tag{3.1}$$

The LEM mapping is

$$v(x,\xi)\equiv \exp\langle \xi, y\rangle\,,\quad \xi=(\xi_1,\xi_2,\dots,\xi_n)\in \Re^n\,. \tag{3.2}$$

This mapping associates to the nonlinear ODS two linear equivalents:

a) a linear PDE, always of first order with respect to x

$$v_t-\langle \xi, f(D)\rangle v=0\,, \tag{3.3}$$

b) a linear, while infinite, first order ODS, that may be also written in matrix form

$$V_t-AV=0,\quad V=\left(V_j\right)_{j\in N}\,,\quad V_j=\left(v_\gamma\right)_{|\gamma|=j}\,. \tag{3.4}$$

In (3.3), the notation $f_j\left(t,\mathrm{D}_\xi\right)$ stands for the formal operator

$$\langle \xi, f(D)\rangle\equiv\sum_{j=1}^{n}\xi_j f_j\left(D_\xi\right)\,,\quad f_j\left(D_\xi\right)=\sum_{|\mu|=1}^{\infty} f_{j\mu}\frac{\partial^{|\mu|}}{\partial\xi^{\mu}}\,. \tag{3.5}$$

The second LEM equivalent, the system (3.4) is obtained from the first one, by searching the unknown function v in the class of analytic in $\boldsymbol{\xi}$ functions

$$v(t,\xi)=1+\sum_{|\gamma|=1}^{\infty}v_\gamma(t)\frac{\xi^\gamma}{\gamma!}\,. \tag{3.6}$$

Let us associate to (3.1) the initial conditions

$$y(t_0)=y_0,\quad t_0\in I\,. \tag{3.7}$$

By LEM, they are transferred to

$$v(t_0,\xi)=\exp\langle \xi, y_0\rangle,\quad \xi\in\Re^n\,, \tag{3.8}$$

a condition that must be associated to the PDE (3.3), and

$$V(t_0)=\left(y_0^\gamma\right)_{|\gamma|\in N}\,, \tag{3.9}$$

indicating an initial condition for the second LEM equivalent.

Let us apply LEM to solve the system (2.6), with attached initial conditions

$$v(0)=\beta,\quad \phi(0)=\gamma\,. \tag{3.10}$$

The equivalent LEM system of (2.6) is written as

$$\begin{gathered} v=\phi_{,T}\,,\\ v_{ijsln,T}=iv_{i-1,j+1,sln}+j\left(T_0\Omega v_{i,j-1,s+1,ln}-qT_0v_{i+1,j-1,sln}+2\lambda T_0 v_{i,j-1,sl,n+1}+\lambda v_{i,j-1,sl,n+2}\right)+\\ +s\Omega v_{ij,k-1,l+1,n}+l\left(-qT_0v_{ij,s+1,l-1,n}+\lambda v_{i+1,js,l-1,n}-2\lambda qT_0v_{ijs,l-1,n+1}-v_{ijs,l-1,n+2}\right)+\\ +n\left(-qv_{i,j+1,sl,n-1}-2\lambda\Omega v_{i,j+1,sln}-q\lambda v_{i,j+1,sl,n+1}\right). \end{gathered} \tag{3.11}$$

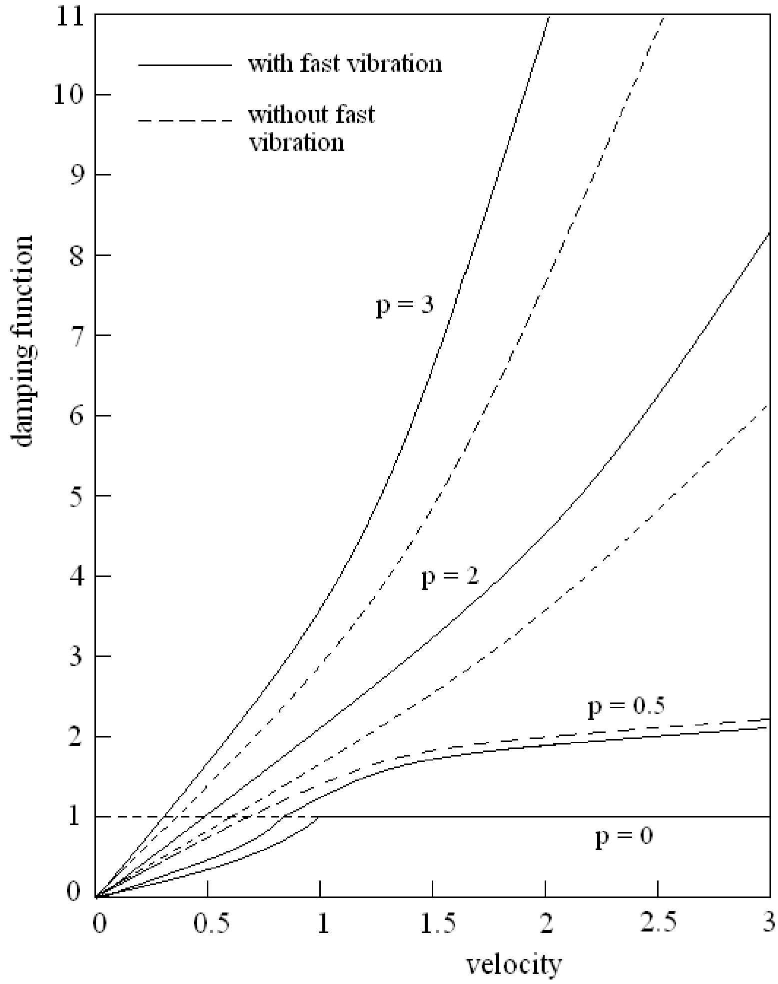

Fig. 2 – Effect of harmonic fast vibration of p^{th} power damping with and without fast vibration.

The solution v is found next

$$v(T) \cong \frac{1}{2}\beta\cos\omega T + \frac{1}{2}\gamma\sin\omega T + A_1 + \gamma^2 A_2 + \beta^2 A_3, \tag{3.12}$$

with

$$\begin{gathered}
A_1 = \frac{1}{2}\big[\beta\cos\varpi T + a_1 T\sin\varpi T + a_2\cos\Omega T\sin\varpi T + \\
a_3(\cos 2\Omega T\sin 2\varpi T + \cos(\Omega-\varpi)T - \cos(\Omega+\varpi)T)\big], \\
A_2 = \frac{1}{2}\big[\gamma\sin\varpi T + b_1 T\cos\varpi T + b_2 T\sin\varpi T + \\
+b_3(2\cos\Omega T - \cos(\Omega-\varpi)T - \cos(\Omega+\varpi)T)\big], \\
A_3 = a_1 b_1\sin\varpi T\sin\Omega T + a_2 b_2 T\sin\varpi T\cos^2\varpi T + a_3 b_3 T\sin^2\Omega T\cos 2\varpi T,
\end{gathered} \tag{3.13}$$

where a_i, b_i, $i = 1,2,3$ are constants which depend on $\beta,\gamma,\Omega,q,\lambda$ and T_0. The solution (3.12) is inserted next in (2.7), and numerically solved.

The effect of fast vibration of soft and hard pth power dampers is understood from Fig.2, where the damping function is plotted for $p=0$, $p=0.5$, $p=2$ and $p=3$ with fast and without fast vibration. It seen that the hard damper ($p=2$, $p=3$) produce higher damping force at high velocity. At low-velocity the force has a low value. The low-velocity damping of soft damper ($p = 0.5$) is reduced by fast excitation. The case $p = 0$ corresponds to friction damping and represents an extreme case of soft damping characteristics. The effect of fast vibration on the transient vibration of the system is displayed in Fig. 3, for $q = 0.002$, $\Omega = 1000$ and $p = 2$, respectively $p = 0.5$.

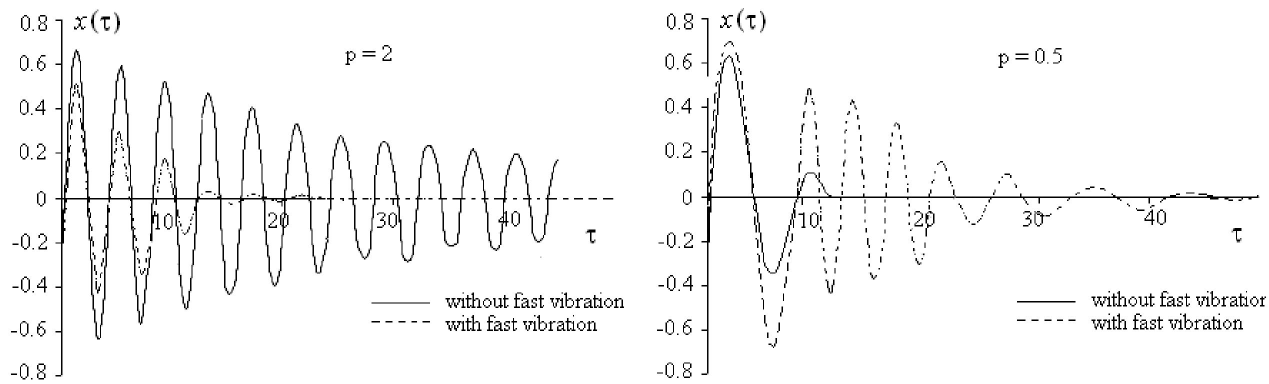

Fig. 3 – Effect of fast vibration on the transient vibration with and without fast vibration.

4. CONCLUSIONS

The effect of the non-trivial effect of high-frequency sinusoidal excitation on the slow dynamics of the *p*th power damped oscillators is discussed. The method of direct partition of motion and the the linear equivalence method are applied to obtain semi analytical solutions of the problem. We show that fast vibration has a significant influence on the damping characteristics of the damper, especially in the low-velocity range. It is concluded that the low-velocity damping of hard damper increases and low-velocity damping of soft damper decreases due to fast vibration.

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