PUMP INDUCER OPTIMIZATION BASED ON CAVITATION CRITERION

Irina Georgeta MOISĂ¹, Romeo SUSAN-RESIGA¹, Sebastian MUNTEAN²

¹“Politehnica” University of Timișoara, Romania
²Center of Advanced Research in Engineering Sciences, Romanian Academy - Timișoara Branch, Romania
E-mail: resiga@mh.mec.upt.ro

The paper presents the optimization procedure of a pump inducer taking into account the cavitation criterion. The pump inducer optimization procedure is based on inverse design method where the inflow and outflow conditions are known. Axial flow is considered as upstream condition while the downstream swirling flow is suited for the pump impeller inlet, respectively. The key ingredient in this procedure is the function of loading shape which provides the distribution of blade loading from leading edge to trailing edge. A new parametric analytical expression is introduced in this paper for the loading shape function. The objective function corresponds to the minimum pressure coefficient with reverse sign, which is minimized within the chosen parameter space. This procedure is applied for optimize a pump inducer with three blades, with improved cavitating performance.

Key words: pump inducer, inverse design method, cavitational optimization.

1. INTRODUCTION

Nowadays, pumping systems account for nearly 20% of the world’s electrical energy demand and range from 25–50% of the energy usage in certain industrial plant operations [1]. The pumping systems are widely used in industry to store energy, to provide cooling and lubrication services, to transfer fluids for processing, and to provide the driving force in hydraulic systems. Clearly, pumping systems consume a significant amount of the total electrical energy. The key element of the pumping system is the pumping unit which includes the electrical motor and the hydraulic pump. In the manufacturing sector, pumps represent 27% of the electricity used by industrial systems [2]. As a result, the improvements of energetic and cavitational performances of the pumps become more and more important. A common technical solution is to install an inducer at the pump impeller inlet in order to improve the cavitational behavior of the pumps [3]. Generally, inducers are axial flow impellers with a small blade numbers and long blades. As a result, an inducer increases the inlet static pressure at the impeller inlet thus improving the overall cavitational performances. Using conventional design the inducers blades with helical surfaces and a straight line or a combination of straight lines and curves is chosen for the camber line [4]. The extensive experimental and numerical investigations have been performed in our group in order to improve the performances of these inducers during last two decades [4–6]. However, the inducer design guidelines are based on the empirical relationship between geometry parameters and performance characteristics [7]. Moreover, the designed inducers based on conventional methods may have leading edge backflow even at the design point. Therefore, the three-dimensional inverse design method developed by Zangeneh et al. [8] was applied to improve the inlet backflow characteristics of highly loaded turbopump inducers for a liquid hydrogen rocket engine [9]. Optimizing the blade loading distribution using the three-dimensional inverse design method eliminates this inlet backflow [10]. Consequently, the numerical and experimental investigations were performed in order to compare the performance of inducers with different blade loadings, [11].

In situ experimental investigations performed by Anton [12] on large double suction storage pumps have revealed serious cavitational problems. As a result, a model inducer (Fig. 1) was designed using inverse design method [13] based on the technical specifications shown in Table 1. The loading function \( \partial (r v_h) / \partial \phi \) imposed along to three sections of the blade: \( S_h \) near the hub, \( S_M \) in the middle and \( S_T \) near the tip and
dimensionless loading \((rv_u)\), along the leading and trailing edges are plotted against normalized abscissa \(x^*\) in Figs. 2 and 3, respectively.

\[
rv_u = \frac{RV_u}{R_{ref} U_{ref}}, \quad x^* = \frac{x - x_{LE}}{x_{TE} - x_{LE}}, \quad x = \frac{X}{R_{ref}},
\]

where the reference velocity is \(U_{ref} = \omega R_{ref} = 15.64\) m/s, \(x_{LE}\) dimensionless abscissa at the leading edge (LE) and \(x_{TE}\) dimensionless abscissa at the trailing edge (TE), respectively. This solution was numerically analyzed [14] and experimentally investigated together with model impeller leading to improved cavitation behavior over an operating range extended up to 20% [15]. This inducer (called “initial”) was designed taking into account the suggestions provided by Ashihara et al. [9–11].

Table 1

<table>
<thead>
<tr>
<th>Design specifications for inducer</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Rotation speed</strong> (n) [rpm]</td>
</tr>
<tr>
<td><strong>Tip diameter</strong> (D_T = D_{ref} = 2R_{ref}) [mm]</td>
</tr>
<tr>
<td><strong>Hub diameter</strong> (D_H) [mm]</td>
</tr>
<tr>
<td><strong>Blade number</strong> (Z) [-]</td>
</tr>
<tr>
<td><strong>Blade thickness</strong> (t) [mm]</td>
</tr>
<tr>
<td><strong>Design flow rate</strong> (Q) [l/s]</td>
</tr>
</tbody>
</table>

An optimization procedure is developed in this paper and it is applied to the model inducer in order to achieve improved cavitation performances. Section 2 presents the three-dimensional inviscid flow analysis on the initial inducer considering negligible blades thickness. Firstly, a short review of the designing method for thin blade cascades is presented in Section 3. Also, a new analytical parameterized expression for the loading function shape is introduced as a key ingredient in this optimization procedure. Secondly, the optimization steps and the objective function are presented and applied for initial inducer in Section 4. Next, the optimized pressure coefficient obtained on three sections of the inducer blade is compared against 3D numerical results in Section 5 in order to assess the cavitation improvements. Finally, the conclusions are drawn in last section.

### 2. THREE-DIMENSIONAL FLOW ANALYSIS

The initial inducer model case is designed using [16] with constant thickness of 3 mm and rounded leading and trailing edges, respectively. The 3D inducer blade shape is generated stacking several 2D stream surface profiles [17]. For such profiles, a camber line and thickness representation is used in which a thickness distribution is superimposed on the mean camber line as shown in [19, Fig. 3]. In this representation, the camber line and the thickness distribution can be varied independently leading to a good...
control for both parameters. Particularly, the hydrodynamic performances of the inducer are given by the camber line while the blade thickness is selected based on structural considerations. Therefore, throughout this paper the blade thickness is considered negligible. The 3D computational domain includes only one inter-blade channel, Fig. 4, since the flow is similar on each channel at design operating point. The computational domain is extended upstream from leading edge with one cascade pitch while twice the pitch extension is considered downstream from trailing edge. Two camber surfaces corresponding to consecutive blades together with periodic surfaces define the computational domain. A structured mesh with H topology [20] having 420k hexahedral cells was generated using Gambit [21]. The 3D inviscid flow is computed using FLUENT code [22] with following boundary conditions: i) uniform axial velocity on the inlet surface; ii) radial equilibrium condition on the outlet surface; iii) periodic condition on periodic surfaces and iv) impenetrable condition on the chamber, hub and tip surfaces, respectively. Also, the rotational speed of 2900 rpm is imposed on the domain and water is selected as working fluid. As a result, the 3D hydrodynamic field is obtained. The pressure coefficient defined in (2) distribution versus normalized abscissa along to three sections on the inducer blade is plotted in Fig. 5.

\[ c_p = \frac{p - p_{\text{in}}}{\frac{1}{2} \rho U_{\text{ref}}^2}, \]

where the water density is 998.2 kg/m³ and the reference static pressure is computed as mass weighted average value on the inlet surface.

One can observe in Fig. 5 that the minimum value of pressure coefficient is located on the suction side, in the neighborhood of tip blade at 15% near the leading edge. A general view reveals that the minimum value of the pressure coefficient on each section has quite the same value and it is located in the same region. The last observation proves that the inducer design is quite good even within this trial and error framework.

3. DESIGN OF THIN HYDROFOIL CASCADES

It was already mentioned, the camber surface of each blade is built stacking several camber lines located from hub to tip at different radii (Fig. 6a). Thereby, a thin hydrofoil cascade is obtained at each radius if the camber surface is cut by cylinder (Fig. 6c). As a result, several thin hydrofoil cascades have to be designed at various radii in order to obtain the camber surface of the blade inducer. The design of thin hydrofoil cascades was tackled during last four decades. Hawthorne et al. [22] have proposed a quasi-analytical method for designing two-dimensional highly loaded cascades. Susan-Resiga et al. [23] have developed an analytical solution and applied for optimization the turbines hydrofoil cascades [24]. Further, a synopsis view of the analytical solution is revised together with it assumptions. The fluid is assumed to be incompressible and inviscid and the blades with negligible thickness. Considering a potential flow through the cascade, one assumes that the thin blades may be represented by bound vortices of strength \( \gamma(x) \)
distributed along camber lines given by \( y - f(x) = 0, \pm s, \pm 2s, \cdots \), where \( s = S/L \) is the dimensionless blade pitch, Fig. 6. All lengths are made dimensionless by the cascade axial extent.

\[
\begin{align*}
\phi D_T & \quad \text{TIP} \\
S_T & \quad \text{HUB} \\
S_m & \quad \text{TIP} \\
S_n & \quad \text{TIP}
\end{align*}
\]

Fig. 6. – The flow through an axial inducer and associated hydrofoil cascade.

The goal of the cascade design is to find the camber line shape \( f(x) \) given the flow direction far upstream, \( \beta_1 \), and downstream, \( \beta_2 \), as well as the vortex strength distribution \( \gamma(x) \). The axial symmetry assumption requires the pitch-averaged velocity defined according to eq. (3)

\[
\bar{W}(x) = \frac{1}{S} \int_0^S W(x, y) \, dy,
\]

while the velocity upstream and downstream the cascade is \( \bar{W}_1 \) and \( \bar{W}_2 \), respectively,

\[
\bar{W}_{y1} = \bar{W}_{y2} = \bar{W}_x, \quad \text{and} \quad \bar{W}_{y2} - \bar{W}_{y1} = \frac{1}{S} \int_0^S \gamma(x) \, dx.
\]

Obviously, for blades with zero thickness the pitchwise mean value of the axial velocity remains constant within the bladed region as well. It is introduced the pitchwise averaged boundary vorticity,

\[
\bar{\gamma}(x) = \frac{\gamma(x)}{s} = \frac{d\bar{W}_y}{dx}, \quad \text{and the pitch-averaged tangential velocity is written as follow}
\]

\[
\bar{W}_\gamma(x) = \bar{W}_{y1} + \int_0^x \bar{\gamma}(t) \, dt, \quad \text{or} \quad \bar{W}_{y2} - \bar{W}_{y1} = \int_0^1 \bar{\gamma}(t) \, dt.
\]

One can see from (5) that for cascade design can be alternatively provided by \( \bar{W}_\gamma \) schedule instead of \( \bar{\gamma} \). An average streamline, starting at origin, can also be found as,

\[
\bar{f}(x) = \int_0^x \bar{W}_\gamma(t) \, dt.
\]

Integral equation for the camberline shape can be written

\[
\frac{df}{dx}(x) = \frac{1}{2} \left( \frac{\bar{W}_{y1}}{\bar{W}_x} + \frac{\bar{W}_{y2}}{\bar{W}_x} \right) + \frac{1}{2} \int_0^1 \frac{\bar{\gamma}(t)}{2} \frac{\sinh(2\pi/s)(x-t) + f'(x) \sin(2\pi/s)[f(x) - f(t)]}{\cosh(2\pi/s)(x-t) - \cos(2\pi/s)[f(x) - f(t)]} \, dt.
\]

By writing the integrand in (7) as

\[
\frac{\bar{\gamma}(t)}{2} \frac{s}{2\pi} \ln \left\{ \cosh(2\pi/s)(x-t) - \cos(2\pi/s)[f(x) - f(t)] \right\},
\]

one can integrate (8). If \( f = 0 \) at the leading edge \( x = 0 \), i.e. the camberline starts at the origin, we obtain
\[ f(x) = \frac{x}{2} \left( \frac{\bar{W}_{y_1}}{\bar{W}_x} + \frac{\bar{W}_{y_2}}{\bar{W}_x} \right) + \frac{s}{2} \int_0^1 \frac{\gamma(t)}{4\pi} \ln \left\{ \frac{\cosh(2\pi/s)(x-t) - \cos(2\pi/s) [f(x) - f(t)]}{\cosh(2\pi/s)t - \cos(2\pi/s) f(t)} \right\} dt. \quad (9) \]

The determination of the blade profile \( f(x) \) requires the integration of equations (8) and (9), each of which has a singularity at \( t = x \). As \( t \to x \), we have \( f(x) - f(t) \to (x-t) f'(x) \) and it can separated the integrand singularities. With \( \bar{W}_x = 1 \), we can rewrite (9) as,

\[ \frac{df}{dx}(x) = \frac{\tan \beta_1 + \tan \beta_2}{2} + \int_0^1 F_1(x,t) dt + \frac{s\gamma(x)}{2\pi} \ln \frac{x}{1-x}, \quad (10a) \]

where

\[ F_1(x,t) = \frac{\gamma(t) \sinh(2\pi/s)(x-t) + f'(x) \sin(2\pi/s) [f(x) - f(t)]}{2 \cosh(2\pi/s)(x-t) - \cos(2\pi/s) f(t)} - \frac{s\gamma(x)}{2\pi} \frac{1}{x-t}. \quad (10b) \]

Now the integrand \( F_1 \) vanishes as \( t \to x \) and the integral can be calculated with standard quadrature rules. The last term in the right-hand side of (10a) is the Cauchy principal value of the singularity integral. The same approach can be used to remove the singularity in (9),

\[ f(x) = x \frac{\tan \beta_1 + \tan \beta_2}{2} + \int_0^1 F_2(x,t) dt + \frac{s\gamma(x)}{2\pi} \left[ \frac{\ln \left( \frac{2\pi}{s} \right)}{2} + \frac{1}{2} \ln \left( \frac{1 + f''(x)}{2} \right) \right] + x \ln x + (1-x) \ln (1-x) - 1, \quad (11a) \]

\[ F_2(x,t) = \frac{s\gamma(t)}{4\pi} \ln \left\{ \frac{\cosh(2\pi/s)(x-t) - \cos(2\pi/s) f'(x) [f(x) - f(t)]}{\cosh(2\pi/s)t - \cos(2\pi/s) f(t)} \right\} - \frac{s\gamma(x)}{4\pi} \ln \left\{ \left( \frac{2\pi}{s} \right)^2 \frac{1 + f''(x)}{2} \right\}. \quad (11b) \]

The equations (10) and (11) are solved in [23] assuming a first guess for the unknown function \( f(x) \) and its derivative \( f'(x) \), then the right-hand side is evaluated. A suitable first guess is provided by the average streamline (7). The new values for \( f(x) \) and \( f'(x) \) are further used to evaluate again the right-hand sides in (10a) and (11a), and the iterative process continues until the solution change from one iteration to the next one is smaller than a given threshold. Since the integrals of \( F_1 \) and \( F_2 \) cannot be evaluated analytically, a discrete representation of the unknown function was considered by Susan–Resiga et al. [23]. First, a set of points \( x_1 < x_2 < \ldots < x_{n+1} = 1 \) are considered on the unit interval. The goal was to find the function values \( f_1 \equiv f(x_1), f_2 \equiv f(x_2), \ldots, f_{n+1} \equiv f(x_{n+1}) \) as well as the function derivatives \( f'_1 \equiv f'(x_1), f'_2 \equiv f'(x_2), \ldots, f'_{n+1} \equiv f'(x_{n+1}) \) using the above iterative method.

The key ingredient of the design and optimization procedure of the thin hydrofoil cascade is

---

**Fig. 7 – Flow chart of the optimization procedure.**
the vortex strength distribution $\overline{\gamma}(x)$ defined on the unit interval. A new loading function shape $g(x, XTE, SLO, FLE)$ with three parameters is introduced by eq. (12): 1) the abscissa, $XTE$; 2) the slope $SLO$; and 3) the load (Fig. 2) at the leading edge, $FLE$ ($\overline{\gamma}(0) = FLE$).

$$g(x, XTE, SLO, FLE) = (FLE + SLO \cdot x) \cdot \text{erf} \left( \frac{\sqrt{x}}{2} \cdot \frac{1 - x}{1 - XTE} \right) \cdot g^\circ(XTE, SLO, FLE), \quad (12)$$

where $g^\circ(XTE, SLO, FLE) = \int_0^1 (FLE + SLO \cdot x) \cdot \text{erf} \left( \frac{\sqrt{x}}{2} \cdot \frac{1 - x}{1 - XTE} \right) \, dx^{-1}$

resulting in

$$\int_0^1 g(x, XTE, SLO, FLE) \, dx = 1.$$

The new function of loading shape is defined in terms of three parameters ($XTE$, $SLO$ and $FLE$) while an expression with two parameters was used by Resiga et al. [23]. The third parameter $FLE$ is directly linked with the camber line curvature at the leading edge. As a result, the camber line curvature at the leading edge is decreased if the load value at leading edge is larger.

4. OPTIMIZATION PROCEDURE

The parametric camber line is integrated in the optimization procedure in order to be found the minimum value of the objective function. The quasi-Newton method and a finite-difference gradient are used to minimize the function with respect to both parameters $SLO$ and $FLE$. This optimization procedure is effective if the problem is not noisy [25]. As a result, the gradients are found relatively easily and only a local search is required. The optimization algorithm, shown in Fig. 7, is implemented in FORTRAN using the UMINF subroutine from IMSL library [26]. The optimization procedure is applied for several thin hydrofoil cascades with prescribed values of $w_y$, $w_z$, and $s$ given in Table 2. Each thin hydrofoil cascade was computed imposing the normalized abscissa $XTE$ yielding the parameters slope $SLO$ and loading at the leading edge $FLE$ as well as the minimum value of the pressure coefficient $c_{p_{\text{min}}}$. The procedure is applied for several thin hydrofoil cascades associated to different sections from hub to tip with variable values of the normalized abscissa XTE.

<table>
<thead>
<tr>
<th>section</th>
<th>$S_H$</th>
<th>$S_M$</th>
<th>$S_T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_1$ = $w_z$</td>
<td>1.2941</td>
<td>1.9448</td>
<td>2.6318</td>
</tr>
<tr>
<td>$w_2$ = $w_z$</td>
<td>0.6591</td>
<td>1.3959</td>
<td>2.1603</td>
</tr>
<tr>
<td>$s = S/L$</td>
<td>0.9674</td>
<td>1.4539</td>
<td>1.9674</td>
</tr>
<tr>
<td>XTE</td>
<td>0.27</td>
<td>0.465</td>
<td>0.59</td>
</tr>
<tr>
<td>SLO</td>
<td>0.12005</td>
<td>0.0645</td>
<td>0.04804</td>
</tr>
<tr>
<td>FLE</td>
<td>0.3681</td>
<td>0.2993</td>
<td>0.2489</td>
</tr>
</tbody>
</table>

The distribution of the minimum pressure coefficient in terms of the normalized abscissa $XTE$ on three sections, near the hub ($S_H$), middle ($S_M$) and tip ($S_T$) are plotted in Fig. 8.
5. NUMERICAL RESULTS

The pressure coefficient distribution on optimized thin hydrofoil cascade of the pump inducer at three sections $S_H$, $S_M$ and $S_T$ is plotted in Fig. 11. A wide flat region at $-0.029$ can be observed in the optimized pressure coefficient distribution on the suction side at all sections leading to an improved cavitation behavior of the pump inducer. The optimized pressure coefficient distribution is compared with initial one (plotted in Fig. 5) in order to assess the improvements. One can see that on the section $S_H$ near the hub the minimum value is increased from $-0.046$ to $-0.029$ (Fig. 12) while on section $S_T$ near the tip from $-0.060$ to $-0.029$ (Fig. 13), respectively. Additionally, the position of the minimum value is shifted from $x^* = 0.11$ to $x^* = 0.39$ on section $S_H$ and on the section $S_T$ from $x^* = 0.285$ to $x^* = 0.33$. It is clearly shown that the optimization procedure shifts the minimum value of the pressure coefficient toward the middle region of the inducer camber surface.
6. CONCLUSIONS

In this paper, a new procedure based on inverse design method was developed to optimize the cavitational behavior of the pump inducer. We prescribe an axial inflow with an outlet swirl suited for the pump impeller inlet. Firstly, the 3D inviscid flow analysis of the inducer with thin blades is performed. Secondly, several thin hydrofoil cascades are optimized based on cavitation criterion but the results are only presented for three of them. The key ingredient in optimization procedure is the new function of loading shape with three parameters introduced in this paper. This function provides an appropriate mathematical model for the distribution of loading from leading edge to trailing edge on each blade section. The procedure is applied for pump inducer with three blades in order to obtain optimized cavitational performances. The numerical value of the minimum pressure coefficient for the optimized case leads to an improved cavitational behavior of the pump inducer. Also, the optimization procedure shifts the minimum value of the pressure coefficient toward the middle region of the inducer chamber surface.

ACKNOWLEDGEMENTS

Ms. Moisa I.G. was partially supported by the strategic grant POSDRU 2009 project ID 50783 of the Ministry of Labour, Family and Social Protection, Romania, co-financed by the European Social Fund – Investing in People. Prof. Susan-Resiga R. and Dr. Muntean S. were supported by the Romanian Academy program “Hydrodynamics Optimization and Flow Control of the Hydraulic Turbomachinery in order to improve the Energetic and Cavitational Performances”.

REFERENCES


Received January 15, 2013