CHANNEL RESONANCES AND REDUCED R-MATRIX

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The reaction dynamics in eliminated channel can be described by the 'equation of channel state', an equation relating the channel reduced R-matrix element to channel logarithmic derivative. The 'channel state equation' accounts for channel coupling resonances originating in bound or quasistationary channel states. The channel coupling results into 'direct compression' of channel resonance decay width and in a shift of resonance pole to real axis. Possible connections of channel resonances to 'channel coupling poles' and to K-matrix 'molecular resonances' are discussed.

Key words: R-matrix, channel resonances, quantum defect.

1. INTRODUCTION

The approach of multichannel problems in terms of effective operators is commonly used in Scattering Physics. The formal basis is, for example, Projector Method developed in Nuclear Physics by Feshbach (1962). The two projection operators are used to divide the set of scattering channels into two subsets, retained channels and eliminated channels. An effective Hamiltonian for retained channels is obtained; the original "bare" interaction is replaced by an effective one, taking into account eliminated channel. The method of channel elimination is historically related to Wigner Reduced R-Matrix (Lane and Thomas, 1958). The Reduced R-Matrix consists from a term describing the retained channels, uncoupled to eliminated ones, and an additional term which accounts for coupling between the two groups of channels. One can extend the concept of reduced operator to Collision Matrix too.

The Reduced Collision Matrix, both for open or closed eliminated channel, depends on channel couplings as well as on phenomena developing in eliminated channel n, described by pole $(1 - \Re_{nn} L_n)^{-1}$, ($\Re_{nn}$ – reduced R-matrix element, L_n – channel logarithmic derivative). The channel equation $1 - \Re_{nn} L_n = 0$ does encompass both bound and quasistationary channel states in multichannel reactions. This channel state, subject of equation $1 - \Re_{nn} L_n = 0$, is responsible for 'channel resonances'; the 'channel resonances' are complementary to multichannel resonances described by R-matrix poles. The 'channel state equation' implies an interplay of inner configuration space parameters to those of channel space, resulting in specific properties as channel renormalization (threshold and channel coupling compressions) of the reduced width and threshold level shift.

2. CHANNEL STATES AND CHANNEL RESONANCES

The channel state is defined, (Robson and Lane, 1967; Lane, 1969), as a state with large overlap to only one channel; it could be either bound (below particle threshold) or quasistationary (above particle threshold) state. The R- matrix prototype of channel state is an R-matrix level with maximum reduced width; its decay reduced widths for complementary reaction channels are negligible. The R-matrix channel state in multichannel systems is subject of mixing to complementary states due to changes in interactions and in channel boundary conditions. The change in boundary conditions could result in that it asymptotes the pure

out wave at state energy, either real or complex. The state which asymptotes pure *out* wave is either bound or quasistationary state. It encompasses both inner configuration space and channel space properties.

An example of channel (single particle) state, having a large overlap to channel wave function, is the bound state described in R- Matrix Theory (Lane and Thomas, 1958) by equation $R^{-1} = S$ (S is logarithmic derivative at negative energy). Bound state in R-Matrix Theory is defined not by a R-matrix pole but rather by equation $R_{nn}^{-1} = S_n$, matching internal logarithmic derivative, R_{nn}^{-1}, to channel logarithmic derivative, S_n, (Lane and Thomas, 1958). The shift-function S_n is logarithmic derivative of Whittaker function which, at its turn, is *out* wave at negative energy.

Quasistationary state or 'radioactive state decaying in reaction channel' is corresponding at positive energy of the negative energy bound state. The quasistationary state is a pure *out* wave; the outgoing wave at infinity corresponds to quasistationary state decay (*e.g.* Sitenko, 1990). By analogy to bound state case, the R-matrix equation for quasistationary state should be $R_{nn}^{-1} = L_n^{>}$, with $L_n^{>}$-channel logarithmic derivative for *out* wave. The logarithmic derivative of outgoing wave $L_n^{>}$ is the corresponding, at positive energy, of the shift function $S_n = L_n^{<}$ defined for negative energy; the two functions are related by analytic continuation. As in case of bound states, this condition yields a set of eigenenergies which now are complex. The boundary conditions for quasistationary state are those of *out* waves at state energy E_λ, not at prescribed energy. The quasistationary level is no more defined by an R-matrix pole but rather by channel equation $1 - R_{nn}(E_\lambda)L_n(E_\lambda) = 0$.

The equation $1 - R_{nn}L_n = 0$ defines pole of reduced R-matrix. We related it here to R-matrix approach to bound state and quasistationary state and to "channel state" defined by Lane. The equation $1 - \Re_{nn}L_n = 0$ defines pole of reduced collision matrix.

A pole at negative energy in collision matrix elements can be obtained by the condition $\Re_{nn}^{-1} = L_n^{<} = S_n$, ($S_n$-shift function below threshold). In non-coupling limit the reduced R- matrix $\Re_{nn}$ reduces to single channel R-matrix element R_{nn}. Or this is just bound state condition of the R-Matrix Theory, (Lane and Thomas, 1958, p. 280). The result $\Re_{nn} - L_{n<}^{-1} = 0$ is an R-matrix proof that the bound state from a closed channel induces resonance in competing open channels of the multichannel system.

A pole in reduced collision matrix at positive energy is obtained by a condition which is analog to the bound state case, $\Re_{nn}^{-1} = L_n^{>}$.

The equation

$$1 - \Re(E_\lambda)L_n(E_\lambda) = 0 \tag{1}$$

defines the channel (bound or quasistationary) state in case of multichannel system. Both $\Re$ and L, energy dependent, are assumed to be analytically continuable from E to complex E_λ.

The energy E_λ of the *n*-channel state λ in multichannel system, subject to equation $1 - \Re_{nn}(E_\lambda)L_n(E_\lambda) = 0$, is complex even at negative energy due to complex-valued reduced-matrix element $\Re_{nn}$; the bound state becomes quasistationary due to coupling to open reaction channels.

As physical examples of multichannel resonances we mention the 'channel' and 'inner' resonances in Multichannel Quantum Defect Theory. The resonances in electron multichannel scattering on atoms or ions originate either in multielectron excitations of electronic inner core or from excitation of Rydberg far-away located states; they are called "inner resonances" and "channel resonances", respectively (Lane, 1986). Adopting this terminology we could have in mind another processes in Scattering Physics, as e.g. nucleon scattering on nuclei. The "inner" and "channel" resonances do correspond to "compound nucleus"- and to "single particle"-resonances, respectively. In the R-Matrix Theory, usually, one considers only compound system multichannel resonances described by poles of all R-matrix elements. The inner multichannel resonances are described by R-matrix poles while the channel resonances are related to channel's logarithmic derivative and reduced R- matrix element.

The 'channel state', subject of equation $1 - R_{nn}L_n = 0$, shares both channel (L_n) and compound system (R_{nn}) characteristics implying specific spectroscopic aspects. In next chapters one discusses multichannel atomic and nuclear phenomena in relation to the 'channel state equation'.

3. CHANNEL RESONANCE RELATED TO POTENTIAL SCATTERING

The level's (real or complex) energy, E_0, in absence of coupling to open channels, is defined by bound or quasistationary state condition $L_n^{-1}(E_0) - R_{nn}(E_0) = 0$.

The actual level's energy, defined by $L_n^{-1}(E_0) - \Re_{nn}(E_0) = 0$, implies level shift and decay width with respect to same parameters of origin state.

The channel state equations

$$L_n^{-1}(E_0) - R_{nn}(E_0) = 0 \tag{2}$$

$$L_n^{-1}(E_0) - \Re_{nn}(E_0) = 0 \tag{3}$$

$$\Re_{nn} = R_{nn} + \Delta_n + i\Gamma_n \tag{4}$$

result into resonance parameters as reduced width. The potential scattering means that $R_{nn}(E)$ and $\Re_{nn}(E)$ are smooth functions on energies; the only function stronger dependent on energy in channel equations is the logarithmic derivative.

The complex level shift is, in first order of Taylor expansion $(L_n^{-1}(E_0) - R_{nn}(E_0) = 0)$

$$E_0 - E_0 = -\frac{\Delta_n(E_0) + i\Gamma_n(E_0)}{(dL_n/dE_0) R_{nn}^2 + R_{nn}'(E_0)} . \tag{5}$$

The R-matrix 'reduced width' (Lane and Thomas, 1958) defined by equation

$$R_{nn}' = R_{nn}^2 / \gamma_n^2 \tag{6}$$

results into 'renormalized reduced width' equation

$$L_n'(E_0) + 1/\gamma_n^2 = 1/\omega_n^2 \tag{7}$$

$$\omega_n^2 = \gamma_n^2 / (1 + L_n'(E_0)\gamma_n^2) = \beta_n \gamma_n^2 . \tag{8}$$

The subunitary factor ($L_n' > 0$)

$$\beta_n = 1/(1 + L_n'(E_0)\gamma_n^2) \tag{9}$$

is R-matrix compression factor and it is effective near threshold ('threshold compression'). The logarithmic derivative is monotone increasing with energy, $\operatorname{Re} L_n' > 0$ and $\operatorname{Im} L_n' > 0$. The real and imaginary parts of R-matrix compression factor are positive and subunitary.

The complex level-shift in terms of renormalized reduced width becomes

$$E_0 - E_0 = -(\Delta_n(E_0) + i\Gamma_n(E_0)) \frac{\omega_n^2}{R_{nn}^2(E_0)} . \tag{10}$$

The decay width $-\operatorname{Im}(E_0)$ component originating in channel coupling is

$$G_n = -\operatorname{Im}(E_0 - E_0) = \Gamma_n(E_0) \frac{\omega_n^2}{R_{nn}^2(E_0)} . \tag{11}$$

In this definition is assumed that logarithmic derivative $L_n(E_0)$ is real (case of negative energy $E_0 < 0$ or case of non-zero partial wave just above threshold $E_0 \approx 0$). Observe that channel resonance width $\Gamma_n(E_0)\omega_n^2/R_{nn}^2$ is factorised into a component depending on single channel parameters ω_n^2/R_{nn}^2 and another one originating in channel couplings Γ_n.

In limit $R'_{nn} \approx 0$ the renormalized reduced width becomes $\omega_n^2 = (dL_n/dE)^{-1}$. Far away from threshold $L_n \approx$ const. and ω_n^2 is very large; no resonance effect. The logarithmic derivative L_n is strongly varying near threshold and renormalized reduced width ω_n^2 becomes smaller; a resonance effect can appear only near threshold.

In statistical limit for background scattering (Lane and Thomas 1958)

$$\Gamma_n = \operatorname{Im} \Re_{nn} = \pi < \gamma_{\lambda n}^2 >/D, \tag{12}$$

where $< \gamma_{\lambda n}^2 >/D$ is n-channel pole strength function, (D - mean level spacing). In the limit $R'_{nn}(E_0) \to 0$ it results $\omega_n^2 \to (dL_n/dE_0)^{-1}$ and $\omega_n^2/R_{nn}^2 \to (dL_n/dE_0)^{-1}R_{nn}^{-2} = L_n(E_0)(dL_n/dE_0)^{-1}R_{nn}^{-1}$

$$G_n = < \gamma_n^2 > \pi/D \times L_n(E_0)/\left(dL_n/dE_0\right) \times R_{nn}^{-1}. \tag{13}$$

Observe that quantity $\pi/D \times L_n(E_0)/\left(dL_n/dE_0\right)$ is dimensionless and positive. In threshold limit $dL_n/dE_0 \cong (L_n(E_0) - L_n(0))/\Delta E = L_n(E_0)/\Delta E$ it is just π provided $\Delta E = D$. Anyway the width G_n is proportional to averaged reduced width $< \gamma_n^2 >$ and from here its physical significance.

The implication of concomitant validity of equations for single channel case (2), and for multichannel case (3) was exploited in this paragraph, proving specific aspects of 'channel coupling resonance' which does not originate in a R-matrix level but rather in interplay of reduced R-matrix element with energy dependent channel logarithmic derivative.

4. QUANTUM DEFECT CHANNEL RESONANCES

The energy-dependent logarithmic derivative, with poles on real axis, is used in Atomic Physics for studying electron Rydberg states, at negative energy. The (reduced) R-matrix element of eliminated channel could be then considered as non-dependent on energy. By applying the channel state equation to electron Rydberg states, one should obtain basic results of Quantum Defect Theory. The electron (closed channel, $n=e$) logarithmic derivative is given by (Baz, Zeldovich and Perelomov, 1971; Landau and Lifschitz, 1980) $L_{e<} = -\cot \pi\sqrt{\alpha^2/(E_\pi - E)}$ ($\alpha^2 = e_1^2 e_2^2 m/2\hbar^2$; e_1, e_2 and m-channel particles electric charges and reduced mass). The Rydberg states, in absence of inner core, are defined by equation $L_{e<}^{-1} = 0$. The level equation $L_{e<}^{-1} = 0$ results into energy of Rydberg states, defined with respect to E_π threshold energy, $E_\pi - E_n = \alpha^2/n^2$, with n an integer number.

If the quantum defect, due to inner core, is taken into account then the principal quantum number n is replaced by effective quantum number ν resulting into level-shift, which, at its turn, is related to quantum defect μ by relation $\nu = n - \mu$ (Seaton, 1983). The electron channel logarithmic derivative below threshold becomes $L_{e<} = -\cot \pi\nu = \cot \pi\mu$. In non-coupling limit, the reduced R- matrix becomes the single channel R-matrix element R_{ee}. The bound state condition, for one electron closed channel, according to $L_{e<}^{-1} - R_{ee} = 0$, is now $\tan \pi\mu = R_{ee}$; it relates one-channel quantum defect, μ, to Rydberg channel R-matrix element, R_{ee}.

The coupling of the closed channel e to open channels results in the resonance equation

$$\tan \pi\nu + \Re_{ee} = 0, \tag{14}$$

$$\tan \pi \tilde{\mu} = \Re_{ee}. \tag{15}$$

It is Seaton theorem for a channel immersed in multichannel system, $\tan \pi \tilde{\mu} = \Re_{ee}$, or approximatively $\pi \tilde{\mu} \approx \Re_{ee}$. These relations result into definition of complex effective quantum number $\nu = n - \tilde{\mu}$ and of 'complex quantum defect' $\tilde{\mu} = \mu_{Re} + i\mu_{Im}$ in terms of reduced R- matrix element.

The Rydberg states, with energies $E_n - E_\pi = -\alpha^2 / n^2$, from the electron closed channel will induce in the electron (elastic) scattering channel set of resonances shifted to new energy positions $E_\nu = E_\pi - \alpha^2 / \nu^2$. The resonance equation, $\tan \pi(\nu - n) + \Re_{ee} = 0$, results in

$$E_\nu = E_\pi - \alpha^2 / \nu^2 \cong E_n - 2\frac{\alpha^2}{n^3}\tilde{\mu}, \tag{16}$$

with $E_n = E_\pi - \alpha^2 / n^2$ (Seaton, 1983; see also Sobelman *et al.*, 1981). The open channel resonance shift and width are determined by reduced R- matrix or complex quantum defect of Rydberg channel

The decay width of Quantum Defect Channel Resonance is

$$-\operatorname{Im} E_\nu = \Gamma_\nu = 2\alpha^2 \frac{\mu_{Im}}{n^3} = 2\frac{\alpha^2}{n^3}\frac{\operatorname{Im}\Re_{ee}}{\pi}. \tag{17}$$

The Rydberg channel logarithmic derivative

$$L_e^< = -\cot \pi\nu, \tag{18}$$

where $\nu = \alpha / \sqrt{(E_\pi - E)}$, and $\alpha = \nu\sqrt{(E_\pi - E)} = const.$, results into,

$$\frac{dL_e}{dE} = \frac{dL}{d\nu}\frac{d\nu}{dE} = \frac{\pi}{\sin^2 \pi\nu}\frac{1}{2}\frac{\nu^3}{\alpha^2}. \tag{19}$$

The decay width of Quantum Defect Channel Resonance in relation to potential scattering

$$-\operatorname{Im}(E_0 - E_0) = \frac{\operatorname{Im}\Re_{ee}}{(dL_e / dE)L_e^{-2}}, \tag{20}$$

$$\frac{L_e^2}{(dL_e / dE)} = \frac{\cot^2 \pi\nu}{(\pi / \sin^2 \pi\nu)(\nu^3 / 2\alpha^2)} = \frac{2\alpha^2}{\nu^3}\frac{\cos^2 \pi\nu}{\pi}, \tag{21}$$

$$-\operatorname{Im}(E_0 - E_0) = \frac{\operatorname{Im}\Re_{ee}}{(dL_e / dE)L_e^{-2}} = \frac{2\alpha^2}{\nu^3}\frac{\cos^2 \pi\nu}{\pi}\operatorname{Im}\Re_{ee}. \tag{22}$$

In the limit $\cos^2 \pi\nu \approx 1$, or $\nu \approx$ integer $= n$ (quasi-genuine Rydberg state), the two formulae of decay width, $-\operatorname{Im} E_\nu$ and $-\operatorname{Im}(E_0 - E_0)$, do coincide. The limit $\nu \approx n$ or $E_0 \cong E_0$ is congruent with validity of first-order Taylor expansion of logarithmic derivative $L_n(E_0)$.

5. DIRECT (COUPLING) COMPRESSION OF CHANNEL RESONANCE

There are two mechanisms of 'decay width compression' of channel resonance. One of them is 'threshold compression' of decay width, (see formulae 8 and 9), mentioned by Lane (Lane, 1970) in relation to threshold effects originating in single particle resonances. Another one, mentioned with respect to 'quasiresonant scattering', (Hategan, Graw and Comisel, 2005), is the 'direct (channel coupling) compression' of decay width due to channel couplings, even to interplay of channel resonance with background (potential) scattering. The last mechanism is here discussed for 'channel (coupling) resonances'.

The effective term of W collision matrix, $\Delta W_N \propto R_{Nn}(L_n^{-1} - \Re_{nn})^{-1} R_{nN}$, is resembling to additional term of $\Re_N$ reduced R-matrix. The only difference is the "bare" R-matrix element R_{nn} of eliminated n-channel is here replaced by its reduced (or effective) counterpart $\Re_{nn} = R_{nn} + \Delta_n + i\Gamma_n$; the n-channel reduced $\Re_{nn}$-matrix element does include also rescattering effects from complementary open channels (N)

$$\Re_{nn} = R_{nn} - R_{nN}(R_{NN} - L_N^{-1})^{-1} R_{Nn}. \tag{23}$$

The physical implication of the reduced R-matrix element $\Re_{nn}$, instead of uncoupled n-channel R- matrix R_{nn}, is obtained by writing the open retained channels component $(R_{NN} - L_N^{-1})^{-1}$ in terms of the W_N^0 collision matrix and T_N^0 transition matrix for open uncoupled channels, $W_N^0 = 1 + 2iT_N^0$. By using natural boundary conditions, $L_N = iP_N$, one obtains $(R_{NN} - L_N^{-1})^{-1} = P_N^{1/2}(T_N^0 - i)P_N^{1/2}$. The reduced $\Re_{nn}$-matrix element of the eliminated channel becomes,

$$\Re_{nn} = R_{nn} + R_{nN} P_N^{1/2} (i - T_N^0) P_N^{1/2} R_{Nn} \tag{24}$$

$$\Delta_n = -R_{nN} P_N^{1/2} \operatorname{Re} T_N^0 P_N^{1/2} R_{Nn} \tag{25}$$

$$\Gamma_n = R_{nN} P_N^{1/2} (1 - \operatorname{Im} T_N^0) P_N^{1/2} R_{Nn}. \tag{26}$$

One has to remark that effective term of reduced R- matrix ('level's shift' Δ_n and 'width' Γ_n) depends not only on coupling strength R_{Nn} but also on rescattering T_N^0 in open channels. The imaginary component Γ_n of reduced R-matrix effective term has meaning of 'decay width'. (In chapter 3 on channel resonance one proves the relation (11) of the corresponding decay width to Γ_n.) In following we discuss a specific property of the 'decay width' related to coupling to and to rescattering in open complementary channels. The Unitarity Condition, $\operatorname{Im} T_{aa} = (TT^+)_{aa}$ results into $(1 - \operatorname{Im} T_{aa}) = (1 - \sum_{b=1}^N |T_{ab}|^2) < 1$. The corresponding component of the width is compressed by term $\sum_{a=1}^N R_{na}(1 - \sum_{b=1}^N |T_{ab}^0|^2)R_{an}$. The width compression is mostly transparent for only one open channel a, $T_{aa}^0 = \sin\delta_a e^{i\delta_a}$ (δ_a-open channel scattering phase shift), $\Gamma_n \propto (1 - |T_{aa}|^2) = \cos^2\delta_a$ and $\Delta_n / \Gamma_n = -\operatorname{Re} T_{aa}^0 /(1 - |T_{aa}^0|^2) = -\tan\delta_a$.

6. DYNAMICAL ASPECTS OF CHANNEL COUPLING RESONANCES

A bound or a quasistationary state, originating in an eliminated channel, induces a resonance in open competing channels. Both the width and level shift are determined by channels couplings and by rescattering in open channels. A channel resonance pole, defined by equation $1 - \Re_{nn} L_n = 0$ but not related to a specific R- matrix level, is subject of motion in complex energy plane, both by proximity threshold and *esp* by couplings to complementary reaction channels. A broad quasistationary state (from eliminated channel) results in a smaller width resonance which, in addition, is shifted to a lower energy. This result can be analytically demonstrated in terms of complex scattering length *via* its relation to channel reduced R- matrix element.

Phenomena developing in threshold channel near zero-energy (bound-unbound transition zone) could be properly described in terms of Scattering Length, (*e.g.* Drukarev, 1978; Burke, 2011) . The scattering length a_n, for neutron *s*-wave, is defined in terms of δ_n scattering phase shift

$$k \cot\delta_n = -1/a_n + 1/2 r_0 k^2, \tag{27}$$

where k and r_0 are channel wave number and effective radius; the Scattering Length defines the Scattering Amplitude just at threshold energy. On the other hand the *s*-wave nuclear scattering phase-shift δ_n is related to R-matrix element R_{nn} and penetration factor P_n through relation $\tan \delta_n = P_n R_{nn}$. In zero-energy limit the scattering length and R_{nn} matrix element, are related by $a_n = -bR_{nn}$ with *b*-channel radius. The relation is extended to complex phase-shift and reduced R-matrix element too, $\tan \tilde{\delta}_n = P_n \mathfrak{R}_{nn}$. The complex scattering length $\tilde{a}_n = a_1 - ia_2 = -b\mathfrak{R}_{nn}$ components are related to those of reduced R-matrix element, $\mathfrak{R}_{nn} = (R_n + \Delta_n + i\Gamma_n)$, namely $a_1 = -b(R_{nn} + \Delta_n)$, and $a_2 = b\Gamma_n > 0$. (The imaginary component $\Gamma_n > 0$ is consequence of subunitary collision matrix element $|W_{nn}| < 1$. The Optical Model Scattering Length has also a negative imaginary component too, $-ia_2$, with $a_2 > 0$ due to absorptive component of optical potential. The Optical Model pole is located at $k = i/\tilde{a}$, either in second or third wave number quadrant). The complex scattering length, $\tilde{a}_n = a_1 - ia_2$ is an alternative way to take into account channels couplings. The complex scattering length is dependent on coupling and rescattering in open channel.

The complex scattering length, in case of only one open channel *o*, depends on corresponding phase shift δ_o as

$$a_1 = a_n - b\Delta_n = a_n + b\Gamma_n \tan\delta_o = a_n + a_2 \tan\delta_o. \tag{28}$$

Therefore $a_1 > a_n$ provided $\tan\delta_o > 0$; also $a_2 = b\Gamma_n$ decreases as effect of coupling and rescattering in open channel.

$$k = i/\tilde{a} = \frac{a_1 + ia_2}{|a_1|^2 + |a_2|^2}. \tag{29}$$

As a_1 increases and a_2 decreases one obtains that both $\operatorname{Re} k \propto a_1/a_1^2$ and $\operatorname{Im} k \propto a_2/a_1^2$ decrease so the pole is shifted to threshold. It is an analytical demonstration that channels couplings result into shift of the channel state pole to the real axis and into decrease of its decay width.

In literature, (Badalyan *et al.*, 1982), one reports on "channel coupling pole" observed in numerical experiments for multichannel scattering; a single channel pole may be driven to physical region of the complex energy plane when channel coupling becomes effective. The "channel coupling resonances" and multichannel resonances originating in quasistationary or bound channel states have similar width property.

This approach to channel (also to inner) resonances can be compared to K-matrix formalism for resonances (Chung *et al.*, 1995). There are two types of resonances which differ in dynamical character; they are parameterized, according to K-matrix, in two distinct forms. Resonances can arise from strongly varying K-matrix elements (pole). These 'normal' resonances correspond to dynamical sources at the constituent level; in our case they correspond to electron 'inner' resonances or 'compound nucleus' resonances. Resonances can appear also from constant K-matrix element provided the energy variation is supplied by phase space. These 'molecular' resonances are assumed to arise from couplings in the reaction channels; in our case the reduced R-matrix element $\mathfrak{R}_{nn}$ does include couplings to complementary channels. The 'channel' resonance, described by channel equation $\mathfrak{R}_{nn} - L_n^{-1} = 0$, originates in constant reduced R-matrix element and in energy dependent logarithmic derivative. The energy variation of channel logarithmic derivative is implied in realization of the quasistationary state condition: $\mathfrak{R}_{nn} = L_n^{-1}$.

7. CONCLUSIONS

The concept of reduced or effective operator, previously designed for R-Matrix, (or K-Matrix), can be extended to Collision/Scattering Matrix. The reason is the Collision Matrix is a primary concept in Scattering Physics, because it is associated with the whole Dynamics of Scattering Process. The Reduced Collision

Matrix consists from Collision Matrix of 'bare' retained channels (uncoupled to eliminated ones) and from an effective term representing the effect of eliminated channel(s) on the retained ones. The effective term contains the retained-eliminated channels couplings as well as a term related to dynamics of collision process in the eliminated channel n. This last term has the form $(L_n^{-1} - \Re_{nn})^{-1}$ with L_n as channel logarithmic derivative and $\Re_{nn}$ as channel reduced R-matrix element. Observe that in the decoupling limit the equation $L_n^{-1} - \Re_{nn} = 0$ describes either the bound state (below n- channel threshold) or quasistationary state (above n-channel threshold). The bound or quasistationary state is not more described by an R-matrix pole but rather by channel state equation $1 - R_{nn} L_n = 0$. If the eliminated channel is coupled to retained ones the equation becomes $1 - \Re_{nn} L_n = 0$. It is the equation which is applied here to describe different atomic or nuclear phenomena: Coupled Channel Resonances.

The channel state equation $1 - \Re_{ee} L_e = 0$ (e-Rydberg channel) is basis for derivation of Complex Quantum Defect and MQD Channel Resonances. The derivations require only the explicit form of Rydberg channel logarithmic derivative.

A bound (closed eliminated channel) or a quasistationary (open eliminated channel) state do induce channel resonances in observed reaction channels; they are Channel Coupling Resonances. The reduced widths of channel resonances are subject of two mechanisms of compression: the R-Matrix Compression factor, effective only near threshold, and Direct (Coupling) Compression factor due to channels couplings. The last mechanism could be put into correspondence with 'Molecular Resonances' or with 'Channel Coupling Pole' observed in numerical experiments for multichannel scattering: a single channel pole may be driven to physical region of the complex energy plane if channel couplings become effective.

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