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DAMPING CHARACTERISTICS IDENTIFICATION OF SELF-REINFORCED POLY-ETHYLENE TEREPHTHALATE

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Abstract. Damping material characteristics are important to be considered in dynamic numerical simulations in order to obtain accurate results. The dependency between the critical damping fraction and frequency is experimentally determined within this article for a novel thermoplastic composite material - poly-ethylene terephthalate fiber reinforced poly-ethylene terephthalate (SrPET). The article explains the theoretical background used for obtaining the values of damping ratio. A curve fitting process is applied in order to identify the mathematical function that best fit the experimental data. It is shown that the damping ratio varies with frequency by following an exponential function. The influence of material damping characteristic is evidenced through a case study where a flat plate of SrPET is subjected to an impulse unit force. The results indicate, as expected, a high sensitivity to damping data.

Key words: damping ratio, poly-ethylene terephthalate, self-reinforced-polymer.

1. INTRODUCTION

An intelligent use of composite materials and structures may provide important benefits with respect to different engineering systems' performance criteria. This may refer to lightweight design which is important in order to reduce energy consumption of the system in use and to save material resources [1]. The performance criteria may also refer to mechanical characteristics as high stiffness and strength or high impact performance [2–4], special thermal properties [5, 6] or electric properties [7]. A large spectrum of characteristics is therefore attainable when combining several materials by following a specific way dictated by the application needs [8].

Although the individual characteristics of polymers may not be attractive when looking at mechanical properties, their properties may be improved for example through reinforcements. Novel composite materials related to high specific performance criteria are continuously proposed. Additional benefits as recycling potential is also of high importance nowadays [9]. One category of such novel materials are the self-reinforced polymers: SrPET – self-reinforced poly-ethylene terephthalate [10, 11] or SrPP – self-reinforced polypropylene [12–14]. The SrPET is a poly-ethylene terephthalate matrix reinforced with poly-ethylene terephthalate fibres. However, their melting temperature is different; details about the manufacturing process of the SrPET are given within the 3rd section of this article.

Especially when working with ductile materials, as the case of the SrPET, the dynamic numerical analysis may be inaccurate if the damping characteristic is not considered [15]. However, this type of data is not usually found on the material supplier and must be determined by the user.

The purpose of this article is to determine the dependency between damping ratio and frequency for the SrPET material. This data is usually required in a tabular form within the dynamic FE analyses. The influence of the damping ratio on the dynamic simulations is afterwards investigated within a numerical case study. Conclusions are drawn eventually based on the obtained results.

2. DAMPED RATIO IDENTIFICATION. THEORETICAL BACKGROUND

The modal parameters can be found using the so-called frequency domain methods. These methods are based on estimators that are frequency response functions defined with some frequency dependent functions as: Auto Spectral Density of the force excitation $S_{ff}(\omega)$, Auto Spectral Density of the response $S_{xx}(\omega)$, the Cross Spectral Density between the response and the force excitation $S_{xf}(\omega)$, and the Cross Spectral Density between the force excitation and the response $S_{fx}(\omega)$. According to Heylen, W. *et. al.* [16], the main estimators used in modal analysis are the following:

$$H_1(\omega) = \frac{S_{fx}(\omega)}{S_{ff}(\omega)}, \tag{1}$$

$$H_2(\omega) = \frac{S_{xx}(\omega)}{S_{xf}(\omega)}, \tag{2}$$

$$H_3 = \sqrt{H_1 H_2} \ . \tag{3}$$

The H_3 estimator is used when the signal to noise ratio is approximately the same at the input and at the output. By considering the influence of the noise, the two estimators given by the equations (1) and (2) can be rewritten as:

$$H_1(\omega) = \frac{S_{fx}(\omega)}{S_{ff}(\omega)+S_{nn}(\omega)}, \tag{4}$$

where $S_{nn}(\omega)$ is the Autospectra of the noise on input signal, and

$$H_1(\omega) = \frac{S_{fx}(\omega)}{S_{ff}(\omega)+S_{nn}(\omega)}, \tag{5}$$

where $S_{nm}(\omega)$ represents the Autospectra of the noise on output signal.

By considering the equations (4) and (5) it yields that the estimator H_1 will suffer most at the resonance; therefore, H_2 is a better indicator near resonance. In addition, the estimator H_1 is used in case of a low noise at input, while H_2 is mainly used when it is a low answer on response.

The quality of the data is evaluated by the coherence function:

$$\gamma^2 = \frac{H_1(\omega)}{H_2(\omega)} \ . \tag{6}$$

In general, the range value of the coherence function is of $0 \le \gamma^2 \le 1$. For $\gamma^2 \to 0$ the signal is pure noise while for $\gamma^2 \to 1$ a signal without noise is obtained. As it is mentioned within the basic theory of modal analysis, the data with a coherence of less than 0.75 are not used in practice [16].

Another method that can be used in damping ratio evaluation is the so called “Power Input Method “ (PIM) that is based on the comparison between the dissipated energy of a system to its maximum total energy under vibration per radians [17]. This method is unbiased at the natural frequencies of the defined modes. In fact, the method allows the loss factors and the damping ratio to be found at different frequencies. The analyzed structure is divided into n areas, each one having a partial mass $m_i \left(i=\overline{1,n}\right)$ [16].

For a structural system, the damping loss factor per cycle, in the frequency band centered at a considered frequency ω, is defined as follows [18]:

$$\eta(\omega) = \frac{E_d}{E_t} \ , \tag{7}$$

where E_d is the dissipated energy by damping, and E_t represents the time averaged total energy of the system (the strain energy).

The dissipated energy E_d can be evaluated using the simultaneous measurements of the force and velocity at the point of energy input with the formula:

$$E_d = \frac{1}{2\omega}\mathrm{Re}\left[H_{xx}(\omega)\right]S_{xx}(\omega), \tag{8}$$

where $H_{xx}(\omega)$ represents the driving point mobility frequency transfer function (input-response in the application point of the force).

Thus, the strain energy cannot be directly found from the measurements of the force; the velocity it is necessary to be replaced with twice the kinetic energy and the system has to be approximated by a summation as opposed to a volume integral [18]:

$$T = \frac{1}{2}\sum_{i=1}^{n} m_i S_{ii}(\omega), \tag{9}$$

where $S_{ii}(\omega)$ is the power spectral density of the velocity response at each measurement location.

By considering that all the n measurement points are uniformly spaced throughout the system and the mass is divided in equal mass ratio, the damping loss factor for the considered system can be found with the relation [18, 19]:

$$\eta(\omega) = \frac{\mathrm{Re}\left[H_{xx}(\omega)\right]}{\omega m \sum_{i=1}^{n} \left|H_{yx}(\omega)\right|^2} \ , \tag{10}$$

where H_{yx} is the frequency transfer function of the mobility between the driving point x and a measuring point y.

Further on, by considering the value of the damping loss factor given by (10), one can calculate the damping ratio from the following relation:

$$\eta(\omega) = 2\zeta(\omega) \ . \tag{11}$$

3. EXPERIMENTAL INVESTIGATION

3.1. The novel material under study

The investigated material is made of poly-ethylene terephthalate reinforced with fibre poly-ethylene terephthalate (SrPET). The SrPET material used in this study was supplied by Comfil®APS and is a commingled balanced 2/2 twill fabric with a weight of 0.75 $\mathrm{kgm^{-2}}$ and 50% reinforcement fibers. It is presented as a plate made of 6 layers of fabric, with the plate thickness of 2.7 mm and a material density of 1 380 $\mathrm{kg{\cdot}m^{-3}}$.

The SrPET composite consists of a low melting temperature poly-ethylene terephthalate (PET) matrix and a high tenacity poly-ethylene terephthalate (PET) fiber material. The melting temperature of the PET matrix is around 170°C while the PET fiber melts at 260°C. A good consolidation is obtained if the temperature is high enough to melt the matrix and wet the fibers but not too high so that the fibers degrade and lose their reinforcing properties. According to a previous study [10] laminates with good mechanical properties can be consolidated at 220°C for 20 min under a pressure of 1.5bar.

3.2. The experimental set-up

The aim of the considered Experimental Modal Analysis (EMA) is to obtain the modal parameters (natural frequencies, damping ratios or damping loss factors and modal constants or modal amplitudes) from measured vibration data. A mathematical relation for the global value of the damping ratio ζ as function of frequency value is searched based on the above theoretical considerations and EMA.

This experimental investigation is carried out in order to determine how the critical damping ratio ζ varies in terms of frequency.

A Brüel & Kjær equipment was used for the modal analysis, consisting of: a shaker type 4810, three accelerometers type 4517-002, PULSE 12 platform connected to a PC. The data acquisition and post processing results were performed using two soft modules type 7709 and type 7770-6.

The dimensions of the tested composite plate were 300 × 300 mm and the plate was supported by a soft polymeric sponge, thus establishing the so-called free-free conditions. A mesh was drawn on the plate (square elements 50 × 50 mm) in order to arrange the accelerometers in the mesh nodes (Fig.1).

The SIMO test was chosen as testing method by considering the shaker fixed in a reference point, the middle of the plate. The applied signal was a true random signal in the range 0–1600 Hz used in parallel with a Hanning window to minimize the leakage. In the same time a spectrum averaging was used in order to excite the non-linearity in the structure.

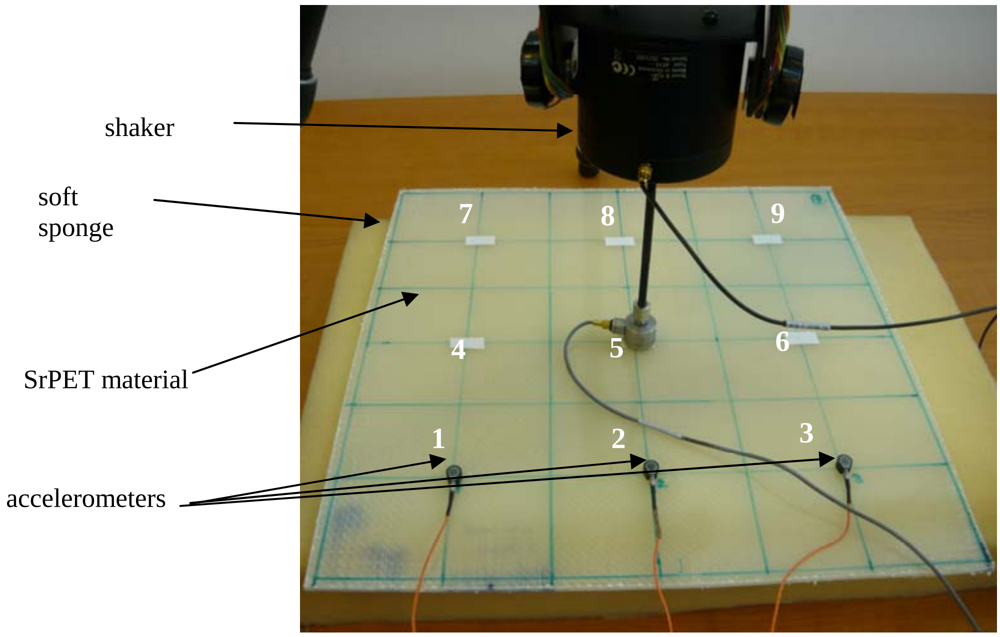
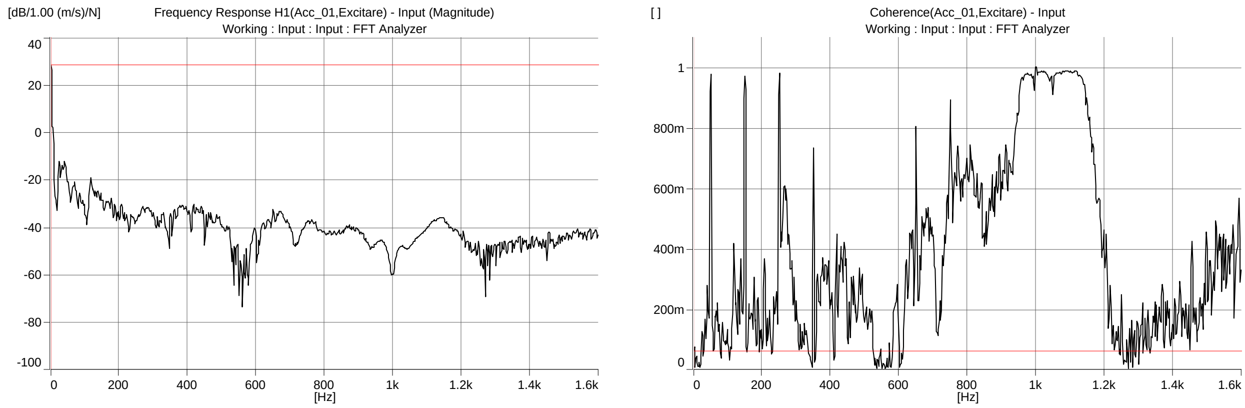


Fig. 2 – Experimental set-up.

The values of interest for modal parameters can be obtained based on the modal analysis identification extraction. The extraction method can be one of the following: the peak amplitude method, the quadrature response method, the maximum quadrature component method, the maximum frequency spacing method, the circle fitting method, or the inverse method. The peak amplitude method was the method used within this investigation. According to this method, at the vicinity of the resonance, the FRF is dominated by the contribution of that vibration mode while the contributions of other vibration modes are negligible. The FRF graph shows the natural frequencies that are the peaks of FRF. By considering the half power method, the modal damping ratios from the sharpness of the peaks are found. The used FRF was the mobility one and the resulted functions and the coherence functions were measured in all 9 nodes of the defined mesh.

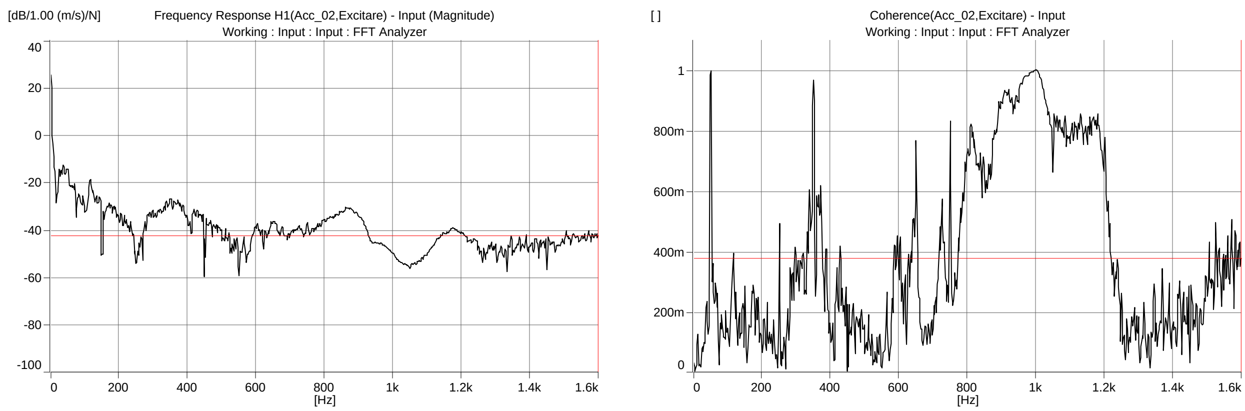
3.3. The results

The obtained set of data was further on processed and interpreted in order to determine a mathematical relation between frequency and the critical damping fraction ζ . The Frequency Response Functions (FRFs) were recorded for all nine points (Fig. 1). In Figs. 2a, 3a, and 4a there are presented the mobility FRFs for the points ①, ②, and ③ from Fig. 1. The measured values were read and for each natural frequency the correspondent value of the coherence was analysed (Figs. 2b, 3b, and 4b). From each diagram one can read, in the considered range, the values of natural frequencies and the values of damping ratio. The considered data are validated by correlation to the coherence values (Fig. 2b, 3b, and 4b).



a) b)

Fig. 2 – Data recorded for the accelerometer 1:
a) frequency response function; b) coherence function.

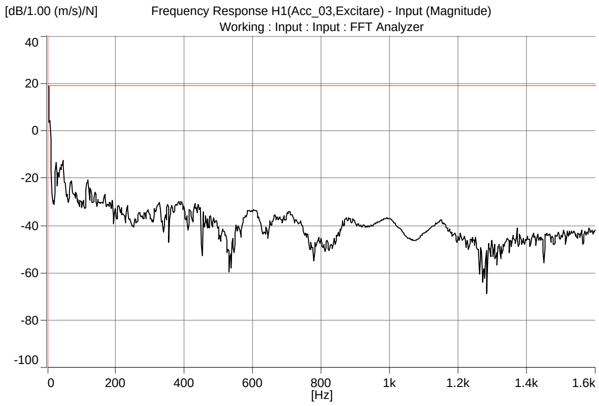


a) b)

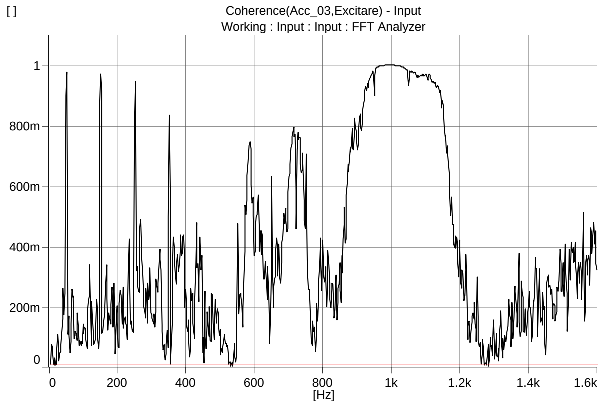
Fig. 3 – Data recorded for the accelerometer 2:
a) frequency response function; b) coherence function.

The proper values of the frequencies and of the damping ratios were chosen by considering the limits of the coherence functions (Fig. 5). A relatively scarce distribution for the measured values of ζ in terms of frequency can be noticed from Fig. 5, i.e. ζ determined on the 9 measured points varies significantly within small intervals of frequencies. This was an expected behavior and it is explained by the structural non-homogeneity of the material. In reality, the tested material is not homogeneous and isotropic, therefore differences in values of the damping ratio measured in different points can be observed.

In order to deal with this problem, an arithmetic mean was calculated for the values of ζ founded within intervals of 20 Hz; the resulted points are illustrated in Fig. 5 as filled circles.



a)



b)

Fig. 4 Data recorded for the accelerometer 3:
a) frequency response function; b) coherence function.

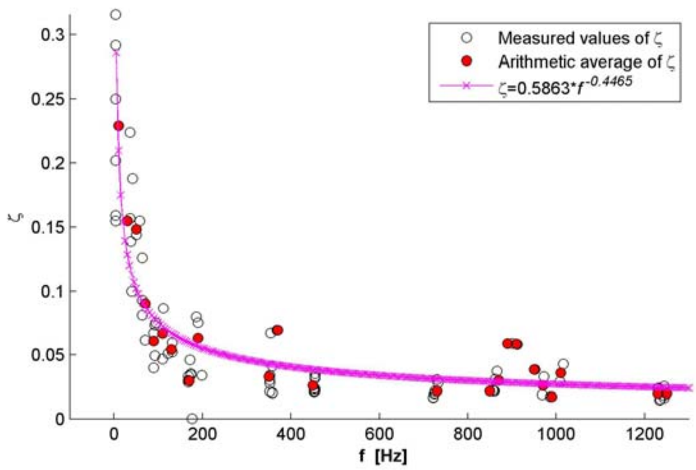


Fig. 5 – Damping ratio as a function of frequency for poly-ethylene terephthalate reinforced with fiber poly-ethylene terephthalate (SrPET).

The mean values are used further on to curve fit data and to find out the mathematical function $\xi(f)$. The best fit is obtained by an exponential function (12):

$$\xi(f) = 0.5863 f^{-0.4465} \, . \tag{12}$$

This relation will be further on implemented within a numerical simulation

4. CASE STUDY

4.1. Numerical model

A numerical approach is further on used in order to study the influence of the damping data on the dynamic response of a structure. The developed numerical model consists of a flat plate, having a thickness of 2.7 mm, clamped at two opposite edges (Fig. 6).

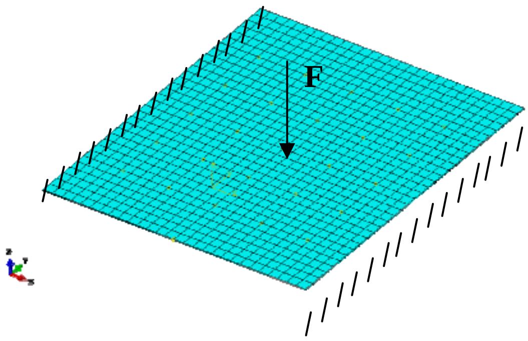


Fig. 6 – Boundary conditions considered within the simulation.

A Dirac delta function is considered to model the excitatory force (unit force). S4R elements are used to create the mesh and the SrPET material is assumed to be isotropic, having the following material properties: $E = 5300$ MPa, $\nu = 0.3$ and $\rho = 1380$ kg/m^3. The analysis is carried out in two steps. Firstly, a frequency step is done, where the first 50 eigenvalues are determined by using the Lanczos eigensolver. Secondly, a modal dynamic step is considered, where the critical damping fraction is defined as a function of frequency using data generated by (12). The model is solved using Abaqus/Standard.

4.2. Results

The vibration response of the plate is recorded over time (for 0.2 seconds), by comparing the displacements of the point where the unit force was previously applied, for both considered cases: damped and un-damped structure (Fig. 7).

As it was expected, while the un-damped structure continuously oscillates, the displacement of the damped structure reduces with time until the structures reach the equilibrium state. The energy generated by the applied unit force is making the plate to oscillate but this energy is lost by intermolecular friction within the material structure. This observation corresponds to the real behavior of structures.

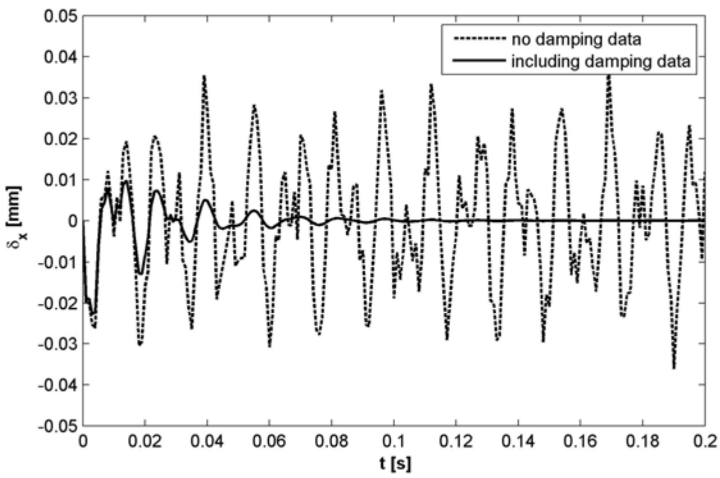


Fig. 7 – Displacement *vs*. time caused by the unit force applied at $t = 0$.

5. CONCLUSION

A novel thermoplastic composite material – poly-ethylene terephthalate fiber reinforced poly-ethylene terephthalate (SrPET) – is analyzed in terms of damping characteristics. The values of the critical damping ratio are found experimentally in terms of frequency. An exponential function is found to best fit the obtained set of experimental data. A numerical model is presented as a case study where dynamic simulations are carried out separately for damped and un-damped structure. The results demonstrate the important influence the damping properties have on the results within numerical dynamic analyses.

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