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ANALYTIC STUDY ON OPTICAL SOLITONS IN A KERR-LAW MEDIUM WITH AN IMPRINTED PARITY-TIME-SYMMETRIC MIXED LINEAR-NONLINEAR LATTICE

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Abstract. The existence of different types of optical solitons in Kerr-law media with parity-time (*PT*)-symmetric mixed linear-nonlinear optical lattices has been investigated by using the inverse engineering method. Four special *PT*-symmetric potentials are considered, namely i) *PT*-symmetric hyperbolic sine potentials for both linear and nonlinear lattices, ii) *PT*-symmetric hyperbolic tangent potentials for both linear and nonlinear lattices, iii) *PT*-symmetric hyperbolic tangent potential for linear lattice and *PT*-symmetric hyperbolic sine potential for nonlinear lattice, and iv) *PT*-symmetric hyperbolic sine potential for linear lattice and *PT*-symmetric hyperbolic tangent potential for nonlinear lattice. The obtained results show that analytical bright and dark optical solitons can be found for all four physical models.

Key words: optical solitons; mixed linear-nonlinear optical lattices; *PT*-symmetry.

1. INTRODUCTION

It is well known that the concept of parity-time (*PT*) symmetric complex-valued potentials was first introduced by Bender and Boettcher in 1998 [1]. The non-Hermitian Hamiltonian $\widehat{H}=(\widehat{p}^2/2)+V(\widehat{x})$ with a complex-valued external potential $V(\widehat{x})$ is PT symmetric if $V(\widehat{x})=V^*(-\widehat{x})$; this means that the real and imaginary parts of the potential are even and odd functions of the spatial coordinate, respectively [1–8]. It is worth mentioning that the energy levels, i.e. the eigenvalues spectrum of such Hamiltonians, can be entirely real [1, 2]. With the application of the *PT*-symmetry concept in nonlinear optics, the research interests on optical solitons have been shifted from the real domain to the complex one [7–18]. The *PT*-symmetric complex-valued external potentials can be implemented in the optics setting by introducing a complex refractive index profile $n(x)=n_R(x)+\mathrm{i}n_I(x)$ in the coupled waveguide arrays, where $n_R(x)$ and $n_I(x)$ are, respectively, even and odd functions of the transverse spatial coordinate x. In such *PT*-symmetric optical systems the gain and loss are exactly balanced [2–8].

Recently, *PT*-symmetric solitons in optical lattices have attracted a lot of interest [7–30]. The possibility of tuning the physical parameters of such optical lattices allows many fascinating soliton characteristics to be put forward as well as the occurrence of new types of solitons. It should be noted that the previous studies were primarily focused upon the solitons' features in either linear or nonlinear lattices with *PT*-symmetric potentials, but studies of existence, stability, and propagation dynamics of solitons in media modulated by *PT*-symmetric mixed linear-nonlinear optical lattices were rarely reported. In this work, we will perform an analytic study of optical solitons in Kerr-law media with imprinted *PT*-symmetric mixed linear-nonlinear optical lattices.

The propagation equation reads

$$\mathrm{i}q_z+\frac{1}{2}q_{xx}+U_l(x)q+U_{nl}(x)|q|^2q+F(|q|^2)q=0, \tag{1}$$

where $q(x, t)$ is the complex-valued optical field, and x and z represent the transverse and longitudinal directions, respectively. The third and fourth terms in Eq. (1) describe, respectively, the *PT*-symmetric linear and nonlinear lattices, and the last term in Eq. (1) gives the non-Kerr law nonlinearity. Here, $U_l(x)=V_1(x)+\mathrm{i}W_1(x)$ and $U_{nl}(x)=V_2(x)+\mathrm{i}W_2(x)$ denote the *PT*-symmetric mixed linear-nonlinear complex-valued potential modulations, in which $V_l(x)=V_l(-x)$ and $W_l(x)=-W_l(-x)$ for $l = 1, 2$.

2. INVERSE ENGINEERING METHOD

In this Section, we will use a direct method to construct analytical soliton solutions of Eq. (1). The tool of integration is the so-called *inverse engineering method* [4, 7, 21].

In order to derive the exact soliton solutions of Eq. (1), we first make the following transformation

$$q(x,z) = A(x)\exp\{\mathrm{i}[\lambda z + \int f(x)\mathrm{d}x]\}, \tag{2}$$

where $A(x)$ is the real amplitude of the complex-valued optical field $q(x, t)$, while λ represents the propagation constant and $f(x)$ gives the phase gradient (the integral in the exponent is the inhomogeneous phase of the nonlinear optical mode of the system).

Substituting the above ansatz (2) into Eq. (1), one obtains

$$\frac{1}{2}\frac{\mathrm{d}^2A}{\mathrm{d}x^2}-[\frac{1}{2}f^2(x)+\lambda-V_1(x)]A+V_2(x)A^3+F(A^2)A=0, \tag{3}$$

$$\frac{1}{2}\frac{\mathrm{d}f}{\mathrm{d}x}A+f(x)\frac{\mathrm{d}A}{\mathrm{d}x}+W_1(x)A+W_2(x)A^3=0. \tag{4}$$

Integrating Eq. (4) and then taking the integration constant equal to zero yields

$$f(x)=-\frac{2}{A^2}\int[A^2(W_2A^2+W_1)]\mathrm{d}x. \tag{5}$$

Below, we will give the steps to get analytical soliton solutions of Eq. (1).

Step 1. We derive the expression of $f(x)$ by substituting the given functions $W_1(x)$, $W_2(x)$, and $A(x)$ into Eq. (5). At the same time, the analytical form of solution (2) will be determined.

Remark 1. We choose $A(x)=A_0\mathrm{sech}(x)$ for bright solitons, $A(x)=A_0\tanh(x)$ for dark solitons, and $A(x)=A_0\coth(x)$ for singular solitons. Here, A_0 gives the strength of the real soliton amplitude $A(x)$.

Remark 2. The imaginary parts of *PT*-symmetric potentials, namely $W_1(x)$ and $W_2(x)$, should be chosen as odd functions.

Step 2. We obtain the relation between $V_1(x)$ and $V_2(x)$ by substituting $\{W_1(x), W_2(x), A(x), f(x)\}$ into Eq. (3).

Remark 1. We can get $V_1(x)$ if $V_2(x)$ is given, and vice versa.

Remark 2. The real parts $V_l(x)$ $(l=1,\ 2)$ of the corresponding *PT*-symmetric mixed linear-nonlinear potentials should be even functions.

3. EXACT SOLITON SOLUTIONS TO EQ. (1) WITH KERR LAW NONLINEARITY

In this Section, we will present an analytic study on the existence of exact soliton solutions in Kerr-law media modulated by *PT*-symmetric mixed linear-nonlinear optical lattices.

In this case, $F(|q|^2)=|q|^2$. Then, Eq. (3) reduces to

$$\frac{1}{2}\frac{\mathrm{d}^2 A}{\mathrm{d}x^2}-[\frac{1}{2}f^2(x)+\lambda-V_1(x)]A+V_2(x)A^3+A^3=0. \tag{6}$$

In what follows we will consider four generic cases for which the imaginary parts of PT -symmetric potentials have specific analytic forms. As a result, exact bright and dark soliton solitons, along with the corresponding real parts of the *PT*-symmetric potentials, will be reported.

3.1. PT-symmetric hyperbolic sine potentials for both linear and nonlinear lattices

We consider that the imaginary parts of *PT*-symmetric linear and nonlinear refractive index modulations are given by the following hyperbolic sine functions:

$$W_1(x)=\omega_1\sinh(x),\; W_2(x)=\omega_2\sinh(x), \tag{7}$$

Where ω_1 and ω_2 are the corresponding amplitudes.

For bright solitons, we choose

$$A(x)=A_0\mathrm{sech}(x), \tag{8}$$

where A_0 is the corresponding strength of the soliton amplitude $A(x)$.

Then, the bright soliton solution of Eq. (1) will be given by

$$q(x,z)=A_0\mathrm{sech}(x)\exp\{\mathrm{i}[\lambda z+\int f(x)\mathrm{d}x]\}. \tag{9}$$

Substituting Eqs. (7) and (8) into Eq. (5), we obtain

$$\begin{aligned} f(x)=&-\frac{2\omega_2}{3}\left[\tanh(x)\sinh(x)+\sinh^2(x)\cosh(x)-\cosh^3(x)\right]\\ &-\frac{2\omega_1}{A_0^2}\left[\sinh^2(x)\cosh(x)-\cosh^3(x)\right]. \end{aligned} \tag{10}$$

Inserting Eqs. (8) and (10) into Eq. (6) gives

$$V_1(x)=\frac{1}{2}f^2(x)+[1-A_0^2-A_0^2V_2(x)]\mathrm{sech}^2(x)+\lambda-\frac{1}{2}. \tag{11}$$

It should be noted that $V_1(x)$ is also an even function due to both $V_2(x)$ and $f(x)$ are even functions. This means that the exact bright one-soliton solution (9) can exist in the Kerr-law media with *PT*-symmetric hyperbolic sine potentials for both linear and nonlinear lattices.

Based on the same steps above, we can get the analytical dark one-soliton solution of Eq. (1):

$$q(x,z)=A_0\tanh(x)\exp\{\mathrm{i}[\lambda z+\int f(x)\mathrm{d}x]\}, \tag{12}$$

where

$$\begin{aligned} f(x)=&-2\omega_2\left[\tanh(x)\sinh(x)+\frac{4}{3}\mathrm{sech}(x)-\frac{8}{3}\cosh(x)+\frac{8}{3}\frac{\cosh(x)}{\tanh^2(x)}\right]\\ &-\frac{2\omega_1}{A_0^2}\left[-\cosh(x)+2\frac{\cosh(x)}{\tanh^2(x)}\right] \end{aligned} \tag{13}$$

and

$$V_1(x) = \frac{1}{2} f^2(x) - [1 + A_0^2 + A_0^2 V_2(x)]\tanh^2(x) + \lambda + 1\,. \tag{14}$$

The explicit singular one-soliton solution is:

$$q(x,z) = A_0 \coth(x) \exp\{\mathrm{i}[\lambda z + \int f(x)\mathrm{d}x]\}, \tag{15}$$

where

$$\begin{aligned} f(x) = -2\omega_2 \left[\cosh(x) - \frac{3}{2}\mathrm{sech}(x) - 3\tanh^2(x)\mathrm{arctanh}(\mathrm{e}^x)\right] - \\ -2\frac{\omega_1}{A_0^2}\left[\cosh(x) - 2\mathrm{arctanh}(\mathrm{e}^x)\right]\tanh^2(x), \end{aligned} \tag{16}$$

and

$$V_1(x) = \frac{1}{2} f^2(x) - [1 + A_0^2 + A_0^2 V_2(x)]\coth^2(x) + 1 + \lambda\,. \tag{17}$$

It should be noted that dark solitons can exist in this setting but not singular solitons. This is because for a given even function $V_2(x)$, the corresponding function $V_1(x)$ is an even function in the case of dark solitons (see Eqs. (13) and (14)), but $V_1(x)$ is not an even function in the case of singular solitons (see Eqs. (16) and (17)).

3.2. PT-symmetric hyperbolic tangent potentials for both linear and nonlinear lattices

We take the following refractive index modulation profiles:

$$W_1(x) = \omega_1 \tanh(x),\; W_2(x) = \omega_2 \tanh(x)\,. \tag{18}$$

Following the same ideas as the Subsection 3.1, we can obtain the exact bright one-soliton solution

$$q(x,z) = A_0 \mathrm{sech}(x) \exp\{\mathrm{i}[\lambda z + \int f(x)\mathrm{d}x]\}, \tag{19}$$

where

$$f(x) = -\frac{\omega_2}{2}\left[\tanh^2(x) + \sinh^2(x)\right] - \frac{\omega_1}{A_0^2}\sinh^2(x) \tag{20}$$

and

$$V_1(x) = \frac{1}{2} f^2(x) + [1 - A_0^2 - A_0^2 V_2(x)]\mathrm{sech}^2(x) + \lambda - \frac{1}{2}\,. \tag{21}$$

The analytical dark one-soliton solution is:

$$q(x,z) = A_0 \tanh(x) \exp\{\mathrm{i}[\lambda z + \int f(x)\mathrm{d}x]\}, \tag{22}$$

where

$$f(x) = \frac{\omega_2}{2}\tanh^2(x) + \omega_1 + \omega_2 + \omega_1 \ln\left[\tanh^2(x) - 1\right] + \frac{\omega_2}{A_0^2}\ln\left[\tanh^2(x) - 1\right] \tag{23}$$

and

$$V_1(x)=\frac{1}{2}f^2(x)-[1+A_0^2+A_0^2V_2(x)]\tanh^2(x)+\lambda+1\,. \tag{24}$$

The explict singular one-soliton solution is:

$$q(x,z)=A_0\tanh(x)\exp\{\mathrm{i}[\lambda z+\int f(x)\mathrm{d}x]\}, \tag{25}$$

where

$$f(x)=-2\omega_2\ln[\sinh(x)]+\omega_2\coth^2(x)-2\frac{\omega_1}{A_0^2}\ln[\sinh(x)] \tag{26}$$

and

$$V_1(x)=\frac{1}{2}f^2(x)-[1+A_0^2+A_0^2V_2(x)]\coth^2(x)+1+\lambda\,. \tag{27}$$

It must be noted that the exact bright and dark solitons can exist in Kerr-law media with *PT*-symmetric hyperbolic tangent potentials for both linear and nonlinear lattices. But, there is no singular soliton in this physical model, see Eqs. (26) and (27).

3.3. PT-symmetric hyperbolic tangent potential for linear lattice and PT-symmetric hyperbolic sine potential for nonlinear lattice

We consider the following *PT*-symmetric optical potentials:

$$W_1(x)=\omega_1\tanh(x)\,,\ W_2(x)=\omega_2\sinh(x)\,. \tag{28}$$

In this case, we can obtain the analytical bright one-soliton solution

$$q(x,z)=A_0\mathrm{sech}(x)\exp\{\mathrm{i}[\lambda z+\int f(x)\mathrm{d}x]\}, \tag{29}$$

where

$$f(x)=-\frac{2\omega_2}{3}\left[\tanh(x)\sinh(x)+\sinh^2(x)\cosh(x)-\cosh^3(x)\right]-\frac{\omega_1}{A_0^2}\sinh^2(x) \tag{30}$$

and

$$V_1(x)=\frac{1}{2}f^2(x)+[1-A_0^2-A_0^2V_2(x)]\mathrm{sech}^2(x)+\lambda-\frac{1}{2}\,. \tag{31}$$

The exact dark one-soliton solution is:

$$q(x,z)=A_0\tanh(x)\exp\{\mathrm{i}[\lambda z+\int f(x)\mathrm{d}x]\}, \tag{32}$$

where

$$\begin{aligned}f(x)=&-2\omega_2\cosh(x)\tanh^2(x)-\frac{8\omega_2}{3}\mathrm{sech}(x)+\frac{16\omega_2}{3}\cosh(x)-\\&-\frac{16\omega_2}{3}\frac{\cosh(x)}{\tanh^2(x)}-2\frac{\omega_1}{A_0^2}\frac{\ln[\cosh(x)]}{\tanh^2(x)}+\frac{\omega_1}{A_0^2},\end{aligned} \tag{33}$$

and

$$V_1(x)=\frac{1}{2}f^2(x)-[1+A_0^2+A_0^2V_2(x)]\tanh^2(x)+\lambda+1. \tag{34}$$

The explicit singular one-soliton solution is:

$$q(x,z)=A_0\coth(x)\exp\{\mathrm{i}[\lambda z+\int f(x)\mathrm{d}x]\}, \tag{35}$$

where

$$f(x)=-2\omega_2\cosh(x)+3\omega_2\mathrm{sech}(x)+6\omega_2\mathrm{arctanh}(\mathrm{e}^x)\tanh^2(x)-\frac{2\omega_1}{A_0^2}\ln[\sinh(x)]\tanh^2(x) \tag{36}$$

and

$$V_1(x)=\frac{1}{2}f^2(x)-[1+A_0^2+A_0^2V_2(x)]\coth^2(x)+1+\lambda. \tag{37}$$

It is emphasized that analytical bright and dark solitons can exist in Kerr-law media with *PT*-symmetric hyperbolic tangent potential for linear lattice and *PT*-symmetric hyperbolic sine potential for nonlinear lattice. However, exact singular soliton does not exist in this case.

3.4. PT-symmetric hyperbolic sine potential for linear lattice and PT-symmetric hyperbolic tangent potential for nonlinear lattice

We take the modulation profiles as follows:

$$W_1(x)=\omega_1\sinh(x),\ W_2(x)=\omega_2\tanh(x). \tag{38}$$

Then, we can construct the following analytical bright one-soliton solution

$$q(x,z)=A_0\mathrm{sech}(x)\exp\{\mathrm{i}[\lambda z+\int f(x)\mathrm{d}x]\}, \tag{39}$$

where

$$f(x)=-\frac{\omega_2}{2}\left[\tanh^2(x)+\sinh^2(x)\right]-\frac{2\omega_1}{A_0^2}\left[\sinh^2(x)\cosh(x)-\cosh^3(x)\right] \tag{40}$$

and

$$V_1(x)=\frac{1}{2}f^2(x)+[1-A_0^2-A_0^2V_2(x)]\mathrm{sech}^2(x)+\lambda-\frac{1}{2}. \tag{41}$$

The exact dark one-soliton solution is:

$$q(x,z)=A_0\tanh(x)\exp\{\mathrm{i}[\lambda z+\int f(x)\mathrm{d}x]\}, \tag{42}$$

where

$$f(x)=-2\omega_2\ln[\cosh(x)]\coth^2(x)+\omega_2+\frac{\omega_2}{2}\tanh^2(x)+\frac{2\omega_1}{A_0^2}\cosh(x)-\frac{4\omega_1}{A_0^2}\cosh(x)\coth^2(x) \tag{43}$$

and

$$V_1(x) = \frac{1}{2} f^2(x) - [1 + A_0^2 + A_0^2 V_2(x)]\tanh^2(x) + \lambda + 1\,. \tag{44}$$

The explicit singular one-soliton solution is:

$$q(x,z) = A_0 \coth(x) \exp\{\mathrm{i}[\lambda z + \int f(x)\mathrm{d}x]\}, \tag{45}$$

where

$$f(x) = -2\omega_2 \ln[\sinh(x)]\tanh^2(x) + \omega_2 - \frac{2\omega_1}{A_0^2}\cosh(x)\tanh^2(x) + \frac{4\omega_1}{A_0^2}\tanh^2(x)\operatorname{arctanh}(\mathrm{e}^x) \tag{46}$$

and

$$V_1(x) = \frac{1}{2} f^2(x) - [1 + A_0^2 + A_0^2 V_2(x)]\coth^2(x) + 1 + \lambda\,. \tag{47}$$

The above results show that only the exact bright and dark solitons can exist in Kerr-law media with *PT*-symmetric hyperbolic sine potential for linear lattice and *PT*-symmetric hyperbolic tangent potential for nonlinear lattice.

4. CONCLUSION

In this work we presented an analytic study on optical solitons that can form in Kerr-law nonlinear media with imprinted *PT*-symmetric mixed linear-nonlinear lattices. By a direct integration technique, namely the inverse engineering method, exact bright, dark, and singular soliton solutions are reported. The obtained analytical results could further enrich and develop the theory of solitons in diverse physical settings, and could promote possible practical applications of *PT*-symmetric lattice solitons in light-controlled switching devices, light routing, and optical communications systems.

In future studies, we will consider the problem of existence of lattice solitons in non-Kerr law media with *PT*-symmetric mixed linear-nonlinear complex-valued external potentials. The stability and propagation dynamics of such *PT*-symmetric lattice solitons will be reported elsewhere.

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