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HIGHER-ORDER RATIONAL SOLUTIONS FOR THE (2+1)-DIMENSIONAL KMN EQUATION

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Abstract. In this paper, we generalize the perturbation (*n*; *M*)-fold Darboux transformation from the (1+1)-dimensional nonlinear partial differential equations to the (2+1)-dimensional nonlinear models. As an example, we investigate the (2+1)-dimensional KMN equation. Some higher-order rational solutions including rogue waves in terms of determinants are obtained by using the Darboux matrix, Taylor expansion, and a limit procedure. The dynamical behaviors of these abundant rational solutions including rogue waves are discussed in detail for different sets of parameters.

Key words: (2+1)-dimensional KMN equation, generalized perturbation (*n*; *M*)-fold Darboux transformation; higher-order rational solutions, rogue waves.

1. INTRODUCTION

Explicit solutions, in particular some types of single- and multi-soliton solutions of nonlinear partial differential equations (NPDEs), may well describe various phenomena in physics and other fields [1–2]. Through these solutions, we get a good insight into the physical aspects of the problems. Recently, rogue waves (RWs) have attracted more and more theoretical and experimental attention [3]. The RWs were first observed in deep oceans and later these studies gradually extended to other fields, such as fiber optics, Bose-Einstein condensates, and capillary waves [3–20]. Nowadays, for better understanding of the physical mechanisms of the RWs, several methods for constructing RW solutions have been received increasing attention, such as the Darboux transformation (DT) method, the similarity transformation, Hirota bilinear method, and so on [3–20]. Actually the RW is a special type of non-singular rational solution.

In this paper, our aim is to apply our proposed generalized perturbation (*n*;*M*)-fold DT method [4,5] to construct higher-order rational solutions including RW solutions of (2+1)-dimensional NPDEs. As an example, we will study the following (2+1)-dimensional Kundu, Mukherjee, and Naskar (KMN) equation [6]

$$\mathrm{i}q_t - q_{xy} - 2\mathrm{i}q(qq_x^{\ *} - q^* q_x) = 0, \tag{1}$$

where the asterisk denotes the complex conjugation. The KMN equation (1) was introduced by Kundu, Mukherjee, and Naskar, see e.g. Ref. [6], as a new extension of the well-known nonlinear Schrödinger (NLS) equation. The first-order RW solution for Eq. (1) has been obtained in Ref. [6] by the one-fold DT from a non-zero "seed" solution, and the associated properties have been also discussed. To the best of our knowledge, there is no study on the higher-order rational solutions including RW solutions of Eq. (1). Thus we will further study Eq. (1) via our proposed generalized perturbation (*n*; *M*)-fold DT.

2. GENERALIZED PERTURBATION (*N*; *M*)-FOLD DT

Equation (1) can be decomposed into the following (1+1)-dimensional focusing NLS equation [14,17]:

$$q_y + iq_{xx} + 2i\,|\,q\,|^2\, q = 0, \tag{2}$$

and the (1+1)-dimensional complex mKdV equation [14, 21, 22]:

$$q_t + q_{xxx} + 6\,|\,q\,|^2\, q_x = 0. \tag{3}$$

We can prove that if $q(x,y,t)$ solves Eqs. (2) and (3), then it also satisfies Eq. (1). The equations (2) and (3) are considered as the compatibility condition of the following linear systems:

$$\varphi_x = U\varphi = \begin{pmatrix} \mathrm{i}\lambda & \mathrm{i}q \\ \mathrm{i}q^* & -\mathrm{i}\lambda \end{pmatrix}\varphi, \tag{4}$$

$$\varphi_y = U\varphi = \begin{pmatrix} 2\mathrm{i}\lambda^2 - \mathrm{i}\,|q|^2 & 2\lambda \mathrm{i}q + q_x \\ 2\lambda \mathrm{i}q^* - q_x^* & -2\mathrm{i}\lambda^2 + \mathrm{i}\,|q|^2 \end{pmatrix}\varphi, \tag{5}$$

$$\varphi_t = U\varphi = \begin{pmatrix} 4\mathrm{i}\lambda^3 - 2\mathrm{i}\lambda\,|q|^2 + qq_x^* - q^*q_x & 4\mathrm{i}\lambda^2 q + 2\lambda q_x - 2\mathrm{i}\,|q|^2\, q - \mathrm{i}q_{xx} \\ 4\mathrm{i}\lambda^2 q^* - 2\lambda q_x^* - 2\mathrm{i}\,|q|^2\, q^* - \mathrm{i}q^*_{xx} & -4\mathrm{i}\lambda^3 + 2\mathrm{i}\lambda\,|q|^2 - qq_x^* + q^*q_x \end{pmatrix}\varphi, \tag{6}$$

where $\varphi = (\phi,\psi)^{\mathrm{T}}$ (the superscript T denotes the vector transpose) is the solution of the system (4–6), λ is the spectral parameter, and $\mathrm{i}^2 = -1$. It is easy to check that the consistency conditions $\varphi_{xy} = \varphi_{yx}$ and $\varphi_{xt} = \varphi_{tx}$ are equivalent to Eqs. (2) and (3), respectively. We consider the gauge transformation

$$\tilde{\varphi} = T(\lambda)\varphi\,,\;\; \tilde{\varphi} = (\tilde{\phi},\tilde{\psi})^{\mathrm{T}}, \tag{7}$$

with the Darboux matrix

$$T(\lambda) = \begin{pmatrix} \lambda^N + \sum_{j=0}^{N-1} A^{(j)}\lambda^j & \sum_{j=0}^{N-1} B^{(j)}\lambda^j \\ -\sum_{j=0}^{N-1} B^{(j)*}\lambda^j & \lambda^N + \sum_{j=0}^{N-1} A^{(j)*}\lambda^j \end{pmatrix}. \tag{8}$$

Here the transformation (7) maps the old eigenfunction φ into the new one $\tilde{\varphi}$, where $\tilde{\varphi}$ is also required to satisfy the system (4–6) with U, V, and W being replaced by $\tilde{U}, \tilde{V}$, and $\tilde{W}$, and the potential q being replacing by $\tilde{q}$. In the matrix (8), N is a positive integer, the $2N$ unknown complex functions $A^{(j)}, B^{(j)}$ $(0 \le j \le$ $\le N-1)$ are determined by solving the following linear algebraic system with $2N$ equations

$$T(\lambda_k)\varphi_k = 0,\;\; (k = 1,2,..,N) \tag{9}$$

where $\varphi_k = \varphi(\lambda_k) = (\phi(\lambda_k),\psi(\lambda_k))^T$ $(k = 1,2,..,N)$ are the solutions of the system (4–6) for the corresponding N distinct spectral parameters λ_k $(k = 1,2,..,N)$ and the initial solution q_0. When N distinct spectral parameters λ_k $(k = 1,2,..,N)$ are suitably chosen such that $2N$ variables $A^{(j)}, B^{(j)}$ (j = 0, 1, ..., $N-1$) in matrix (8) can be uniquely determined, therefore, the transformation (7) is uniquely given. By using the above facts, we can prove the following theorem.

THEOREM 1. *The system* (4–6) *is covariant with respect to the transformations (7) and*

$$\tilde{q}_{N-1} = q_0 - 2B^{(N-1)}, \tag{10}$$

where $B^{(N-1)}$ *is given by the linear algebraic system* (9), i.e., $B^{(N-1)} = \dfrac{\Delta B^{(N-1)}}{\Delta_N}$ *with*

$$\Delta_N = \begin{vmatrix} \lambda_1^{N-1}\phi_1 & \lambda_1^{N-2}\phi_1 & \cdots & \phi_1 & \lambda_1^{N-1}\psi_1 & \lambda_1^{N-2}\psi_1 & \cdots & \psi_1 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \lambda_N^{N-1}\phi_N & \lambda_N^{N-2}\phi_N & \cdots & \phi_N & \lambda_N^{N-1}\psi_N & \lambda_N^{N-2}\psi_N & \cdots & \psi_N \\ \lambda_1^{*(N-1)}\psi_1^* & \lambda_1^{*(N-2)}\psi_1^* & \cdots & \psi_1^* & -\lambda_1^{*(N-1)}\phi_1^* & -\lambda_1^{*(N-2)}\phi_1^* & \cdots & -\phi_1^* \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \lambda_N^{*(N-1)}\psi_N^* & \lambda_N^{*(N-2)}\psi_N^* & \cdots & \psi_N^* & -\lambda_N^{*(N-1)}\phi_N^* & -\lambda_N^{*(N-2)}\phi_N^* & \cdots & -\phi_N^* \end{vmatrix}$$

and $\Delta B^{(N-1)}$ *is given by the determinant* Δ_N *by replacing its* (N + 1)*-th column by the column vector*

$(-\lambda_1{}^N\phi_1, -\lambda_2{}^N\phi_2, .., -\lambda_N{}^N\phi_N, -\lambda_1{}^{*N}\psi_1{}^*, -\lambda_2{}^{*N}\psi_2{}^*, .., -\lambda_N{}^{*N}\psi_N{}^*)^{\mathrm{T}}$.

The transformations (7) and (10) are referred to as the *N*-fold DT of Eq. (1). The multi-soliton solutions (or breather solutions) of Eq. (1) can be derived by the *N*-fold DT with the initial solution $q_0 = 0$ (q_0 is an initial plane-wave solution). In the following, we will extend the above-found *N*-fold DT to the generalized perturbation (*n, M*) – fold DT through Darboux matrix (8), Taylor expansion, and a limit procedure such that higher-order rational solutions in terms of determinants of Eq. (1) can be found. In fact, we can change the number of spectral parameter λ in (9); here we only use n ($1 \le n \le N$) distinct spectral parameters λ_i (i = 1,2,...,n) and their corresponding m_i- order (m_i = 0, 1, 2,...) perturbation derivatives, where n, m_i satisfy $N = n + \sum_{i=1}^{n} m_i = n + M$ with $M = \sum_{i=1}^{n} m_i$. For the spectral parameters λ_i (i = 1, 2,..., n) with $n < N$, we can obtain 2n linear algebraic equations that contain the 2N unknown variables $A^{(j)}, B^{(j)}$ (j = 0, 1,...,$N-1$). To determine the 2N unknown variables $A^{(j)}, B^{(j)}$ uniquely we must add the 2($N-n$) constraint equations for the variables $A^{(j)}, B^{(j)}$, and for this reason, we expand the following expression at $\varepsilon = 0$:

$$T(\lambda_i)\varphi|_{\lambda_i=\lambda_i+\varepsilon} = T(\lambda_i+\varepsilon)\varphi(\lambda_i+\varepsilon) = 0, \qquad (i = 1, 2, .., n), \tag{11}$$

where $\varphi(\lambda_i+\varepsilon) = \varphi^{(0)}(\lambda_i) + \varphi^{(1)}(\lambda_i)\varepsilon + \varphi^{(2)}(\lambda_i)\varepsilon^2 + ... + \varphi^{(k)}(\lambda_i)\varepsilon^k + ...$ with $\varphi_i^{(k)}(\lambda_i) = \frac{1}{k!}\frac{\partial^k}{\partial\lambda_i^k}\varphi_i(\lambda)|_{\lambda=\lambda_i}$, $\varphi_i^{(0)}(\lambda_i) = \varphi_i(\lambda_i)$, and $T(\lambda_i+\varepsilon) = \sum_{k=0}^{N} T^{(k)}(\lambda_i)\varepsilon^k$ in which $T^{(0)}(\lambda_i) = T(\lambda_i)$, $T^{(N)}(\lambda_i) = I$ (I is an unit matrix). Thus we have

$$T(\lambda_i+\varepsilon)\varphi(\lambda_i+\varepsilon) = \sum_{k=0}^{N}\sum_{j=0}^{k} T^{(j)}(\lambda_i)\varphi^{(k-j)}(\lambda_i)\varepsilon^k + \sum_{k=1}^{\infty}\sum_{j=0}^{N} T^{(j)}(\lambda_i)\varphi^{(N-k-j)}(\lambda_i)\varepsilon^{N+k} = 0. \tag{12}$$

Let $\varphi_k = (\phi(\lambda_k), \psi(\lambda_k))^{\mathrm{T}}$ $(k = 1, 2, .., n)$ be n distinct column vector solutions of the system (4–6) for the corresponding n distinct spectral parameters λ_k $(k = 1, 2, .., n)$ and the initial solution q_0, respectively. To determine the 2N unknown variables $A^{(j)}, B^{(j)}$ (j = 0, 1, ..., $N-1$), let

$$\lim_{\varepsilon\to 0}\frac{T(\lambda_i+\varepsilon)\varphi(\lambda_i+\varepsilon)}{\varepsilon^{k_i}} = 0 \ (i = 1, 2, .., n) \tag{13}$$

with $k_i = 0, 1, .., m_i$, so that we can obtain the linear algebraic system with 2N equations:

$$\begin{cases} T^{(0)}(\lambda_i)\varphi^{(0)}(\lambda_i) = 0, \\ T^{(0)}(\lambda_i)\varphi^{(1)}(\lambda_i) + T^{(1)}(\lambda_i)\varphi^{(0)}(\lambda_i) = 0, \\ ..., \\ \sum_{j=0}^{m_i} T^{(j)}(\lambda_i)\varphi^{(m_i-j)}(\lambda_i) = 0. \end{cases} \qquad (i = 1, 2, .., n). \tag{14}$$

The 2N unknown variables $A^{(j)}, B^{(j)}$ ($0 \le j \le N-1$) are determined by the system (14) via the Cramer's rule, so we have the following theorem.

THEOREM 2. *The system* (4–6) *is covariant with respect to the transformations (7) and*

$$\tilde{q}_{N-1} = q_0 - 2B^{(N-1)}, \tag{15}$$

where $B^{(N-1)}$ *is given by the linear algebraic system* (14), *i.e.,* $B^{(N-1)} = \frac{\Delta B^{(N-1)}}{\Delta_N}$ *with*

$$\Delta_N = \det([\Delta^{(1)}_{m_1+1}, \Delta^{(2)}_{m_2+1}, ..., \Delta^{(n)}_{m_n+1}]^{\mathrm{T}}),$$

$$\Delta_{m_i+1}^{(i)} = (\Delta_{j,s}^{(i)})_{2(m_i+1)\times 2N}$$

$$= \begin{vmatrix} \lambda_i^{N-1}\phi_i^{(0)} & \lambda_i^{N-2}\phi_i^{(0)} & \cdots & \phi_i^{(0)} & \lambda_i^{N-1}\psi_i^{(0)} & \lambda_i^{N-2}\psi_i^{(0)} & \cdots & \psi_i^{(0)} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \Delta_{m_i+1,1}^{(i)} & \Delta_{m_i+1,2}^{(i)} & \cdots & \phi_i^{(m_i)} & \Delta_{m_i+1,N+1}^{(i)} & \Delta_{m_i+1,N+2}^{(i)} & \cdots & \Delta_{m_i+1,2N}^{(i)} \\ \lambda_i^{*(N-1)}\psi_i^{(0)*} & \lambda_i^{*(N-2)}\psi_i^{(0)*} & \cdots & \psi_i^{(0)*} & -\lambda_i^{*(N-1)}\phi_i^{*(0)} & -\lambda_i^{*(N-2)}\phi_i^{*(0)} & \cdots & -\phi_i^{*(0)} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \Delta_{2(m_i+1),1}^{(i)} & \Delta_{2(m_i+1),2}^{(i)} & \cdots & \psi_i^{(m_i)*} & \Delta_{2(m_i+1),N+1}^{(i)} & \Delta_{2(m_i+1),N+2}^{(i)} & \cdots & -\phi_i^{(m_i)**} \end{vmatrix}, \quad (16)$$

where $\Delta_{j,s}^{(i)} (1 \le j \le 2(m_i+1), 1 \le s \le 2N)$ *are given in the form*:

$$\Delta_{j,s}^{(i)} = \begin{cases} \sum_{k=0}^{j-1} C_{N-s}^{k} \lambda_i^{N-s-k} \phi_i^{(j-1-k)}, & \text{for } 1 \le j \le m_i+1, 1 \le s \le N \\ \sum_{k=0}^{j-1} C_{2N-s}^{k} \lambda_i^{2N-s-k} \psi_i^{(j-1-k)}, & \text{for } 1 \le j \le m_i+1, N+1 \le s \le 2N \\ \sum_{k=0}^{j-N-1} C_{N-s}^{k} \lambda_i^{*(N-s-k)} \psi_i^{*(j-N-1-k)}, & \text{for } m_i+2 \le j \le 2(m_i+1), 1 \le s \le N \\ -\sum_{k=0}^{j-N-1} C_{2N-s}^{k} \lambda_i^{*(2N-s-k)} \phi_i^{*(j-N-1-k)}, & \text{for } m_i+2 \le j \le 2(m_i+1), N+1 \le s \le 2N \end{cases}$$

and $\Delta B^{(N-1)}$ *is given by the determinant* Δ_N *by replacing its* (N + 1)*-th column by the column vector* $(b^{(1)},\ldots,b^{(n)})^{\mathrm{T}}$, *where* $b^{(i)} = (b_j^{(i)})_{2(m_i+1)\times 1}$ *with*

$$b_j^{(i)} = \begin{cases} -\sum_{k=0}^{j-1} C_N^{k} \lambda_i^{N-k} \phi_i^{(j-1-k)}, & \text{for } 1 \le j \le m_i+1, \\ -\sum_{k=0}^{j-N-1} C_N^{k} \lambda_i^{*(N-k)} \psi_i^{*(j-N-1-k)}, & \text{for } m_i+2 \le j \le 2(m_i+1). \end{cases}$$

Remark. Here we call the transformations (7) and (15) as the generalized perturbation (*n,M*)-fold DT of Eq. (1). When $n = N$ and $m_i = 0$ (i = 1, 2, .., n), THEOREM 2 reduces to the THEOREM 1, i.e., the (*n* ,*M*)-fold DT is the extension of the usual *N*-fold DT. When n = 1 and $m_1 = N-1$, THEOREM 2 reduces to the generalized perturbation (1, *N*−1)-fold DT that is used to obtain the higher-order rational solutions in the next Section.

3. HIGHER-ORDER RATIONAL SOLUTIONS

In this Section, we only discuss the generalized perturbation (1, *N*−1)-fold DT with the same single spectral parameter. We now consider the ‘seed’ plane wave solution of Eq. (1) in the form $q_0 = ce^{i[ax+(a^2-2c^2)y+(a^3-6ac^2)t]}$, where the real-valued constant a is the wave number, the real-valued non-zero constant c is the amplitude of the plane wave, and the wave numbers in *x, y* directions are a and a^2-2c^2, respectively. It is known that the phase velocities in *x, y* directions are $6c^2-a^2$ and $(6ac^2-a^3)/(a^2-2c^2)$, respectively, and the group velocities are $6c^2-3a^2$ and $3c^2-3a^2/2$, respectively. We know that $|q_0| \to |c|$ as $|x|, |t| \to \infty$. Substitution of the initial plane wave solution $q_0 = ce^{i[ax+(a^2-2c^2)y+(a^3-6ac^2)t]}$ into the system (4–6) can give the eigenfunction solution with fixed λ as follows:

$$\varphi=\begin{pmatrix}(C_2\mathrm{e}^{A}+C_1\mathrm{e}^{-A})\mathrm{e}^{B}\\ (\frac{a-2\lambda+\sqrt{a^2-4\lambda a+4\lambda^2+4c^2}}{2c}C_2\mathrm{e}^{A}+\frac{a-2\lambda-\sqrt{a^2-4\lambda a+4\lambda^2+4c^2}}{2c}C_1\mathrm{e}^{-A})\mathrm{e}^{-B}\end{pmatrix} \tag{17}$$

with

$$A=\frac{i}{2}\sqrt{a^2-4\lambda a+4\lambda^2+4c^2}[x+(a+2\lambda)y+(4\lambda^2+2\lambda a+a^2-2c^2)t+\Theta(\varepsilon)],$$

$$\Theta(\varepsilon)=\sum_{k=1}^{N}(b_k+id_k)\varepsilon^{2k},\qquad B=\frac{i}{2}[ax+(a^2-2c^2)y+(a^3-6ac^2)t],$$

where C_1, C_2, b_k, d_k are real parameters and ε is a small parameter. Next, we fix the spectral parameter $\lambda_1=\frac{1}{2}a-ic$, and set $\lambda=\lambda_1+\varepsilon^2$, then expand the vector function $\varphi(\varepsilon^2)$ in Eq. (17) as a Taylor series at $\varepsilon=0$. Here we choose $C_1=C_2=1$ to simplify the expansion expression of φ, so we have

$$\varphi(\varepsilon^2)=\varphi^{(0)}+\varphi^{(1)}\varepsilon^2+\varphi^{(2)}\varepsilon^4+\varphi^{(3)}\varepsilon^6+.... \tag{18}$$

in which $\varphi^{(0)}=\begin{pmatrix}\phi^{(0)}\\ \psi^{(0)}\end{pmatrix}=\begin{pmatrix}2\mathrm{e}^{\frac{i}{2}[ax+(a^2-2c^2)y+(a^3-6ac^2)t]}\\ 2i\mathrm{e}^{-\frac{i}{2}[ax+(a^2-2c^2)y+(a^3-6ac^2)t]}\end{pmatrix}$ and $\varphi^{(i)}=(\phi^{(i)},\psi^{(i)})^{\mathrm{T}}\quad (i=1,2,3,...)$ are so complicated and will be omitted here.

To understand the obtained exact solutions of Eq. (1) via the generalized perturbation (1, $N-1$)-fold DT, we will discuss the solution (15) for four cases with N = 1, 2, 3, 4. The case N = 1 is not interesting because it does not generate any new solutions except for the trivial plane wave solution.

(I) When N = 2, based on the generalized perturbation (1, 1)-fold DT, we have

$$\tilde{q}_1=q_0-2B^{(1)}=ce^{i[ax+(a^2-2c^2)y+(a^3-6ac^2)t]}-2\frac{\Delta B^{(1)}}{\Delta_2}. \tag{19}$$

The analytical expression of first-order rational solution for Eq. (1) is given as below:

$$\tilde{q}_1=q_0-2B^{(1)}=ce^{i[ax+(a^2-2c^2)y+(a^3-6ac^2)t]}[1+2(12iac^2t+4ic^2y)/(1+18a^4c^2t^2+12a^2c^2xt+8c^4y^2+ 24a^3c^2yt+8ac^2xy+8a^2c^2y^2+12c^3t-12cx+2c^2x^2+72c^6t^2-4acy-6a^2ct-24c^4xt)]. \tag{20}$$

For solution (20) with two arbitrary parameters a, c when choosing t as a constant, $\tilde{q}_1$ is the RW solution of NLS Eq. (2) with $y=t$; the related properties are omitted here and the reader can refer to [10]. When taking y as constant, $\tilde{q}_1$ is the rational solution of the complex mKdV Eq. (3), the relevant properties are omitted here, and the reader can refer to [21, 22]. If $a=0$ it is necessary to mention that the solution (20) reduces to the rational solution given in [6].

(II) When N = 3, based on the generalized perturbation (1, 2)-fold DT, we can derive the second-order rational solution with four arbitrary constant parameters a, c, b_1, d_1 as below:

$$\tilde{q}_2=q_0-2B^{(2)}=ce^{i[ax+(a^2-2c^2)y+(a^3-6ac^2)t]}-2\frac{\Delta B^{(2)}}{\Delta_3}. \tag{21}$$

The analytical expression of $\tilde{q}_2$ is so complicated and will be omitted here. Next, we discuss some special structures of the second-order rational solution for the following two cases:

- For solution (21) with parameters $a=b_1=d_1=0$, $c=1$ with different values of t, the evolution structures are shown in Fig. 1. When the time $|t|$ is larger than a small constant, the second-order RW is split into three first-order RWs and displays a triangle structure. When $|t|\to 0$, three first-order RWs crowd round the origin $(x, y)=(0,0)$ and strongly interact.
- For solution (21) with parameters b_1 = 100, $a=d_1=0$, c = 1 with different values of y, the evolution structures are shown in Fig. 2. We can see that the second-order rational solution exhibits a one-soliton

structure for $y < 0$, then the one-soliton becomes a two-parallel-soliton structure with increasing y, and the amplitude of two-parallel-soliton first increases, then decreases and finally vanishes.

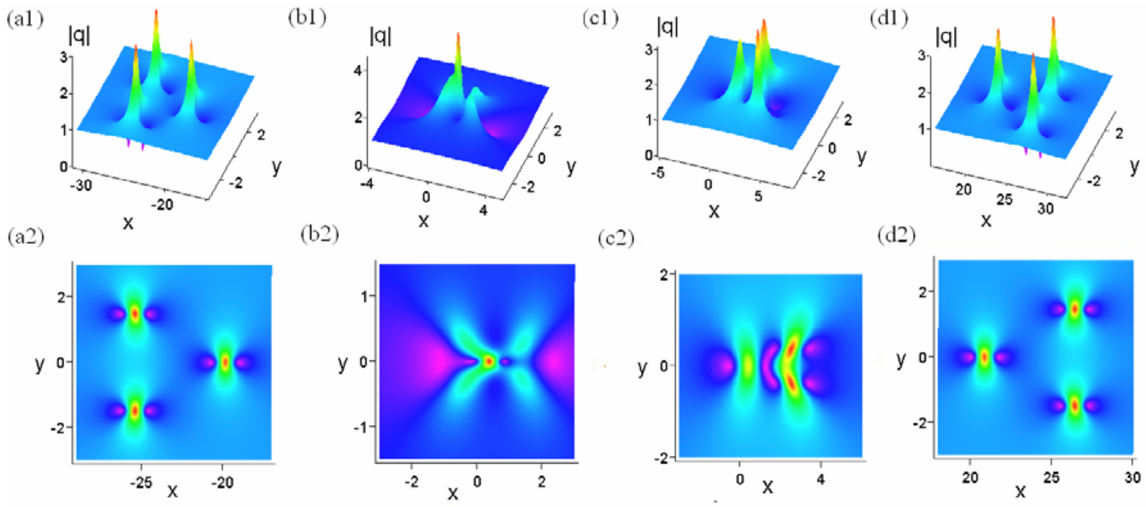

Fig. 1 – The second-order rogue wave solution (21) with $a = b_1 = d_1 = 0$, c = 1 and different values of t: (a1–a2) $t = -4$; (b1–b2) $t = 0$; (c1–c2) $t = 0.2$; (d1–d2) $t = 4$.

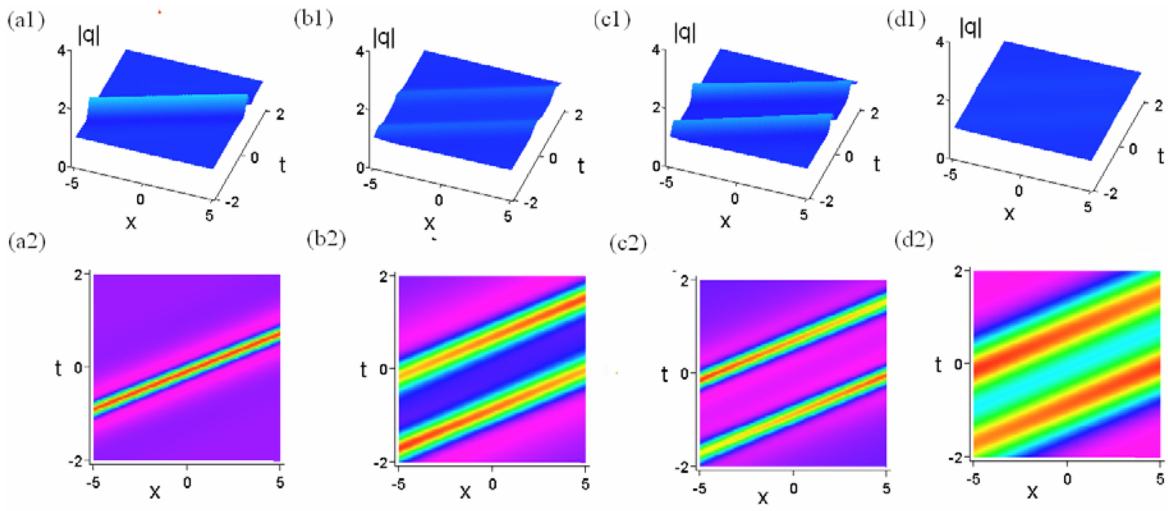

Fig. 2 – The second-order rational solution (21) with $b_1 = 100$, $a = d_1 = 0$, $c = 1$ and different values of y: (a1–a2) $y = -2$; (b1–b2) $y = 0$; (c1–c2) $y = 2$; (d1–d2) $y = 4$.

(III) When N =4, based on the generalized perturbation (1, 3)-fold DT, we can derive the third-order RW solution with six arbitrary constant parameters a, c, b_1, d_1, b_2, d_2 as below:

$$\tilde{q}_3 = q_0 - 2B^{(3)} = ce^{i[ax+(a^2-2c^2)y+(a^3-6ac^2)t]} - 2\frac{\Delta B^{(3)}}{\Delta_4}. \tag{22}$$

Here we omit the analytical expression of $\tilde{q}_3$ because it is so tedious. Next, we discuss some special structures of the third-order rational solution for the following two cases:

- For solution (22) with parameters $a = b_1 = d_1 = b_2 = d_2 = 0$, $c = 1$ (see Fig. 3), we can see that $\tilde{q}_2 \to 1$ by letting $x \to \infty, y \to \infty$ with the fixed time t. When $|t| \neq 0$, the third-order RW is split into six first-order RWs, which arrange into a triangle structure. When $|t| \to 0$, the six first-order RWs crowd round the origin and strongly interact.

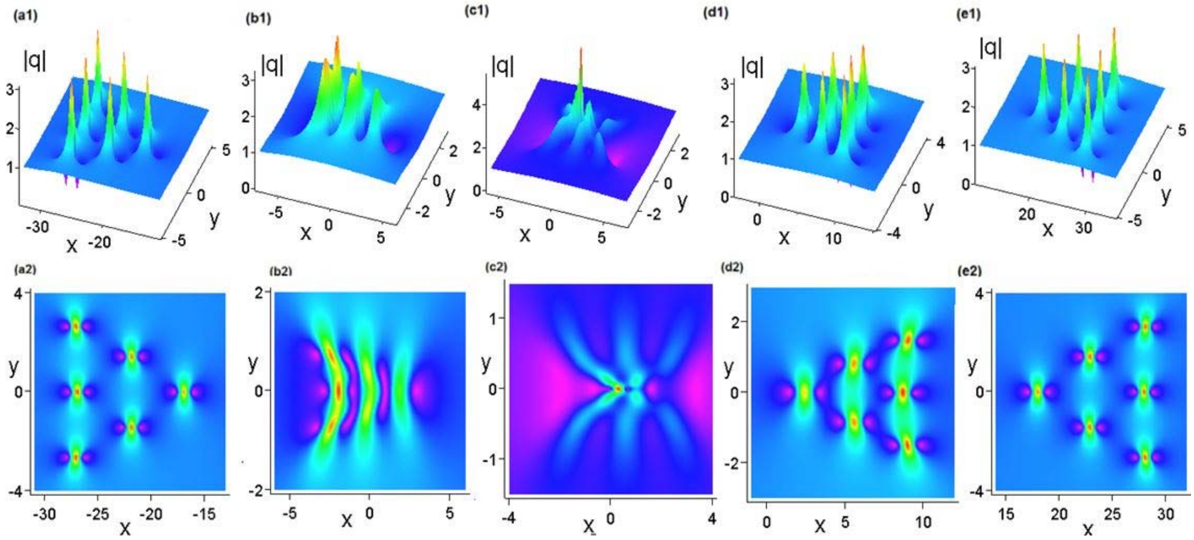

Fig. 3 – The third-order RW solution (22) with parameters $a = b_1 = d_1 = b_2 = d_2 = 0$, c = 1 and different values of t: (a1–a2) $t = -4$; (b1–b2) $t = -0.2$; (c1–c2) $t = 0$; (d1–d2) $t = 1$; (e1–e2) $t = 4$.

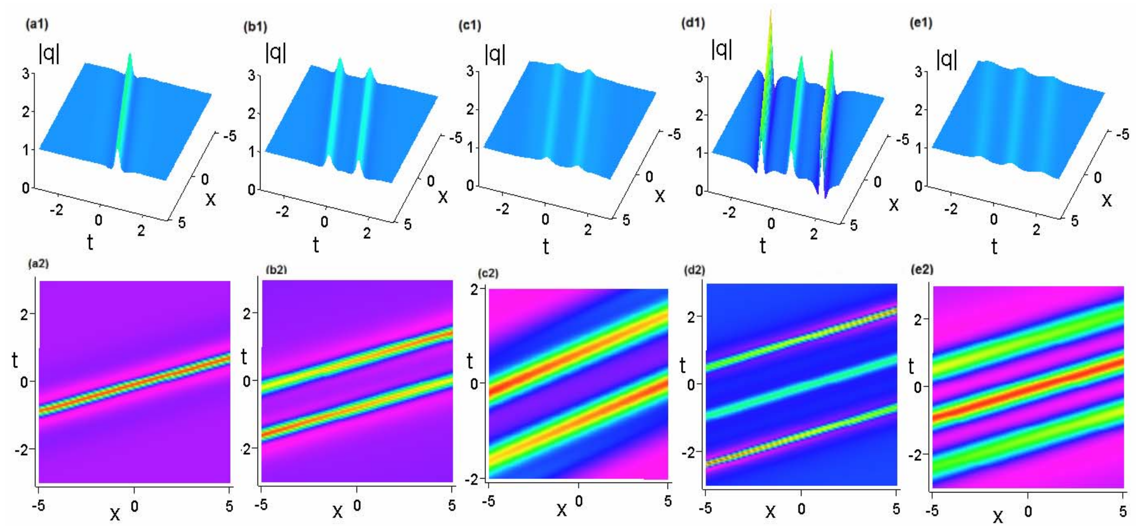

Fig. 4 The third-order rational solution (22) with $b_1 = 100$, $a = d_1 = b_2 = d_2 = 0$, $c = 1$ and different values of y: (a1–a2) $y = -4$; (b1–b2) $y = -2$; (c1–c2) $y = 0$; (d1–d2) $y = 2$; (e1–e2) $y = 4$.

- For solution (22) with parameters $b_1 = 100$, $a = d_1 = b_2 = d_2 = 0$, $c = 1$ (see Fig. 4), we can see that the third-order rational solution shows one-soliton structure for $y = -4$, then the one-soliton becomes a two-parallel soliton for $y = -2$ and $y = 0$. For $y > 0$, the two-parallel soliton becomes a three-parallel soliton, first of all the amplitude of the three-parallel soliton becomes larger, then becomes smaller and finally vanishes. When $b_1 = 0$ and for different values of y, the third-order rational solution generates three-soliton interaction phenomena, and the parameter b_1 can change the number and space arrangement of solitons.

4. CONCLUSIONS

In brief, we generalized the perturbation (n; M)-fold DT method from (1+1)-dimensional NPDEs to (2+1)-dimensional NPDEs. In particular, the generalized perturbation (1, $N-1$)-fold DT method with a single spectral parameter allows us to calculate the higher-order rational solutions including RW solutions in terms of determinants in a unified way without iteration procedure. Comparing with the known results for Eq. (1), we gave the detailed method of construction of the generalized perturbation (n; M)-fold DT for Eq. (1) to obtain higher-order rational solutions including RW solutions, in terms of determinants. In particular, the results reported in Ref. [6] are special cases of ours for N=1. We believe that our method is rather general and could be applied to other physically interesting nonlinear wave models as well.

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