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# SWIRL2D: AN INTERFACE TRACKING ALGORITHM FOR COMPUTING THE TWO-DIMENSIONAL SWIRLING FLOWS WITH STAGNANT REGION

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**Abstract.** The fluid engineering problems require tailored numerical computation algorithms and deep mathematical insight. The computation of swirling flows has been an intense field of investigation during the last three decades. Swirling flows may develop stagnant regions separated from the main flow by an interface that is not a-priori known. The interface localization approaches can be: interface capturing techniques (ICaT) and interface-tracking techniques (ITrT). The main contribution brought by the present paper is related to an augmented functional for computing the fluid interface boundary using a mesh that "tracks" the stagnant region of swirling flows with the help of an in-house ITrT software. The variational formulation and the developed software system are detailed. The numerical solutions obtained with conventional commercial codes are compared against the SWIRL2D in-house solutions. The SWIRL2D algorithm allows for more rapid, robust and accurate assessment of the swirling flow when compared with commercial codes.

***Key words***: two-dimensional swirling flow, stagnant region, interface tracking algorithm, variational formulation.

## 1. INTRODUCTION

The computation of swirling flows has been an intense field of investigation during the last three decades. Such flows are involved in many aspects of basic fluid mechanics as well as in engineering problems (e.g. hydraulic turbines). In particular, the draft tube hydrodynamics is mixed due to the combination of swirling flow deceleration with flow direction and cross-section shape/area changes [1]. Most of the pressure recovery occurs in the draft tube cone, also called discharge cone. The main purpose of a hydraulic turbine draft tube is to decelerate the flow exiting the runner, thereby converting the excess of kinetic energy into static pressure [2]. Extensive experimental [3] and numerical [4] investigations have offered a comprehensive understanding of this flow phenomenon, with accurate evaluation of the main parameters, as well as various details of the hydrodynamic field. Nishi *et al*. [5] introduced a qualitative model for the precessing vortex rope, based on their experimental investigations. They suggest that the circumferentially averaged velocity profiles in the cone could be represented satisfactorily by a model comprising a dead (quasi-stagnant) water region surrounded by the swirling main flow.

All these considerations led to the conclusion that the spiral vortex core observed in the draft tube of a turbine at part load is a rolled-up vortex sheet which originates between the central stalled region and the swirling main flow. Resiga *et al*. [6] have implemented a stagnant region model on top of an incompressible, axi-symmetric flow solver, within the framework of the interface capturing techniques (ICaT), and their numerical results for the flow with precessing vortex rope in a turbine discharge cone were in very good agreement with the LDV measured axial and circumferential velocity profiles. Moreover, it was shown that the vortex rope is located on the boundary interface between the central stagnant region and the swirling flow computed with interface capturing technique, Figure 1, thus successfully confirming Nishi's [5] model.

In swirling flow problems the boundary between the main flow and the stagnant region is a fluid interface. Finding the correct interface location is a challenge for the numerical algorithms. Interface localization techniques could be classified in two large classes: interface capturing (ICaT) and interface-tracking (ITrT).

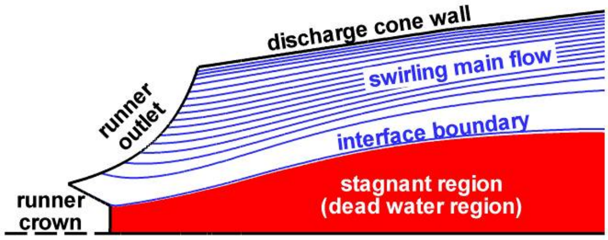


Fig. 1 – Two-dimensional axi-symmetric flow numerical simulation with stagnant region computed with ICaT [6].

Interface capturing techniques (ICaT) compute the flow on a fixed grid, within the whole computational domain with fixed boundaries. The interface is then "captured" and identified by maximum flow gradients (as in compressible flow shocks) or by simply examining the velocity magnitude and cut-out the region with vanishing velocity as in Fig. 1. Other representatives of this class of approaches are the level set and the Volume of Fluid method (since it is essentially based on the transport of the local volume fraction of the liquid) [7–9]. However, this flexible interface description provides challenges regarding mass conservation and the treatment of discontinuities across the interface.

On the other side, interface-tracking technique (ITrT) requires meshes that "track" the interface [10]. In ITrT the interface is a free boundary of the computational domain. Hence, the correction of the interface position requires a new grid generation at each iteration [11, 12]. Interface tracking approaches are known to provide great accuracy, yet their applicability is limited in the case of severe interface motion. The front tracking methods and the marker and cell (MAC) methods belong to the class of interface tracking methods, where markers are used to represent and track the interface.

This paper presents our ongoing efforts for developing mathematical models, and associated numerical algorithms, for a robust description of the swirling flow phenomena. The main contribution to be presented in this paper is related to an augmented functional for computing the interface boundary. We set our goal to develop a theoretical framework for computing the swirling flow starting with the Bragg-Hawthorne equation [13] as the basic model for turbine blade-less regions. However, instead of solving the differential equation we are using the equivalent variational formulation as introduced by Benjamin [14] with a finite element discretization for numerical solution. This novel variational approach is built by admitting a continuous static pressure across the fluid interface between the main swirling flow and the inner stagnant region, Section 2. The software infrastructure selected for implementation of the SWIRL2D code is described in Section 3. The problem setup and the numerical solutions computed with interface tracking technique (ITrT) using SWIRL2D code are compared against numerical solutions obtained with interface capturing technique (ICaT) using FLUENT [15] in Section 4 while the conclusions are drawn in last section.

## 2. A NOVEL TWO-DIMENSIONAL VARIATIONAL APPROACH

The two dimensional axi-symmetrical swirling flow model employs the following assumptions: (i) the fluid is incompressible and inviscid; (ii) the flow is steady. The axial-symmetry hypothesis allows the use of the Stokes' streamfunction $\psi(x, r)$ to express the velocity field in cylindrical coordinates $(x, r, \theta)$ using unit vectors $(\mathbf{x}, \mathbf{r}, \boldsymbol{\theta})$ as given in eq. (1)

$$\mathbf{v} = v_x\,\mathbf{x} + v_r\,\mathbf{r} + v_\theta\,\boldsymbol{\theta} = -\frac{\boldsymbol{\theta}}{r}\times \operatorname{grad}\psi + \frac{\boldsymbol{\theta}}{r}k(\psi). \tag{1}$$

In the case of steady, axisymmetric swirling flows of inviscid and incompressible fluids the Euler equations collapse into a single partial differential equation (2) [16], known as the Bragg-Hawthorne equation [13],

$$\operatorname{div}\left(\frac{\operatorname{grad}\psi}{r}\right) + \frac{k(\psi)}{r^2}\frac{\mathrm{d}k(\psi)}{\mathrm{d}\psi} - \frac{\mathrm{d}h(\psi)}{\mathrm{d}\psi} = 0, \tag{2}$$

where $\psi(x, r)$ is the dimensionless streamfunction, $k(\psi) \equiv r\, v_\theta$ the dimensionless circulation function and $h(\psi) \equiv p + v^2/2$ the dimensionless total enthalpy.

The variational formulation for the Bragg-Hawthorne equation requires the *minimization* of the following functional within the meridian half-plane of the flow domain $D_{\mathrm{f}}$:

$$\mathbf{F}(\psi) \equiv \int_{D_{\mathrm{f}}} \left[ \frac{1}{2}\left(\frac{\mathrm{grad}\psi}{r}\right)^2 - \frac{\kappa^2(\psi)}{2r^2} + h(\psi) \right] r \mathrm{d}D \cdot \tag{3}$$

When stagnant region is present, the functional (3) must be extended to account for the possible contributions of the stagnant pressure [17]. The swirling flow is confined to the annular section in order to satisfy the differential equation (2). It must minimize the functional (3) with respect to the streamfunction $\psi$ subject to the boundary conditions $\psi_b = 0$ and $\psi_w = q/2$. As a result, the extended flow force functional $\mathbf{F}^*(r_{\mathrm{s}})$ in eq. (4) will embed the contribution of the static pressure $p_{\mathrm{s}}(x)$ on the stagnant region,

$$\mathbf{F}^*(r_{\mathrm{S}}) \equiv \min_{\psi} \mathbf{F}(\psi) + \int_{D_{\mathrm{s}}} p_{\mathrm{S}}(x)\, r \,\mathrm{d}D \cdot \tag{4}$$

It is considered that the boundary between a stagnant region and the swirling flow is generally represented by a vortex sheet with possible jumps in both velocity components. However, since this is a fluid interface, the pressure across the vortex sheet must remain continuous while the evolution of the static pressure on the stagnant region boundary $p_{\mathrm{S}}(x)$ is accounted in the meridian half-plane of the stagnant region domain $D_{\mathrm{S}}$ as follows in eq. (5):

$$\int_{D_{\mathrm{s}}} p_{\mathrm{s}}(x) \;\; r \;\; \mathrm{d}D \cong \frac{1}{2}\int_0^L p_{\mathrm{s}}(x) \;\; r_{\mathrm{s}}^2(x) \;\; \mathrm{d}x, \tag{5}$$

where the static pressure on the stagnant region boundary $p_{\mathrm{s}}(x)$ is computed on the stagnant region boundary (interface) $r_{\mathrm{s}}(x)$ using eq. (6).

$$p_{\mathrm{s}} = h(0) - \frac{v_m^2}{2} - \frac{v_\theta^2}{2} = -\frac{1}{2}\left(\frac{1}{r}\frac{\partial \;\psi}{\partial \;n}\right)_{\mathrm{s}}^2 \cdot \tag{6}$$

As a result, the functional (4) reaches a maximum for the correct stagnant region extent. Conclusively, the numerical approach for computing the swirling flow can be summarized as follows: *find the value of the* $r_{\mathrm{s}}(x)$ *which maximizes the functional* $\mathbf{F}^*(r_{\mathrm{s}})$ *while* $\mathbf{F}(\psi)$ *is minimized.* This is the main theoretical development which allows the correct computation of the swirling flow with stagnant region.

## 3. NUMERICAL IMPLEMENTATION

The two-dimensional variational approach presented in the previous section is implemented in our in-house code called SWIRL2D shown in Fig. 2 together with the software packages containing both legacy FORTRAN77 [18] and C99 [19] modules connected with the PETSc Toolkit [20]. The PETSc Toolkit [20] is Libre software [21] which provides a suite of data structures and routines for the scalable solution of scientific applications modeled by partial differential equations [22].

The boundaries of the two-dimensional computational domain are built in the *preprocessing()* routine. The line segments are used to define the inlet, the outlet and the wall boundaries, respectively. The fluid boundary is defined using a *spline* function with several *knots*. The parameters of the problem correspond to the radial coordinates of the *knots* while the axial coordinates are imposed. The *spline* function is provided by the GNU GSL package as a C library [23]. The initial guess of the fluid boundary is determined using the solutions of 1D swirling flow problems [24] for each axial coordinate of the *knot*. The mesh nodes on each boundary are obtained by interpolation using the GNU GSL package. The unstructured mesh with triangle elements is generated on the two-dimensional computational domain using the TRIANGLE package [25]. As a result, the unstructured mesh is generated based on the boundary nodes together with the list of the connectivity which specifies the way a given set of nodes is associated to each triangle element. A data structure type of *triangulateio* implemented in TRIANGLE package is considered for the unstructured mesh in Fig. 3.

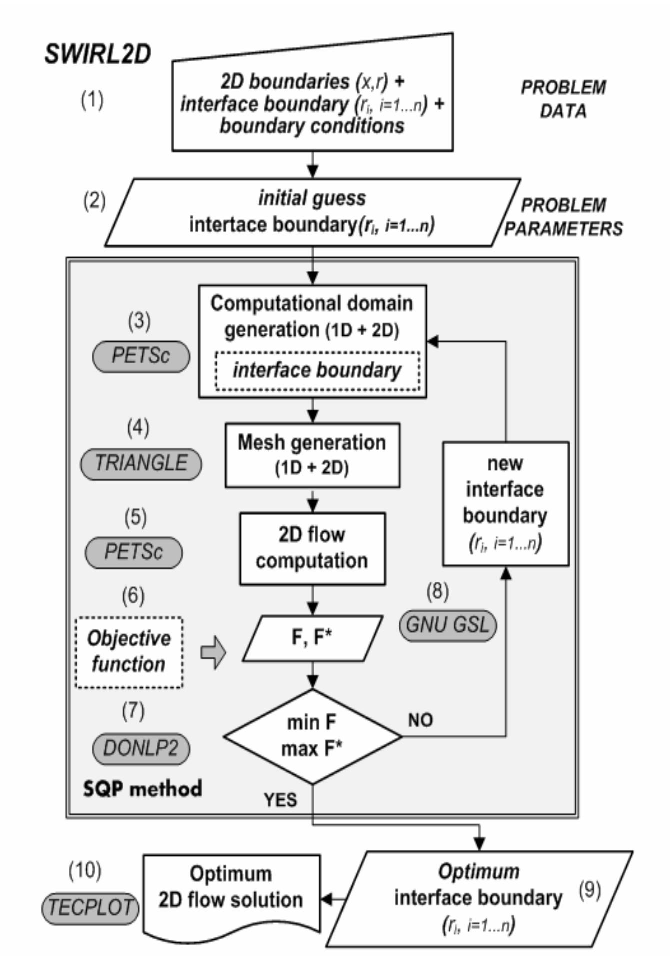


Fig. 2 – Algorithm for computing the swirling flow with stagnant region implemented in SWIRL2D code.

The Finite Element Method (FEM) is considered to discretize the flow equation based on auxiliary variable (stream-function) then the linear system of equations is built and iteratively solved using Krylov method (*KSPsolve()* routine) implemented within the PETSc Toolkit package [18]. As a result, a streamfunction value is obtained in each node of the mesh and the velocity components are straight-forwardly computed. Next, both Bragg-Hawthorne functional (3) over the flow domain and the extended functional (4) over the flow domain with stagnant region are assessed for location of the fluid boundary.

A sequential quadratic programming (SQP) method for the numerical solution of constrained nonlinear optimization problems (NLP) [26, 27] developed by Prof. Peter Spelluci in DONLP2 package [28] and implemented in FORTRAN77 library is used to find the extreme value of each functional. The SQP method uses an iterative procedure to find the location of the fluid boundary which satisfies the conditions. In the end, the SQP method delivers the radial coordinates of the *knots* associated to the optimum solution of the fluid boundary. In order to link the DONLP2 FORTRAN77 package with the main code developed in C, the GNU family of compilers is used for both parts of the code, the GNU FORTRAN compiler and the GNU C compiler [29].

Unlike FORTRAN the C language does not provide any mechanism for calling function arguments by their name. Subroutine arguments in FORTRAN are passed with address references while C language by default passes function arguments by copying the parameter values. Because of the C and FORTRAN linkage all parameters are passed with C pointers. Also variable blocks are grouped in global structures for compatibility. This is transparently handled by the in-house developed DONLP2IF interface package.

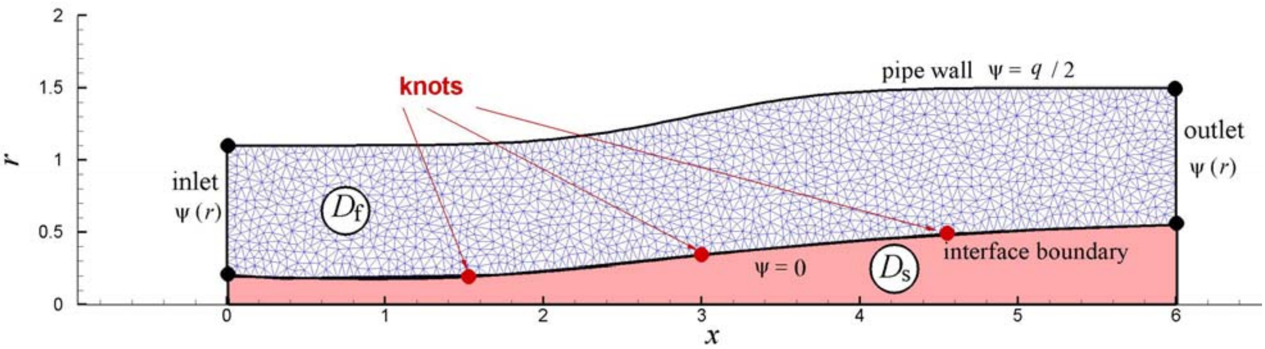


Fig. 3 – Two-dimensional axisymmetric computational domain and triangle mesh generated with SWIRL2D code.

The algorithm for determining the fluid boundary with interface tracking technique implemented in the SWIRL2D package is shown in Fig. 2. The stagnant region is computed starting with an initial educated guess of the coordinates of the points. The rest of the path is obtained by interpolation providing a complete geometry for FEM analysis. A mesh is generated within the computational domain and the flow equations are solved using FEM. The numerical results are used to evaluate the functional yielding its value. The numerical procedure is repeated until the functional value reaches the maximum value. This provides both the path of the stagnant region and the numerical solution within the computational domain.

The final plot of the solution is obtained using external visualization software (e.g. TECPLOT [30]). Although software for converting FORTRAN language to C is available, the original FORTRAN77 version is preferred because it is the native language for scientific computing and it is the original language used for developing the DONLP2 package [28]. A different solution using the F2C [31] converter is tested against the provided benchmarks [29] and the same numbers are obtained. However FORTRAN automatically converted to C99 looks messy and is much harder to debug therefore the use of native language programming packages is preferred.

The PETSc solver is called up until the DONLP2 package [28] determines that the value of the functional is maximal. This is done using the sequential quadratic programming (SQP) algorithm explained in [26] and [27] and boundary constraints consisting of a set of equalities and inequalities. The PETSc Toolkit provides elegant directives for processing the specialized data structures for the solution of large linear systems of equations. The next section presents numerical results obtained using this implementation.

## 4. PROBLEM SETUP. NUMERICAL RESULTS

The two-dimensional, axi-symmetric, steady, inviscid swirling flow in a diffuser is considered in our approach. The computational domain corresponds to the diffuser shape with the wall radius given by

$$r_w(x) = \sqrt{\left[\tfrac{1}{2}\left(R_{w\,inlet}^2 + R_{w\,outlet}^2\right) + \tfrac{1}{2}\left(-R_{w\,inlet}^2 + R_{w\,outlet}^2\right) ERF\left(x - \tfrac{L}{2}\right)\right]} \quad \text{with} \quad 0 \le x \le L = 6\,, \tag{7}$$

where the wall radius of the inlet section is $R_{w\,inlet} = 1.1$, the wall radius of the outlet section is $R_{w\,outlet} = 1.5$ and *ERF(x)* is error function (also called the Gauss error function) defined as

$$ERF\left(x\right) = \frac{2}{\sqrt{\pi}} \int_0^x \mathrm{e}^{-t^2} \mathrm{d}t\,. \tag{8}$$

A computational domain is shown in Figure 3, with the inlet boundary corresponding to an annular section delimited by the hub and the shroud radii, respectively. For computational convenience, the actual outlet section is selected in order to comply with the parallel flow assumption (negligible radial velocity) on it. The Neumann boundary conditions imposed on the inlet and outlet sections are computed using SWIRL1D code [24, §4.1]. The Dirichlet boundary conditions are imposed on the wall ($\psi = q/2$) and the fluid boundary ($\psi = 0$), respectively.

The inlet boundary conditions imposed for FLUENT2D problems correspond to a solid body rotation swirling flow, with velocity components given by

$$v_{x\ inlet} = 1; \quad v_{r\ inlet} = 0; \quad v_{\theta\ inlet}(r) = \omega r, \tag{9}$$

where $\omega$ is the angular velocity, the wall radius of the inlet section is $R_{winlet} = 1.1$ while the hub radius of the inlet section of $R_{b\ inlet} = 0.2$ is selected. Two cases are selected in order to verify the numerical solutions computed with SWIRL2D code. The swirl intensity is increased as $\zeta \equiv 2\omega R_{winlet}/v_{x\,inlet} = 1$ and $2$ [36].

Figure 4 shows the streamlines in a meridian half-plane for low swirl intensity $\zeta = 1$. The numerical solution computed using interface tracking technique (ITrT) with SWIRL2D code is plotted in the upper meridian half-plane while 2D axi-symmetric inviscid flow field obtained using interface capturing technique (ICaT) with FLUENT2D is presented in the lower meridian half-plane, respectively. One can observe a good qualitative agreement between both two-dimensional numerical solutions.

The validation of the numerical results obtained with the SWIRL2D code against the results obtained with FLUENT2D is performed using the velocity components. In this case, the axial and tangential velocity components are computed based on the streamfunction using:

$$v_x \equiv \frac{1}{r}\frac{\partial \psi}{\partial r} \text{ and } v_\theta \equiv 2\omega \frac{\psi}{r}. \tag{10}$$

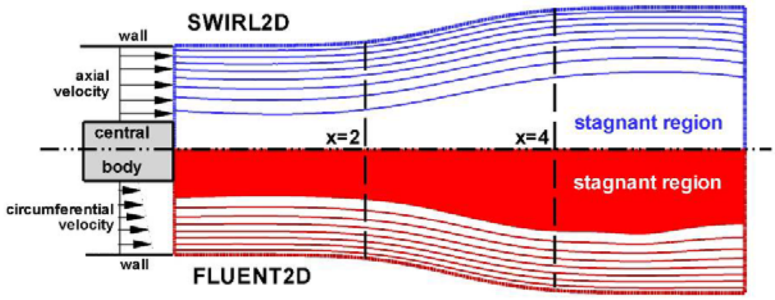


Fig. 4 – Streamlines for swirling flow with stagnant region for swirl intensity of $\zeta = 1$ : SWIRL2D solution (upper meridian half-plane) and FLUENT2D axi-symmetric inviscid solution (lower meridian half-plane).

Figure 5 presents the numerical result obtained with the SWIRL2D code against the results obtained with FLUENT2D on the first case ( $\zeta = 1$ ) on two radial sections displaced at $x = 2$ and $x = 4$, respectively.

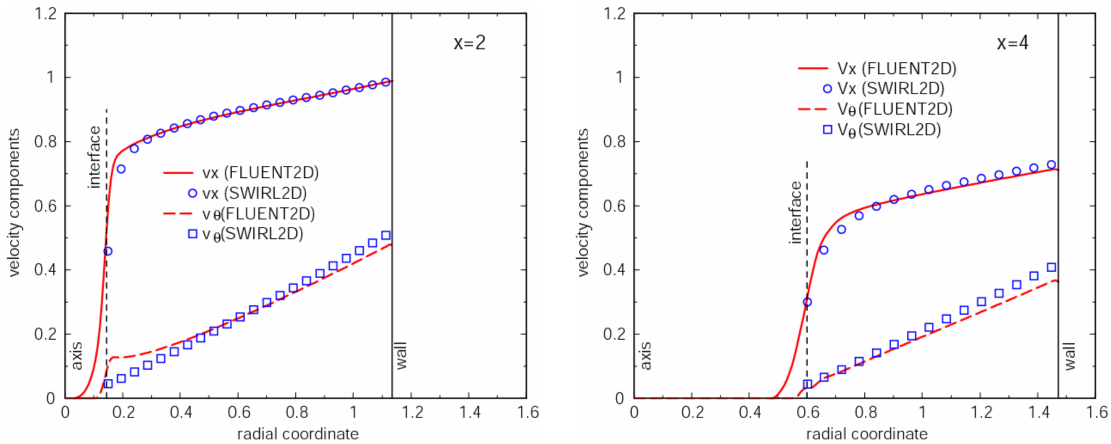


Fig. 5 – Streamlines for swirling flow with stagnant region for swirl intensity of $\zeta = 1$ : SWIRL2D solution (blue dots) and FLUENT2D axi-symmetric inviscid solution (red lines).

The streamlines in a meridian half-plane for swirl intensity of $\zeta = 2$ are plotted in Fig. 6. Also, the numerical solution computed using interface tracking technique (ITrT) with SWIRL2D code (upper meridian half-plane) is qualitatively compared against 2D axi-symmetric inviscid flow solution obtained using interface capturing technique (ICaT) with FLUENT (lower meridian half-plane).

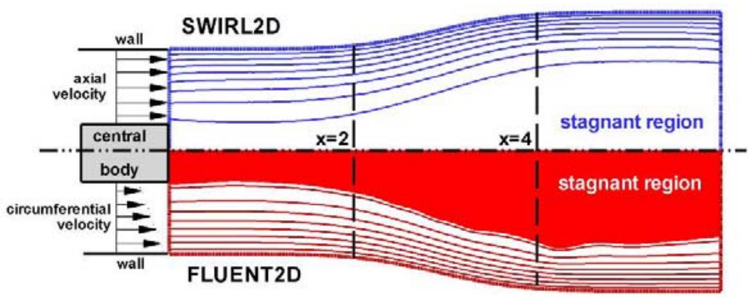


Fig. 6 – Streamlines for swirling flow with stagnant region for swirl intensity of $\zeta = 2$ : SWIRL2D solution (upper meridian half-plane) and FLUENT2D axi-symmetric inviscid solution (lower meridian half-plane).

A good comparison between the numerical results obtained with the SWIRL2D code against the results obtained with FLUENT2D on the second case ($\zeta = 2$), on both radial sections displaced at $x = 2$ and $x = 4$, is shown in Fig. 7.

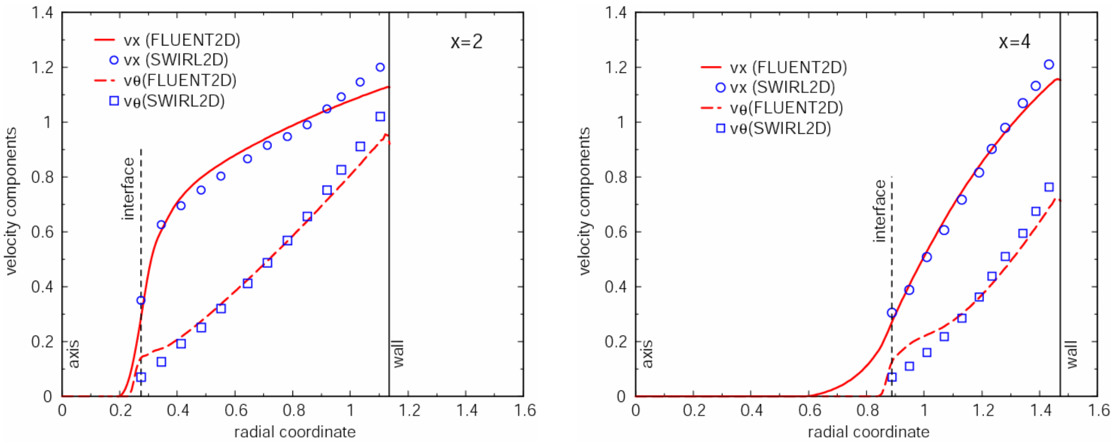


Fig. 7 – Streamlines for swirling flow with stagnant region for swirl intensity of $\zeta = 2$ : SWIRL2D solution (blue dots) and FLUENT2D axi-symmetric inviscid solution (red lines).

## 5. CONCLUSIONS

The paper presents a tailored algorithm for computing the two-dimensional swirling flows based on interface tracking approach. An augmented functional for computing the fluid interface boundary in the two-dimensional inviscid swirling flows is developed. The novel variational approach is built by admitting a continuous static pressure across the fluid interface between the main annular swirling flow and the inner stagnant region. The algorithm is implemented in the SWIRL2D package using a numerical platform. The inlet boundary condition incorporates the kinematical constraint on the relative flow angle at the runner outlet, as well as integral constrains corresponding to the operating point (i.e. discharge). The inlet boundary conditions are implemented within the streamfunction formulation of the Bragg-Hawthorne equation. This is not possible within the regular commercial codes (e.g. FLUENT), where all velocity components need to be prescribed at the inlet section. The numerical solutions computed with actual algorithm agree well with ones obtained with FLUENT code. As a result, the next investigations can exclusively focus on the parametric studies related to the configuration of the geometry diffuser since rapid and accurate assessment of the swirling flow is possible using this customized swirling flow solver.

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