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OPTIMAL MASS DESIGN OF A SINGLE-STAGE HELICAL GEAR UNIT WITH GENETIC ALGORITHMS

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The problem of minimum mass design of a single-stage helical gear unit represents a real interest, since the most commonly encountered mechanical power transmission require low weight. This paper presents an optimal design mass minimization problem of a single-stage helical gear unit, complete with the sizing of shafts, gearing and housing using genetic algorithms (GAs). It can be observed that the proposed optimal design with GAs has the potential to yield considerably better solutions than the traditional heuristics. At the same time, the GAs offer a better understanding of the trade-offs between various objectives (such as service life and mass).

Key words: automated optimal mass design, genetic algorithms, single-stage helical gear unit.

1. INTRODUCTION

Designing a mechanical power transmission such as a single-stage helical gear unit is a very complex task. The complexity arises from strong and often intractable interdependencies between the design variable defining its subsystems. In other words, an optimal reducer is generally not an assembly of components optimized in isolation, a fact overlooked by many conventional design heuristics. Furthermore, it is know that designing of a reducer is an iterative process in which it is necessary to make some tentative choices, and to determine which parts of the design are critical. For example, the impact of certain choice of the coefficient of face width of the pinion may yield to a minimum mass gearing, but the selection of this coefficient may cascade through subsequent steps of the design process (sizing of shafts, radial seals, tapered roller bearings, housing etc.) that ultimately lead to a heavier reducer than if a slight compromise had been made on the choice of the gearing. Of course, in a few trivial cases from all of them, it is almost impossible to tell what the first compromise should have been done, let alone what any subsequent choices should have been made with the overall goal in mind, instead of concerning on the subsystem instead. Moreover, for solving such complex real design problem, conventional optimization techniques are very difficult to consider, taken into account the large number of design variables and the complexity of the interactions between them and the highly non-linear nature of the constraints and the objectives. For the last decades, evolutionary algorithms such as genetic algorithms (GAs) are getting increasing attention to solve the complex mechanical power transmission design problems among the scientific and engineering community. At the same time, the simple trial and error type methods which are used to tackle this design problem are used more rarely. The potential replacements have begun in the shape of computer programs and expert systems. Thus Madhusudan and Vijayasimha in [11] presented a computer program in order to design a required type of gear under a specified set of working conditions. A new computer-aided method for automated gearbox design was described in [10]. An interactive physical programming was developed in [8] in order to optimize a three-stage spur gear reduction unit. An expert system for designing and manufacturing a gearbox is described [1]. Li et al. [9] carried out a study for minimizing the centre distance of a helical gear using American Gear Manufacturers Association (AGMA) procedures. An optimal weight design problem of a gear with an improved GA is presented in [17]. A non-dominated sorting genetic algorithm (NSGA-II) was used in [3] in order to solve a multi-objective optimization of a multi-speed gearbox. Thompson et al. in [15] presented a generalized optimal design of two-stage and three-stage spur gear reduction units in a formulation with multiple objectives. The benefits of the particle swarm searches in resolving different engineering designs are shown in [12]. Two advanced optimization algorithms known as particle swarm optimization (PSO) and simulated annealing (SA) are used in [14] for minimizing the weight of a spur gear train. The results of the proposed algorithms were compared with the results obtained in [17]. In [5] GAs were applied in order to minimize the volume of a two stage helical gear train. A complete automated optimal design of a two-stage helical gear reducer using a two-phase evolutionary algorithm is presented in [16].

The motivation behind the work described in this paper is that evolutionary computing technology has now reached the level where, we believe, it is computationally feasible to consider an automated optimal design of a complete single-stage helical gear unit. The studies referenced above have been instrumented in order to highlight the importance of using modern global optimization techniques in mechanical power transmission design (as opposite to conventional, trail and error type methods), even when considering certain subproblems.

In this paper we deal with a single-stage helical gear reducer, whose every element (gears, shafts, radial seal, tapered roller bearing, the shape of the housing etc.) is subjected to changes during the optimization process. The industrial relevance of the exercise shown in this article is ensured by the consideration of all design constraints typically encountered in practice. We bound the design space by a total of 45 constraints. In Section 3 we discuss this formulation in detail. Section 4 contains an effective example of optimal design followed by a discussion and a comparison between optimal design with GAs and traditional design (when we used a commonly trial and cut error procedure). Eventually, some suggestions regarding the possible extensions of the results of this study are presented.

2. THE GENETIC ALGORITHM

The genetic algorithm is perhaps the most well known of all evolution-based search algorithms [2]. The basic concepts of GAs were developed by John Holland [7] in an attempt to explain and describe the biological processes that can be appreciated in *Nature*, and to design new *artificial evolutionary systems* based upon these natural processes. GAs maintain a population of solutions, then allow the fitter individuals to reproduce, and let the less fit individuals die off [13]. Each individual consists of a *genotype* (i.e. the search space of coded solutions) and a corresponding *phenotype* (i.e. the solution space). *Phenotypes* usually are collections of parameters (for example in our 'single-stage helical gear unit', such parameters might define the number of teeth on pinion, the centre distance, the radial input/output shafts sealing, the tapered roller bearings corresponding on input/output shafts etc.). *Genotypes* consist of coded versions of these parameters. A coded parameter is normally referred to as a *gene*. A collection of genes in one genotype is often held internally as a string, and is known as a *chromosome* [13]. The GA used in solving our single-stage helical gear unit optimal design problem (Section 3) is shown in Fig. 1.

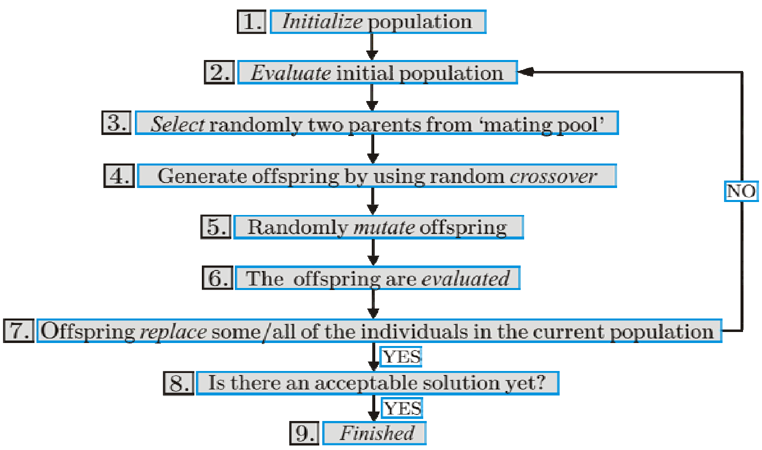

Fig. 1 – The genetic algorithm.

This algorithm works as follow: (1) the *genotype* of every individual (as it can be seen in Section 3, the genotype of the reducer is described by a set of 11 *genes*) in the population is randomly initialized. Then the main loop of the algorithm begins. (2) The *phenotype* of every individual from the initial population is evaluated using the *fitness function* (i.e. the mass of the reducer). Next, (3) two parents are randomly selected for reproduction based on the fitness values of the individuals. Offspring are created (4 – 5) by applying the genetic operators: *crossover* (merges information from two parents into one or two offspring) and *mutation* (acts on a single offspring and works by applying some variation to one or more genes in the offspring's chromosome). The new generated individuals are evaluated (6) using the fitness measure. After evaluation the offspring replace some/all of the individuals in the current population (7). This entire process of evaluation and reproduction continues until either a satisfactory solution emerges or the GA has run for a specified number of generations (9).

3. PROBLEM FORMULATION

3.1. The 'genotype' of the single-stage helical gear unit

The single-stage helical gear unit we are considering in this work (see for example Fig. 2) is highly standardised, both in terms of its layout and in terms of design of its gearings, shafts, radial sealing and tapered roller bearings. The set of 11 *genes* that define the single-stage helical gear unit unequivocally reflects this, with standardization imposing discrete values sets on most of them (as it can be observed in Table 1, from the set of 11 design variables 8 are standardized).

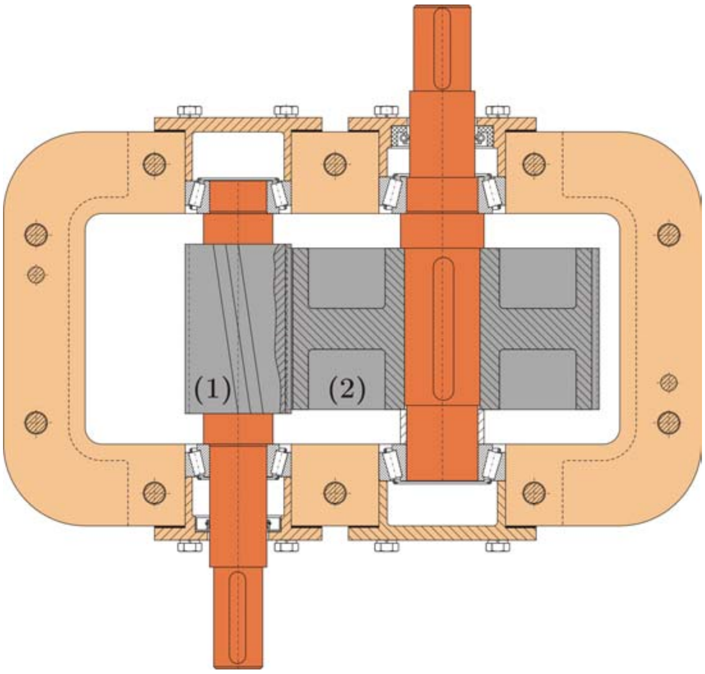

Fig. 2 – The sketch of the single-stage helical gear unit.

Table 1

The 11 genes describing the single-stage helical gear unit

Symbol	Range	Description
z_1	{25 ,…, 56}	Number of teeth on the pinion (1). Integer values.
a_w (mm)	{56 ,…, 315}	Center distance of the single-stage gear unit. Standardized, discrete, real values.
x_{n1}	{–0.6,…,1}	Tooth normal addendum coefficient of pinion (1). Discrete, real values.
β (°)	[4, 19.75]	Helix angle measured at the pitch diameters. Discrete real values.
ψ_a	[0.2,…,0.8]	Gear (2) width to center distance ratio coefficient. Real values.
i_1	{0,…,63}	Catalogue index of standardized end for the input shaft. Integer values.
i_2	{0,…,127}	Catalogue index of radial shaft seal for the input shaft. Integer values.
i_3	{0,…,127}	Catalogue index of tapered roller bearings for the input shaft. Integer values.
i_4	{0,…,63}	Catalogue index of standardized end for the output shaft. Integer values.
i_5	{0,…,127}	Catalogue index of radial shaft seal for the output shaft. Integer values.
i_6	{0,…,127}	Catalogue index of tapered roller bearings for the output shaft. Integer values.

3.2. The objective function

The problem of minimum mass design of simple and multi-stage spur gear trains has been a subject of considerable interest, since many high-performance power transmission applications (e.g. automotive, aerospace, machine tools, etc.) require low mass. For this reason, in this study, minimization of single-stage helical gear unit mass is the objective function and it is defined as shown in Eq. (1).

$$Obj: \quad F(x) = M_1 + M_2 + M_3 = \sum_{i=1}^{5}\left(V_{11i} + V_{12i}\right)\rho_1 + \sum_{j=1}^{2}\left(V_{21j} + V_{22j}\right)\rho_2 + \sum_{k=1}^{7}\left(V_{31k} + V_{32k}\right)\rho_2 \rightarrow \min, \qquad (1)$$

where: V_{11i} is the volume of the reducer's housing body, V_{12i} represents the volume of the reducer's housing cover, V_{21j} is the volume of the pinion, V_{22j} is the volume of the wheel (2), V_{31k} is the volume of the input shaft, V_{32k} is the volume of the output shaft, ρ_1 is the density of cast iron (i.e. $7.2 \cdot 10^{-6}$ mm^3/kg), ρ_2 is the density of steel (i.e. $7.85 \cdot 10^{-6}$ mm^3/kg). These terms are illustrated in Fig. 3.

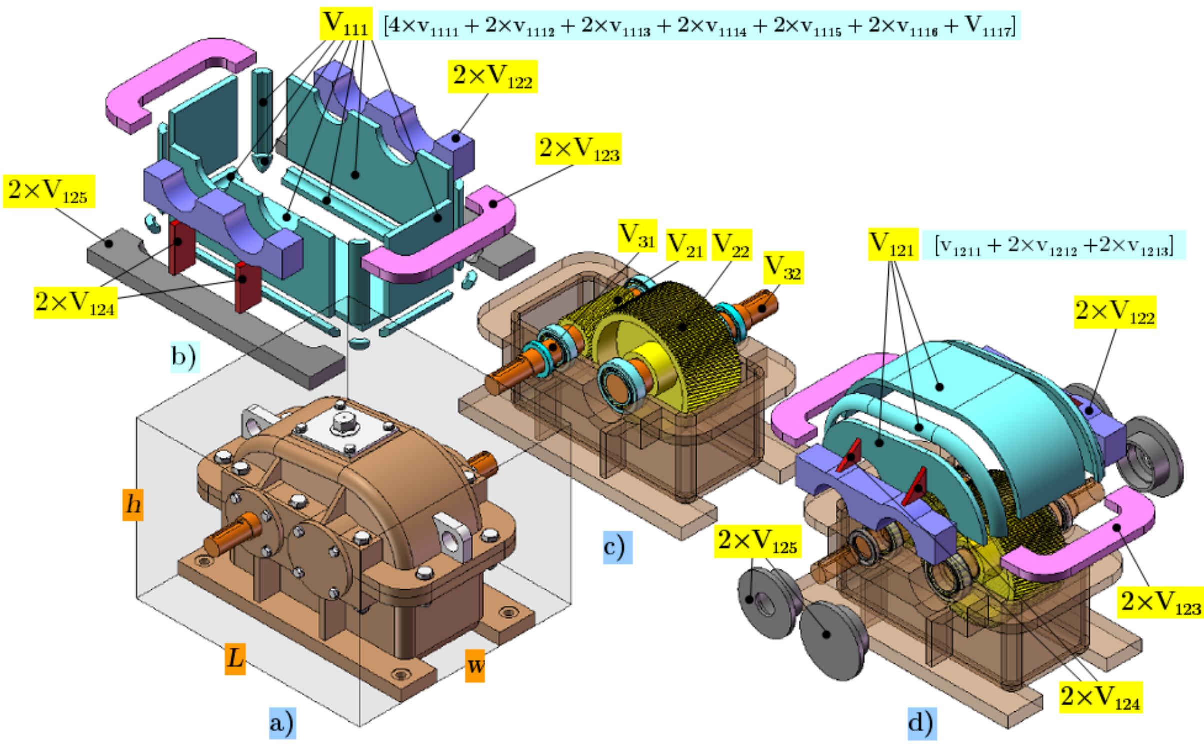

Fig. 3 – a) Completely assembled reducer; b) –exploded view of the reducer's housing body; c) –sub-assembly of the input/output reducer's shafts; d) –exploded view of the reducer's housing cover.

3.3. Constraints

Now is the right moment to turn our attention to the constraints of our optimal design problem. These constraints are all of inequality type, involving strength, geometrical or structural considerations. There are a total of 45 constraints. For conciseness we shall not dwell on the details of their calculation. The interest reader may find all the details of the gearing calculations in the relevant industrial standard document DIN 1987 [18], and the calculation methods used for tapered roller bearings and radial seals correspond to the SKF online catalog (http://skf.com accessed on 21 November 2011); note that they are mostly based on a combination of lookup tables and simple analytical models. It is worth mentioning here that, to enable the calculation of some constraints (geometrical constraints, or the operating temperature), the housing of the reducer was also designed automatically once the virtual space of the gearing was generated. Obviously, a feasible solution of the optimization problem should satisfy those 45 constraints (all the values of these constraints have to be negative or at least zero). The value g_i of a constraint is defined as $g_i = a_i / b_i - 1$, where the constraints is of the form $a_i < b_i$.

With reference to the sketch presented in Fig. 2, the following list of constraints should be viewed.

C1–*The relative difference between the required and the actual gearing ratio must be within the range of* [-2.5% ... +2.5%]. **C2**–*The Hertzian contact pressure on the teeth of gears must not exceed a specified value*. **C3, 4**–*The bending stress on the teeth of gears must not exceed a specified value.* **C5, 6**–*The teeth on pinion* (1) *and wheel* (2) *must not be undercut.* **C7, 8**–*The top land of the teeth on pinion* (1) *and wheel* (2) *must not vanish.* **C9.** *The contact ratio of the gearing must be greater than a specified value.* **C10**–*The addendum coefficient of the wheel* (2) *should be in the range of* [-0.6, 1]. **C11-16** *A set of measurability constraints of the gears.* **C17, 18**–*Constraints related to gears manufacture.* **C19**–*The numbers of teeth of both gears must be relative primes.* **C20, 21**–*The input and output shaft ends must have sufficient diameter step to allow the mounting of a belt wheel.* **C22, 23**–*The inside diameter of the tapered roller bearings on the input and output shafts must be less than the mounting diameter of the seal.* **C24, 25**–*Geometrical constraint related to the space required by the outside ring of the tapered roller bearings on the input and output shafts.* **C26**–*The minimum distance between adjacent tapered roller bearings must be greater than* 15 *mm.* **C27, 28** *Equivalent von Misses maximum stresses experienced by the input and output shafts should be less then the allowable bending stresses.* **C29–31** *The bending strains on the input and output shaft must be below certain threshold values to enable the correct functioning of the gearings and the bearings.* **C32, 33**–*The fatigue life safety factors on the two shafts must not fall below a specified value.* **C34, 35–***The torsional strains in the shafts must be below a threshold value.* **C36, 37**–*The service life of the tapered roller bearings must exceed a specified value*. **C38–41** *The shearing and crushing stresses must not exceed a specified value on the keys and keyways of the input and output shafts.* **C42, 43**–*The shearing and crushing stresses must not exceed a specified value on the key and keyway for mounting the wheel* (2) *on the output the shaft.* **C44.** *The operating temperature of the reducer must not exceed a specified value.* **C45**–*Lubrication constraint—the margin between the minimum and maximum allowable lubricant levels should be no less than* 10 *mm*.

4. A SINGLE-STAGE HELICAL GEAR UNIT MASS MINIMIZATION PROBLEM

Let now us consider the following optimal design problem. A 6.3 kW single-stage helical gear unit (see for example Fig.2) is to be designed for minimum mass and a service life of 8000 h, given an input speed of 750 RPM and a transmission ratio of 3.15. The single-stage helical reducer gears should be based on an ISO 53 basic rack profile. Taken into account the goal of our work, gears materials have a major impact on the reducer's mass. For this reason we considered useful and interesting to run our GA (described earlier in Section 2) for two different types of materials. In this vein we considered the following two situations:

- *Case*–1 when the pinion (1) and the wheel (2) were made of quenched and tempered alloy steel 42CrMo4 and 41Cr4 respectively;
- *Case*–2 in which the gears were made from case hardened alloy steel, pinion (1) – 17CrNiMo6, and wheel (2) – 17Cr3. In this case the number of teeth on the pinion (1) (i.e. the first gene of our optimization problem – Table 1) takes values in the range of 14 – 21 [4].

Now, to solve the optimization design problem for both cases, the proposed GA has been applied. An AMD Turion 64 × 2, 1.6 GHz, 2 GB of RAM laptop was used for computation. The following parameters (which were selected after a set of various trials) were used for our GA:

- Initial population: 50;
- Crossover probability: 0.75;
- Mutation probability: 0.15;
- Total trials: 10000000.

Running the algorithm described earlier in Section 2 (with the above settings) led to a single-stage helical gear unit weighing **33.192 kg** in case–1 and **26.331 kg** in case–2. The values of all considered genes, after optimization (for both cases), are given in Table 2.

Table 2

The values of the genes obtained after optimization (for both cases)

Symbol	z_1	a_w (mm)	x_{n1}	β (°)	ψ_a	i_1	i_2	i_3	i_4	i_5	i_6
Case–1	37	112	0.7102	15.75	0.5024	17	33	5	26	71	23
Case–2	20	90	0.9496	19.25	0.6252	17	34	7	26	71	23

In Fig. 4 the values of the constraints for both cases are represented. The ‘×’ symbols indicate the values of the constraints of the problem at the optimum design, and with black dots are highlighted the constraints whose boundaries are closest to that optimum.

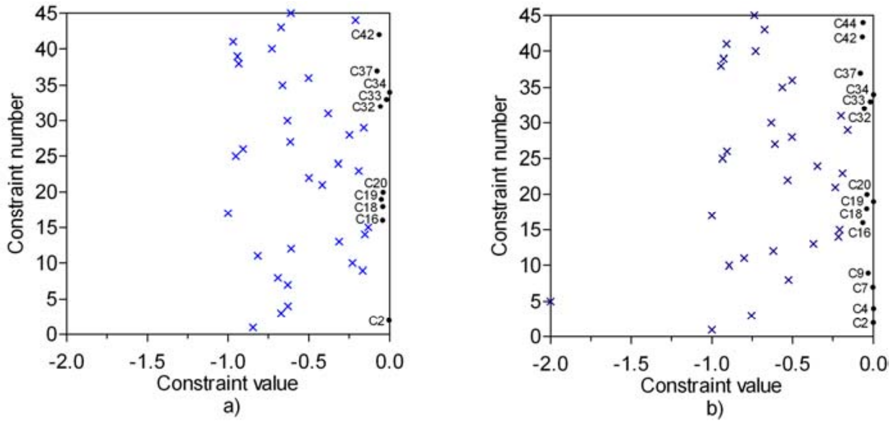

Fig. 4 – Constraints values optimization problem representation: a) case–1; b) case–2.

In Table 3, the main characteristics of the single-stage helical gear unit (traditional design and optimal solutions for both cases) are shown side-by-side.

Table 3

Traditional and optimal design solutions comparison

Symbol	Traditional design		Optimal design	
	Case–1	Case–2	Case–1	Case–2
Gears characteristics				
z_1	33	37	21	20
z_2	104	117	67	63
i_{12}	3.1515	3.1621	3.1904	3.15
m_n (mm)	1.75	1.375	2.25	2
a_w (mm)	125	112	100	90
x_{n1}	0.669	0.7102	0.15	0.9496
x_{n2}	0.8238	0.8275	-0.2531	0.1701
b_1 (mm)	54	60	57	60
b_2 (mm)	50	56	53	56
d_{f1} (mm)	57.00666	51.3751	42.8889	40.5672
d_{f2} (mm)	184.5745	165.9888	145.865	128.5424
d_{w1} (mm)	60.2189	53.8181	47.7272	43.3734
d_{w2} (mm)	189.781	170.1818	152.2727	136.6265
d_{a1} (mm)	64.5504	57.3236	53.0099	49.8575
d_{a2} (mm)	192.1183	171.9373	155.986	137.8327
σ_H (MPa)	558.0433	583.8551	744.3064	739.2687
σ_{HP} (MPa)	593.9152	584.484	745.8103	738.3015
σ_{F1} (MPa)	134.9747	158.3257	180.1192	141.7389
σ_{F2} (MPa)	149.297	173.9785	185.4118	163.6468
σ_{FP1} (MPa)	479.9631	480.703	569.0124	576.2905
σ_{FP2} (MPa)	467.9097	468.0115	493.7047	501.4035
Shafts characteristics				
d_{es1} (mm)	25	20	25	20
d_{es2} (mm)	35	30	38	30
d_{1es1} (mm)	30	24	26	24
d_{1es2} (mm)	40	35	40	40
d_{trb1} (mm)	30	25	30	25
d_{trb2} (mm)	40	35	40	35
σ_{es1} (MPa)	36.6887	72.7944	41.3545	72.7944
σ_{es2} (MPa)	37.9119	59.9687	34.8636	59.738
δ_{11} (mm)	0.0209	0.0546	0.0284	0.0523

Table 3 (continued)

δ_{12} (mm)	0.0006	0.0013	0.0019	0.0027
δ_{21} (mm)	0.0009	0.0018	0.0014	0.0023
F_{lsfs1}	5.4658	2.9958	4.226	3.2001
F_{lsfs2}	4.5608	3.0073	5.6873	3.0189
Overall dimensions of the reducer				
L (mm)	388	361	347	322
w (mm)	326	285	308	316
h (mm)	132	231	203	191
Objective function				
Obj (kg)	37.6619	33.0568	32.9693	26.331

As it can be observed from Table 3, the optimal design solution offers a mass reduction of 12.2% in case–1 and 20.13% in case–2. If a large series of production is considered, the advantages are obviously and manufacturing costs are significantly diminished. For example at 10 (case–1) and 5 (case–2) reducers produced, 1 is for free taking into account the material.

In contrast to the successful determination of this global optimum found in an ‘awkward’ corner defined by several constraint boundaries, consider the outcomes of a set of benchmark experiments. For example, the impact of the required service life of the single-stage helical gear unit on the mass (our objective function in this study). This is a high level relationship shrouded by a plethora of low level connections between the design variables and the constraints, whose analytical calculation can be considered, for all practical purposes, intractable. We can, however, obtain discrete handholds on this relationship by simply running the optimizer for different values of the service life and the results of such a study (for both cases) are shown in Fig. 5.

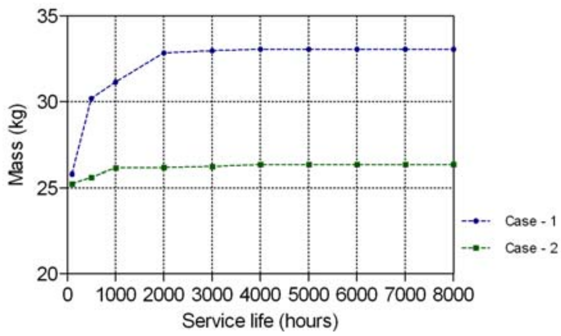

Fig. 5 –Minimum mass single-stage helical gear unit designed for various service life spans for both cases.

From this type of study we can draw some typical conclusions, as shown on the plot (Fig. 5). In case–1 if were aiming for, saying, 1000 h, we would need to sacrifice ≈ 90% of this service life for a ≈ 2 kg (representing roughly 6%). In case–2, for the same period, we need to sacrifice ≈ 95% of service life for 0.7 kg (roughly 2.7%). It is clearly that for both cases, if weight is our sole concern, there would be no point in making any service life sacrifices, if we cannot allow it to drop below 1000 h, as no weight saving would result.

5. **CONCLUSIONS**

In this paper we have shown how a GA, can be used to solve a complex structural design problem of a single-stage helical gear unit. The objective of our optimal design is the minimization of the whole reducer’s mass. Taken into account the goal of our work, gears materials have a major impact on the mass. For this reason we made a study in which we considered two different types of materials for the reducer’s gears described as case–1 (when the gears were made of quenched and tempered alloy steel) and case–2 (in which we considered case hardened alloy steel). Optimal design solutions obtained for both cases were compared to traditional design (i.e. a trial and cut error procedure). In both cases the objective function was subjected to a set of 45 constraints. The design variables considered in the optimization are of mixed nature i.e., continuous, integer, and discrete in total of 11. The results obtained by using GA show significant improvement over the results obtained by traditional design (in case–1, a 12.2% mass reduction was obtained while in case–2 the mass was diminished with 20.13%). Comparing the two optimal design solutions for both cases revealed a reduction of the mass with 20.34%. Also an interesting trade-off between the mass and the service life for both cases was presented. From this study we can conclude that the required service life has to be reduced to less than 1000 h before any significant mass reductions will result. The proposed GA could be easily modified to suit multi-objective design optimization of multi-stage gear units. Also, in the same vein, other objective functions could be considered–manufacturing cost is a simply potential example.

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