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NUMERICAL ANALYSIS OF LIGHT PROPAGATION IN SELF-INDUCED WAVEGUIDES

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We make a short overview of recent results obtained in recording of soliton waveguides in lithium niobate crystals. We numerically investigate the light propagation in waveguides with properties similar to those of the soliton waveguides self-induced in congruent lithium niobate crystals. The influence on the guiding properties of several parameters as the refractive index contrast and transversal profile, the input beam size and the waveguide size, and the wavelength of the guided beam is considered in numerical simulations and discussed.

Key words: soliton waveguides, optical propagation, photorefractive crystals, lithium niobate.

1. INTRODUCTION

During the last decades there was an intense study in the field of optical spatial and temporal solitons. The spatial solitons [1] are non-diffractive light beams for which the natural diffraction is compensated in propagation by the optical nonlinearity of a particular medium. The compensation of beam diffraction is obtained by carefully controlling the nonlinear response of a material at the light excitation. The temporal solitons [2, 3] refer to dispersion compensated light pulses that do not change their temporal profile during propagation. The compensation of both diffraction and dispersion has as result the so called “light bullets” that is a fascinating theoretical concept [4–8], which was also experimentally proven on short propagation distances [9]. The research in the temporal solitons domain led to important developments in optics, like propagation over long distances in optical fibers [10] or supercontinuum generation [11]. In the last years solitons in few-cycle optical pulses have been intensively studied [12–15].

Spatial solitons can be of many different types: bright, dark, gray, discrete, etc. [1, 16–19] and can be obtained as result of different nonlinear mechanisms like Kerr, quadratic, photorefractive, etc. [1, 20–23]. Photorefractive spatial solitons have the advantage that can be experimentally generated with relatively low powers [24]. They have been intensively studied in materials like Strontium-Barium Niobate (SBN) [22–33], Lithium Niobate (LN) [34–45], and Bismuth Silicon Oxide (BSO) [46–50]. The generation of spatial solitons in LN had important consequences in the study of self-induced soliton waveguides (SWGs). A SWG is a waveguide created during the generation of the spatial soliton in a nonlinear material by changing its refractive index. The lifetime of the SWG is dependent on the material used for soliton generation and on the soliton nature. SWGs with short lifetime are important for switching applications (fast response), while SWGs with a long lifetime are important for optical interconnects (long term guiding). In photorefractive materials the lifetime of the SWG is given by the dielectric relaxation time. In LN this time is of the order of one year [45], which makes LN an important material for long term guiding applications. The process of 2D SWGs recording in LN has been intensively analyzed and characterized in the last years [45, 51–57]. An important enhancement in the recording speed has been obtained by using blue-violet light from Blu-ray laser diodes [54]. The generation of bright spatial solitons in LN requires an additional static electric field to compensate the photovoltaic field generated by illumination. This electric field was usually obtained by applying an external high voltage along the crystal c-axis. An important improvement in the recording process was the replacement of this external field by the pyroelectric field generated inside the crystal by changing its temperature with few degrees [52, 55]. An important aspect of the experimentally generated photorefractive SWGs is that they rely on quasi-steady-state solitons, a mathematical solution that approximates a soliton solution but can be obtained in much more relaxed experimental conditions. Quasi-steady-state solitons allow the recording of SWGs with a good transversal profile of the refractive index, which can be close to a Gaussian shape or a step-function shape, depending on the recording parameters [26].

Recently, a lot of work has been done in the domain of surface solitons [58–63]. Surface SWGs can be very useful in sensing applications. The propagation in surface SWGs can be easily observed by capturing the scattered light near the surface [59]. When recording SWGs in the volume of the material the monitoring of the recording process consists of observing the input and output transversal optical mode profile. The evolution of the transversal output mode profile has been experimentally investigated for SWGs recorded at 405 nm wavelength [57]. It is more difficult to analyze the mode profile along the propagation direction in volume SWGs due to the low-loss (laterally scattered light) characteristics of these waveguides.

In this paper we make a numerical analysis of the light propagation in waveguides with a transversal profile of different shapes and with a refractive index contrast that can be obtained by SWGs recording process in LN. We consider typical values of the beam size and waveguide size that can be experimentally obtained by generation of SWGs. The analysis of the results suggests the parameters that have to be chosen or that can be obtained for particular applications.

2. SIMULATIONS OF OPTICAL PROPAGATION IN SOLITON WAVEGUIDES

We use the split step Beam Propagation Method in the Fourier domain [64] to model the light propagation in waveguides with different sizes and shapes. Waveguide parameters are selected in the range of the values that can be experimentally obtained in a typical SWG recording process in LN. To estimate the guided optical power after propagating through a waveguide, we have analyzed the power in a window of defined size, transversal to the propagation direction. It is difficult to select a window that includes all the propagating optical power for any waveguide since this window depends on the waveguide properties. We have defined the size of this window corresponding to the full width at half maximum (FWHM) of the waveguide refractive index profile. In some situations of waveguides with high contrast of the refractive index an amount of power, which is, at the input of the waveguide, outside of the considered window, can be also efficiently guided, being trapped in the considered window during the guided propagation. In this case, the power at the output of the waveguide can slightly exceed the power at the input, in the considered window of a fixed size. It is worth mentioning that when we talk about the input diameter of the guided beam or the waveguide diameter, these are also considered at FWHM of the beam or waveguide transversal profiles.

In Fig. 1 is shown the computed dependence of the output power on the refractive index contrast, for a Gaussian beam guided by a SWG having different transversal profiles of the refractive index. The output power is normalized to the input one. The considered propagation length is 10 mm. The input and output powers are estimated in a window considered as described above. Two wavelengths of the guided beam have been considered in simulations, 405 nm and 1550 nm, respectively. These wavelengths have a particular relevance. The first one (405 nm) allows a very fast recording of SWGs in LN, as it was recently demonstrated [53], whilst the second one (1550 nm) is near the central wavelength of telecom C-band, which is of particular interest for optical communications. Three different transversal refractive index profiles of the waveguides, Gaussian, sech^2, and step-index have been considered in simulations. The maximum refractive index change has been chosen in the range of values between $0.5\cdot10^{-4}$ and $10\cdot10^{-4}$, which can be obtained in the recording process of SWGs in LN. The diameter of the input beam was chosen 10 µm and of the waveguide, 15 µm, respectively. These values are typical for SWGs recording in LN at 405 nm and guiding beams at 1550 nm through them. For lower values of the input beam diameter, or lower waveguide diameter, guiding at 1550 nm would require a higher refractive index change, but this is difficult to obtain in a typical SWG recording process in LN. A better guiding could be also obtained by increasing the input beam diameter but this would also increase the losses due to the coupling. It should also be noticed that in practice additional coupling losses could occur if the coupling of the input beam to the guide is not done with a very precise optical system.

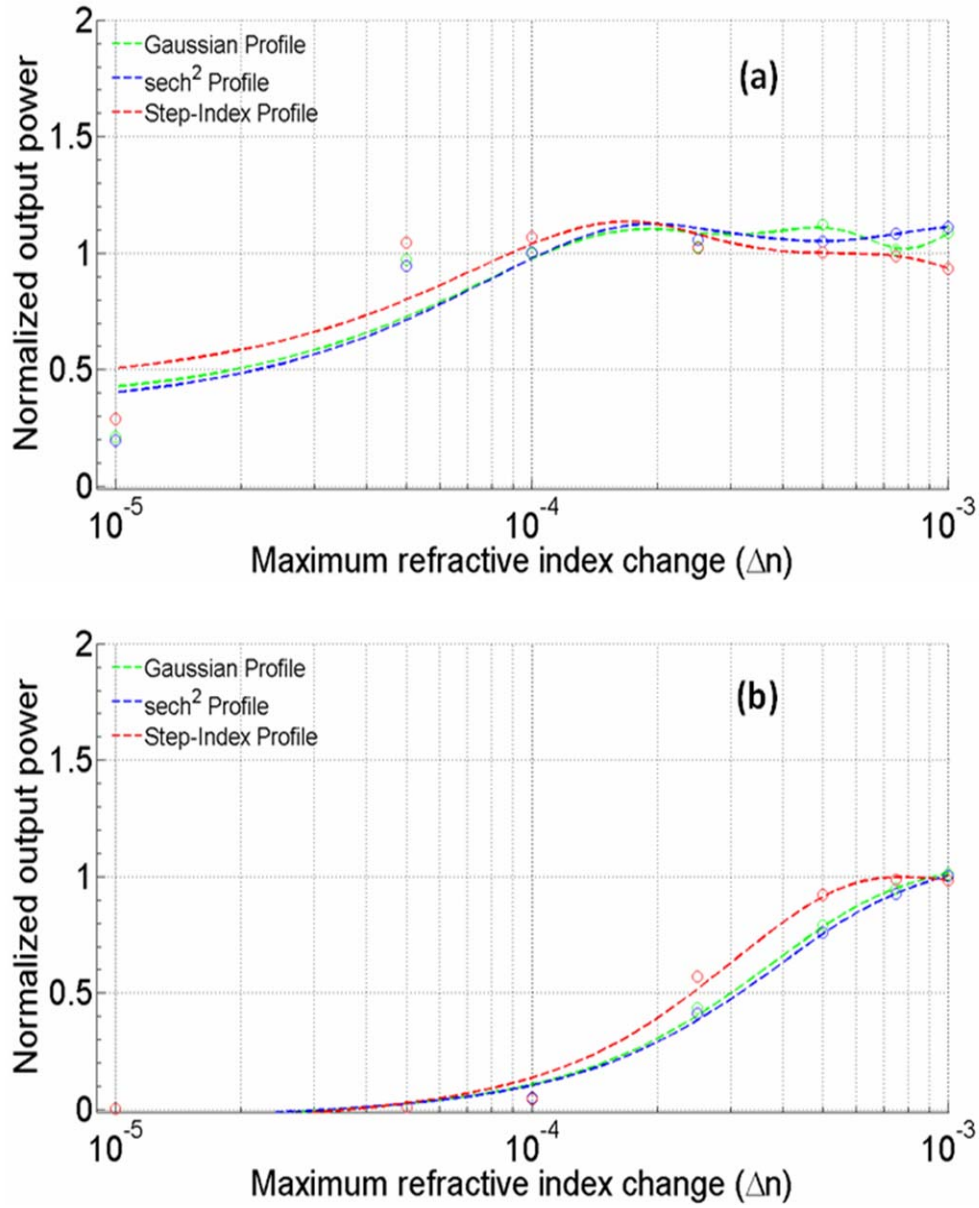

Fig. 1 – The dependence of the normalized output power on the refractive index contrast, for a Gaussian beam with FWHM of 10 µm guided by a SWG of 15 µm diameter and different transversal profiles of the refractive index. The wavelengths considered for the guided beam are 405 nm (a) and 1550 nm (b), respectively. The open circles are the results obtained from simulations and the lines are shown as eye-guides only. The propagation length is 10 mm.

In Fig. 1 it can be seen that the differences in the guided power for different transversal refractive index profiles at the same refractive index contrast are quite low. So, in our further analysis we consider only one transversal refractive index profile of the waveguide, the Gaussian one.

In order to have a closer look on the beam propagation process we have represented the transversal profile of the optical intensity along the propagation direction, considering four different values of the refractive index contrast, in the range $0.5 \cdot 10^{-4}$–$10 \cdot 10^{-4}$ (Fig. 2). The diameters of the waveguide and of the input beam are the same as considered in Fig. 1. Due to the diffraction there is a big difference in propagation between beams at 405 nm (Figs. 2a, c, e, g) and beams at 1550 nm wavelength (Figs. 2b, d, f, h). For example, at the refractive index contrast of $1 \cdot 10^{-4}$, we can have multimode propagation at 405 nm (Fig. 2c), while having a very lossy propagation at 1550 nm (Fig. 2d). To have lower losses at 1550 nm we need a higher refractive index contrast (Figs. 2f, 2h) of the waveguide or to guide larger beams.

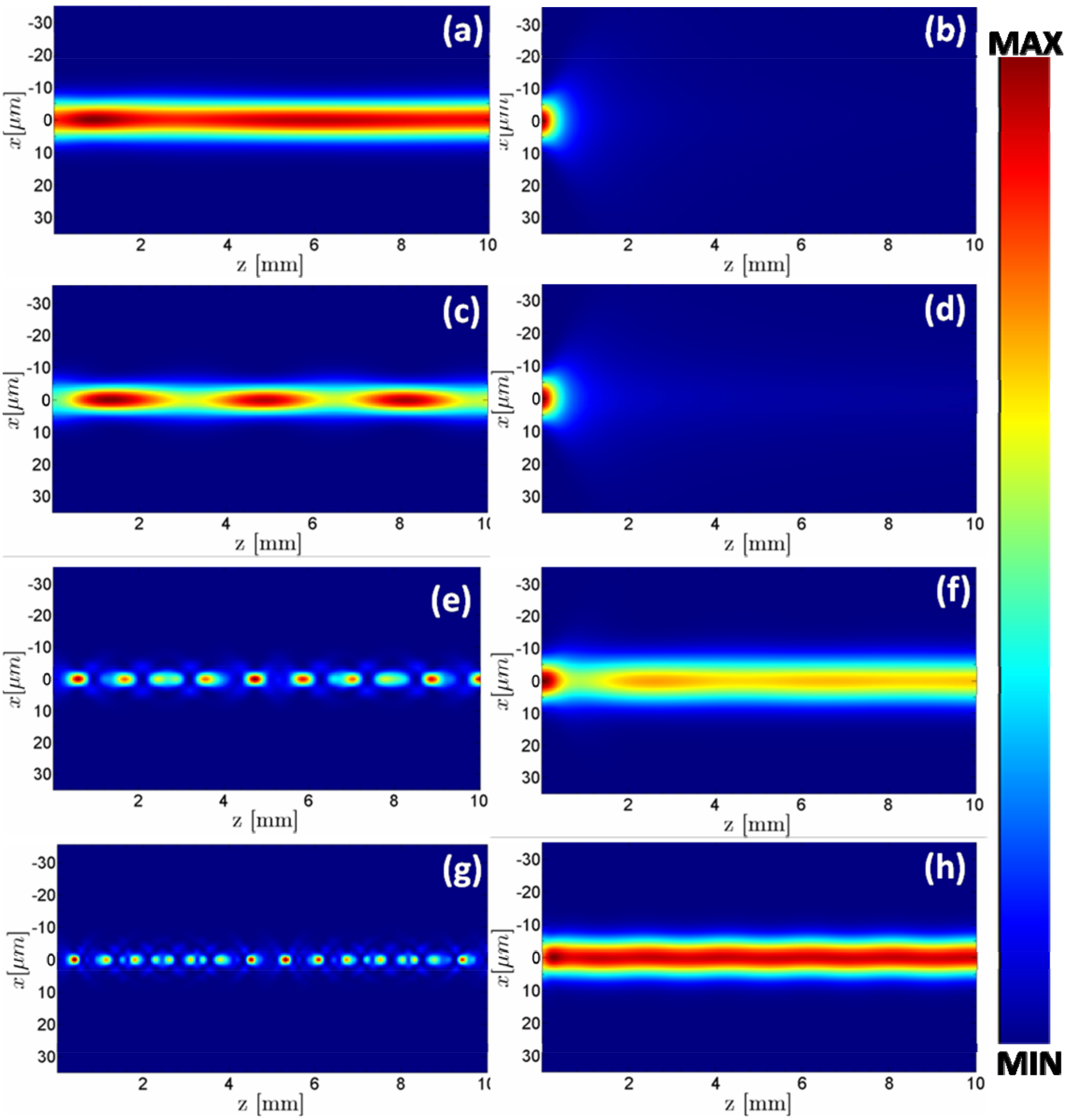

Fig. 2 – Propagation in waveguides of different refractive index contrasts: $0.5{\cdot}10^{-4}$ (a, b), $1{\cdot}10^{-4}$ (c, d), $5{\cdot}10^{-4}$ (e, f), $10{\cdot}10^{-4}$ (g, h). The input beam diameter is 10 μm and the waveguide diameter is 15 μm. Left and right columns are for the wavelength of the guided beam of 405 nm and 1550 nm, respectively.

The influence of the input beam size on the beam guiding was also investigated. It is illustrated in the Fig. 3, in which we have represented the intensity profile of a beam at 1550 nm wavelength along its propagation path in a waveguide with 15 μm diameter and a refractive index contrast of $2.5{\cdot}10^{-4}$. Three different input diameters, 10 μm (Fig. 3a), 15 μm (Fig. 3b), and 20 μm (Fig. 3c), have been considered.

It can be seen that the guiding of a beam at the considered wavelength (1550 nm) is better (more guided light) for larger input beams, despite larger coupling losses that appear in this case due to the non-matching of the input beam size to the waveguide size. The input coupling losses can be reduced by considering larger waveguides.

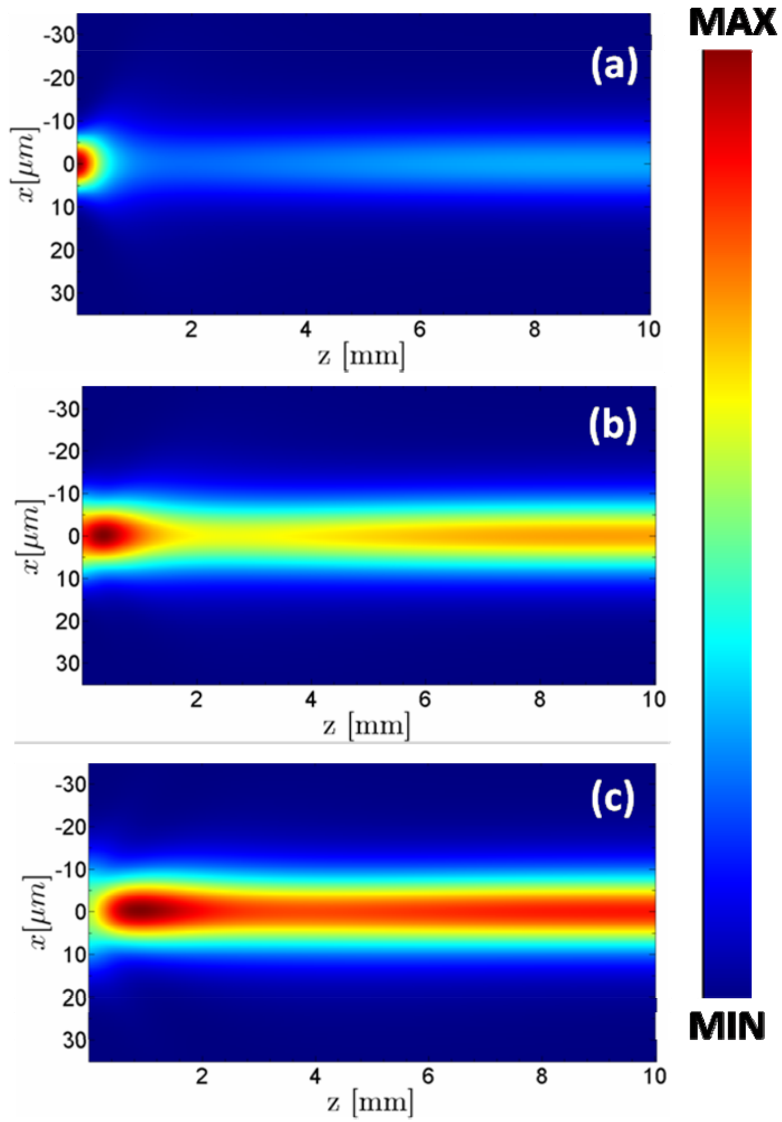

Fig. 3 – Propagation of an input beam (1550 nm wavelength) of 10 μm (a), 15 μm (b), and 20 μm (c) diameter, respectively, through a waveguide of 15 μm diameter and $2.5 \cdot 10^{-4}$ refractive index contrast.

3. CONCLUSIONS

A short overview of recent results in recording soliton waveguides in lithium niobate crystals is given. The improvements in the recording process, based on the use of the blue-violet light and of the pyroelectric effect, are briefly discussed. Using the split step Beam Propagation Method in the Fourier domain we have analyzed the light propagation in waveguides with properties similar to those of soliton self-induced waveguides in lithium niobate. The influence on the guiding properties of several parameters of the waveguide (the refractive index contrast and its transversal profile, the waveguide diameter) and of the guided beam (the wavelength and input diameter) has been considered in numerical simulations and has been discussed. These results give an indication of the proper experimental parameters that have to be chosen to record soliton waveguides with good guiding properties in lithium niobate crystals.

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