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A NEW APPROACH OF TRACKING TRAJECTORY CONTROL OF NONLINEAR PROCESSES

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In this paper, we propose an approach that permits to compute a tracking trajectory control law of continuous nonlinear systems. This approach is based on the study of stability of motion of a given process, by the second method of Lyapunov and using the Lyapunov candidate function to follow a given instruction. We treat also some cases where this control law does not function directly.

Key words: Control, stability, non linear systems, Lyapunov candidate function, tracking trajectory.

1. INTRODUCTION

The control of the evolution of non linear systems is very important, taking into account the complexity of computation they generate.

According to [5], a control of continuous nonlinear systems with good performances in term of tracking trajectory was proposed in 1992 using the flatness property but this approach needs to determine the flat output which is very often difficult. An other solution consists to use a sliding mode control [9].

In this work we propose an approach that permits to compute the tracking trajectory control of a process whose evolution is described by its state equation.

This control is realized from available information concerning the process and its desired evolution [3].

Let consider the following equation of evolution of the system:

$$\begin{cases} \dot{x}(t) = f(x,t) + G(x,t)u(t) \\ y(t) = h(x,t) \end{cases}, \tag{1}$$

where $t \in \tau = [0,+\infty[$ represents the time;

$x \in \mathbb{R}^n$ the state vector of the process;

$u \in \mathbb{R}^l$ the control law;

$y \in \mathbb{R}^m$ the output of the process, with $l \geq m$ and in general we have $l = m$.

The proposed approach is based on the stability of motion and the use of the second Lyapunov method [7, 8, 10].

The tentative Lyapunov function is expressed on the quadratic form

$$v(x,t) = \frac{1}{2}(y_c(t) - y(t))^t (y_c(t) - y(t)), \tag{2}$$

where $y_c(t)$ denotes the desired evolution of the output *y(t)*. The objective is to find a control law $u(x,t,y_c(t))$ that makes $\frac{\mathrm{d}v(x,t)}{\mathrm{d}t}$ negative given the constraints on u and the dynamic of the desired output acceptable by the system.

2. COMPUTATION OF THE TRACKING CONTROL LAW FOR NONLINEAR PROCESS

Let us consider the process whose evolution is described by the state equation

$$\begin{cases} \dot{x}(t)=f(x,t)+G(x,t)u(t) \\ y(t)=h(x,t) \end{cases} \tag{1}$$

and the candidate Lyapunov function [1, 2, 6]

$$v(x,t)=\frac{1}{2}(y_c(t)-y(t))^t(y_c(t)-y(t)), \tag{2}$$

it comes by derivation

$$\frac{\mathrm{d}v(x,t)}{\mathrm{d}t}=(\dot{y}_c(t)-H_x\dot{x}(t))^t(y_c(t)-y(t)), \tag{3}$$

The tracking of the trajectory defined by $y_c(t)$ needs to have

$$\frac{\mathrm{d}v(x,t)}{\mathrm{d}t}\le-\Psi(\|y_c(t)-y(t)\|), \tag{4}$$

where $\Psi(\|y_c(t)-y(t)\|)$ is a definite positive function of $\|y_c(t)-y(t)\|$. Under the condition $H_xG(x,t)$ invertible, this propriety can be obtained by choosing the control law:

$$u(t)=(H_xG(x,t))^{-1}\left[\dot{y}_c(t)-H_xf(x,t)+q(x,t)\right], \tag{5}$$

which implies

$$\frac{\mathrm{d}v(x,t)}{\mathrm{d}t}=q^t(x,t)(y_c(t)-y(t)). \tag{6}$$

The choice of

$$q(x,t)=-\alpha(y_c(t)-y(t)), \tag{7}$$

induce, with $\alpha>0$

$$\frac{\mathrm{d}v(x,t)}{\mathrm{d}t}=-\alpha(y_c(t)-y(t))^t(y_c(t)-y(t)), \tag{8}$$

$$\frac{\mathrm{d}v(x,t)}{\mathrm{d}t}=-\alpha\|(y_c(t)-y(t))\|^2<0 \qquad \forall y(t)\ne y_c(t). \tag{9}$$

It comes:

$$u(t)=(H_xG(x,t))^{-1}\left[\dot{y}_c(t)-H_xf(x,t)-\alpha(y_c(t)-y(t))\right], \tag{10}$$

Example of simulation

To implement this method of non linear system control, we consider the following second order system (*S*), defined by its state equation:

$$(S)\begin{cases} \dot{x}(t)=\begin{bmatrix}-x_2+x_1x_2\\ 2x_1-3x_2\end{bmatrix}+\begin{bmatrix}1+x_1^2\\ 2\end{bmatrix}u. \\ y(t)=x_1+x_1x_2+x_2 \end{cases} \tag{11}$$

We choose the desired trajectory to follow:

$$y_c(t) = \sin(at)\sin(bt), \tag{12}$$

where : $a = 0.1$, $b = 0.2$ and $\alpha=1$. It comes out

$$H_x = [1 + x_2, 1 + x_1], \tag{13}$$

$$H_x G(x,t) = 2x_1 + (x_1^2 + 1)(x_2 + 1) + 2, \tag{14}$$

$$H_x f(x,t) = x_1(2x_1 + x_2(x_2 + 1) + 2) - x_2(3x_1 + x_2 + 4), \tag{15}$$

$$u(t) = \frac{\dot{y}_c - \alpha\left[x_2 - y_c + x_1(x_2 + 1)\right] + x_2(3x_1 + x_2 + 4) - x_1(2x_1 + x_2(x_2 + 1) + 2)}{2x_1 + (x_1^2 + 1)(x_2 + 1) + 2}, \tag{16}$$

and the use of the control law defined in relation (5) leads to a good tracking of the desired trajectory as it appears on Fig. 1

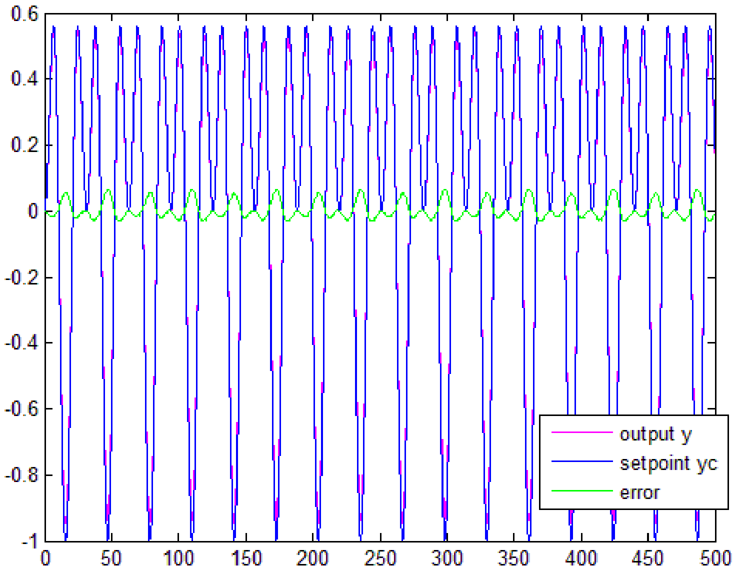

Fig. 1 – Evolution of the system for trajectory tracking y_c.

We can see on Fig. 1 that the system tracks perfectly the trajectory y_c.

3. CASE WHERE ($H_X(X,T)G(X,T)$) IS NOT INVERTIBLE

Let (S) a state system defined in a canonical controllable form

$$(S)\begin{cases} \dot{x}(t) = \begin{bmatrix} 0 & 1\cdots & 0 \\ \vdots & \ddots & \vdots \\ -a_0 & \cdots & -a_{n-1} \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ \vdots \\ 1 \end{bmatrix} u(t) \\ y(t) = \begin{bmatrix} b_0 & \cdots & a_{n-2} & 0 \end{bmatrix} x(t) \end{cases}. \tag{17}$$

We found that for the case where $h(x) = (b_0\ b_1\ \cdots\ b_{n-1})$ and where $b_{n-1} = 0$, the term $H_x G(x,t)$ is non invertible, that's to say that, for example, in the case of a linear system of order n for which the numerator degree is less than $(n-1)$, it is necessary to introduce integrations which modify the original control [4]. Indeed, if the evolution of our system is defined by the relation:

$$\sum_{i=0}^{n} a_i y^{(i)}(t) = \sum_{i=0}^{m} b_i u^{(i)}(t)\,, \tag{18}$$

we replace *u(t)* by :

$$u(t) = \sum_{i=0}^{n-m-1} \beta_i v^{(i)}(t)\,, \tag{19}$$

where $\beta_{n-m-1} \neq 0$ which leads to the equation:

$$\sum_{i=0}^{n} a_i y^{(i)}(t) = \sum_{i=0}^{n-1} b_i^{'} v^{(i)}(t)\,. \tag{20}$$

Hence the controllable canonical form of the equivalent system:

$$(S')\begin{cases} \dot{x}(t) = \begin{bmatrix} 0 & 1\cdots & 0 \\ \vdots & \ddots & \vdots \\ -a_0 & \cdots & -a_{n-1} \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ \vdots \\ 1 \end{bmatrix} v(t) \\ y(t) = \begin{bmatrix} b'_0 & \cdots & b'_{n-2} & b'_{n-1} \end{bmatrix} x(t) \end{cases}, \tag{21}$$

with $b_{n-1}^{'} \neq 0$.

From this system, we calculate the control *v(t)* by the method presented previously, then we deduce the initial control *u(t)* and we replace in (*S*).

Examples of simulation

In order to illustrate this approach, for the examples, we have chosen the case of linear systems.

3.1. Example 1

We consider the second order system (*S*) defined by (17):

$$(S)\begin{cases} \dot{x}(t) = \begin{bmatrix} 0 & 1\cdots & 0 \\ \vdots & \ddots & \vdots \\ -a_0 & \cdots & -a_{n-1} \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ \vdots \\ 1 \end{bmatrix} u(t) \\ y(t) = \begin{bmatrix} b_0 & \cdots & a_{n-2} & 0 \end{bmatrix} x(t) \end{cases}$$

Let us pose $u = \beta_0 v + \beta_1 v^{'}$ where *v* verifies:

$$(S)\begin{cases} \dot{x}(t) = \begin{bmatrix} 0 & 1 \\ -a_0 & -a_1 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} v(t) \\ y(t) = \begin{bmatrix} b_0^{'} & b_1^{'} \end{bmatrix} x(t) \end{cases}.$$

We consider a desired evolution described as: $y_c(t) = \sin(r_1 t)\sin(r_2 t)$

According to the relation (5) of the proposed control in (3.1) we write (10):

$$v(t) = (H_x G(x,t))^{-1}\left[\dot{y}_c(t) - H_x f(x,t) + q(x)\right]$$

as (11):

$$q(x) = -\alpha(y_c - y),$$

so:

$$v = \frac{y_c - b_0' x_2 + \alpha(y_c - b_0' x_1)}{b_1'} + a_0 x_1 + a_1 x_2 - \alpha x_2, \quad (22)$$

$$\dot{v} = x_2(a_0 - \alpha\frac{b_0'}{b_1'}) + \frac{1}{b_1'}\ddot{y}_c + \frac{\alpha}{b_1'}\dot{y}_c + (\alpha - a_1 + \frac{b_0'}{b_1'})(a_1 x_2 - \frac{1}{b_1'}(y_c - \alpha(b_0' x_1 - y_c + b_1' x_2) - x_2(b_0' - a_1 b_1'))) \quad (23)$$

and from (19): $u = \beta_0 v + \beta_1 \dot{v}$. Consequently

$$u = \beta_1(x_2(a_0 - \alpha\frac{b_0'}{b_1'}) + \frac{1}{b_1'}\ddot{y}_c + (\alpha - a_1 + \frac{b_0'}{b_1'})(a_1 x_2 - \frac{1}{b_1'}(\dot{y}_c - \alpha(b_0' x_1 - y_c + b_1' x_2) - x_2(b_0' - a_1 b_1'))) + \frac{\alpha}{b_1'}\dot{y}_c) +$$
$$+\frac{\beta_0}{b_1'}(\dot{y}_c - \alpha(b_1' x_1 - y_c + b_2 x_2) - x_2(b_0' - a_1 b_1') + a_0 b_1' x_1). \quad (24)$$

By choosing the computing parameters as: $\alpha = 0.2$; $a_0 = 0.1$ and $a_1 = 0.66$; $\beta_0 = 0.5$ and $\beta_1 = 0.5$; $r_1 = 0.1$ and $r_2 = 0.3$ It comes $b_0' = 2$ and $b_1' = 0.1$

The simulation result is shown in Fig. 2, where it shows that the system can track reasonably the desired trajectory.

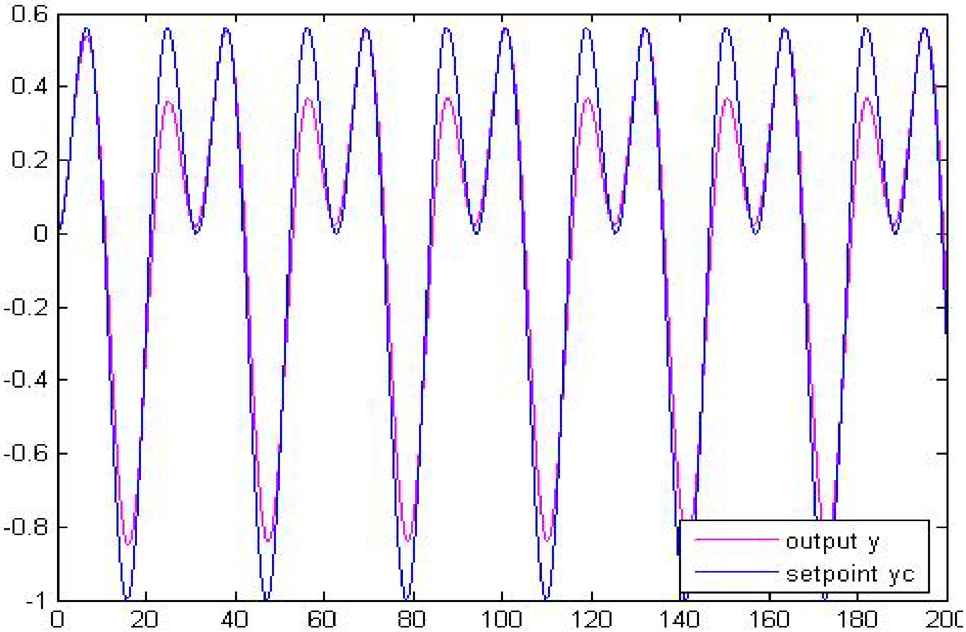

Fig. 2 – Output evolution by tracking the desired trajectory y_c using integrations of the original control.

3.2. Example 2: Case of a very oscillating system with an instable zero:

We consider the same system (*S*), with :
$\alpha = 0.1$; $a_0 = 0.1$ and $a_1 = 0.2$; $\beta_0 = 1$ and $\beta_1 = -1$; $r_1 = 0.1$ and $r_2 = 0.3$. It comes $b_0' = 2$ and $b_1' = 1$.

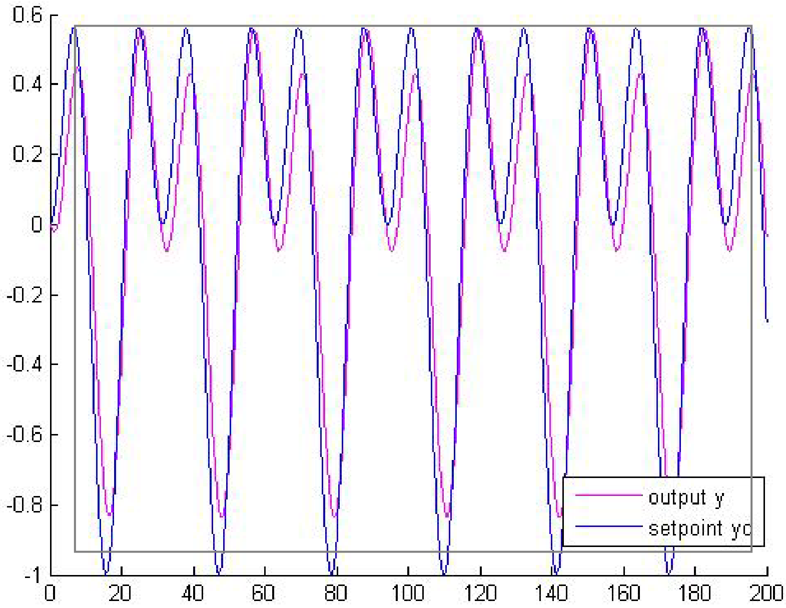

Fig. 3 – Output evolution by tracking the desired trajectory y_c using integrations of the original control, for the system with an instable zero.

3.3. Example 3: Case of a very oscillating unstable system

Let consider the second order system (*S*) defined by (17), where $a_0 = 0.1$ and $a_1 = -0.3$. It is clear that the system is instable. The choice of $u = \beta_0 v + \beta_1 v'$ leads to (21):

$$(S)\begin{cases} \dot{x}(t) = \begin{bmatrix} 0 & 1 \\ -a_0 & -a_1 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} v(t) \\ y(t) = \begin{bmatrix} b_0' & b_1' \end{bmatrix} x(t) \end{cases}.$$

We consider a desired evolution described as: $y_c(t) = \sin(r_1 t)\sin(r_2 t)$, where $\alpha = 10;$ $\beta_0 = 0.5$ and $\beta_1 = 0.5;$. It comes $b_0' = 1$ and $b_1' = 5$.

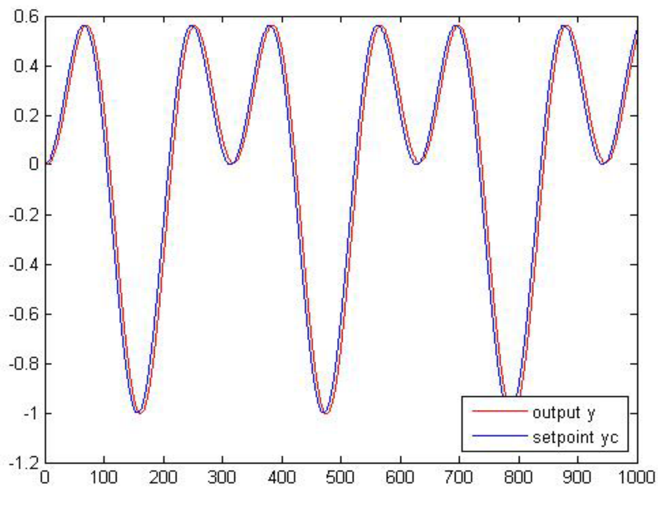

Fig. 4 – Output evolution by tracking of the desired trajectory y_c using integrations of the original control.

4. CONCLUSION

In this work we have presented a new approach that permits to compute a tracking trajectory control of continuous non linear systems. This approach is based on the use of Lyapunov candidate function which enables us to compute the stabilizing control. Illustrating examples have been given to show the robustness of the proposed approach.

For the case when the direct application of the method is not possible, we have proposed a modified approach to find a solution.

Three different examples have been presented to show the efficiency of this approach. The first one treats a stable system that tracks the given trajectory. The second example is a case of a very oscillating system with an instable zero, that we can stabilize on the trajectory and finally for the third example, a very oscillating unstable system and we can see that it reach perfectly the desired trajectory.

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