THE PUBLISHING HOUSE
OF THE ROMANIAN ACADEMY

PROCEEDINGS OF THE ROMANIAN ACADEMY, Series A,
Volume 18, Number 3/2017, pp. 239–247

ELASTIC-PLASTIC ANALYSIS FOR CRACK AT THE HORIZONTAL BRACE OF SEMI-SUBMERSIBLE PLATFORM LOADED BY TENSION

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Abstract. The article present an elastic-plastic theoretical analyse method to calculate the mechanical characteristics of semi-submersible platform's horizontal brace with a circumferential through-crack lies at the boundary between the horizontal brace and column loaded by tension. The solution is clear and closed form solution is found, which is especially suitable solving problems with complicated boundary conditions and could give satisfactory precision in practical engineering application. The variation tendency of the horizontal brace's cracked section which is divided into crack zone, tensile plastic zone, elastic zone and compressive plastic zone as well as the influence of different tension and angles of the crack to them are analysed in this article with a practical engineering application example which could give good suggestion to semi-submersible platform designers and managers.

Key words: elastic-plastic analysis, horizontal brace, circumferential through-crack, semi-submersible platform.

1. INTRODUCTION

Semi-submersible platform, one of the most widely used reusable exploitation platform due to their mobility and ability to operate in deep-water, have gained popularity in recent decades with on-going development of deep-water oil and gas exploitation. The horizontal brace is one of the main structure in semi-submersible platform which serves as the supporting structure especially when the platform encounter horizontal tension load in ocean engineering. Although the safety design standards for this kind of structure of the semi-submersible are quit strict, historical records [1, 2] show that disastrous events especially the crack-induced total losses of the semisubmersibles such as Sedco in 1967, Alexander L. Kielland in 1980, Ocean Ranger in 1982, respectively, cannot be completely avoid.

To ensure the safety and reliability of the semi-submersible platforms which would be subject to very harsh marine environment during their service life, analysis of the mechanical characteristics of a cracked horizontal brace is necessary. The problem of brace structure having a crack has been investigated by a number of authors [3, 4, 5]. In most of the cited articles the crack has been assumed to be surface crack or lies far away from boundary. However, a circumferential through-crack in a horizontal brace usually initial near a joint or similar discontinuity such as in the disastrous event of semi-submersible platform Alexander L. Kielland caused by the failure of a horizontal brace (D-6) having a circumferential through-crack near the joint of the brace and then the remaining braces failed by overloading rapidly. The presence of such cracks at critical locations of horizontal brace can compromise the safety of the whole semi-submersible platform. As for the method of computational analysis, the most popularly applied method to analyse the characteristics of cracked brace structures is the finite element method whose effectiveness has been accepted by the engineering community. Nevertheless, the finite element method will be carried out for every specific and structure system with some local defective elements or nonlinear calculations will be inefficient and spend significant resources which should not be neglected. In this sense, the theoretical analysis is still necessary.

The present article is aimed to investigate an elastic-plastic theoretical analysis method calculating the mechanical characteristics of semi-submersible platform's horizontal brace having a circumferential through-crack which lies at the boundary between the horizontal brace and column by tension.

2. GOVERNING EQUATIONS

A horizontal brace of semi-submersible platform with a circumferential through-crack which lies at the boundary between the horizontal brace and pontoon or column of the semi-submersible platform, which can be assumed to be considerably stiffer than the horizontal brace, subjected to tension load is illustrated in Fig. 1, where a coordinate system and direction of the load are shown.

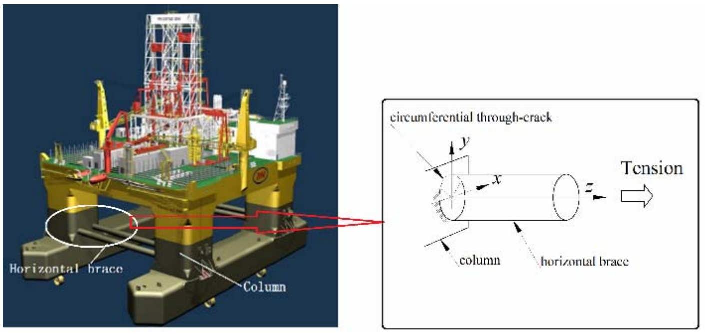

Fig. 1 –The horizontal brace of the semi-submersible platform loaded by tension.

The horizontal braces of the semi-submersible platform are of medium length which means that the radius of the cross section is significantly smaller than the length of the horizontal braces. From the viewpoint of the shell theory, the semi-submersible platform's horizontal braces belong to the mid-long cylindrical shell category. Regarding the length of circumferential through-cracks on the brace structures, historical records [4–8] show that the cracks are of very long. Take the disastrous even of semi-submersible platform Alexander L. Kielland as an example, the circumferential through-crack near the joint of the horizontal brace have propagated to almost 67% of circumferential length of the brace before fracture [9]. Then, the characteristic equation of the horizontal brace could be expressed under a semi-membrane state [3], in which characteristic functions varies slowly in the z direction but not too rapidly in the θ direction, for a complex-valued variable $\Omega = w + i\zeta$ as follow,

$$\frac{\partial^2}{\partial\theta^2}\left(\frac{\partial^2\Omega}{\partial\theta^2}+\Omega\right)-\mathrm{i}\varepsilon^{-2}\frac{\partial^2\Omega}{\partial z^2}=0. \tag{1}$$

Complex characteristic functions Φ and φ both satisfy Eq. (1) here and are related to each other by

$$\frac{\partial^2\Phi}{\partial z^2}=\varepsilon^2\varphi. \tag{2}$$

Here, ε is a small parameter given by $\varepsilon^2 = (h/R)\,[12(1-v^2)]^{-1/2}$ where h and R is the thickness and radius of the horizontal brace of the semi-submersible platform, ν is Poisson's ratio. Then, the expression of dimensionless complex displacements u, v, and w, stress functions χ_z, χ_θ, and ζ, membrane stresses N_z, N_θ, and $N_{z\theta}$ and bending stresses M_z, M_θ, and $M_{z\theta}$ can be given in terms of Φ and φ by

$$\begin{aligned}
&\varepsilon^2 u=\frac{\partial}{\partial z}\left(\frac{\partial^2\Phi}{\partial\theta^2}\right);\ \varepsilon^2 v=-\frac{\partial^3\Phi}{\partial\theta^3};\ w=-\frac{\partial^2\Phi}{\partial\theta^2}+\mathrm{i}\varphi\\
&\varepsilon^2\chi_z=-\mathrm{i}\frac{\partial}{\partial z}\left(\frac{\partial^2\Phi}{\partial\theta^2}\right);\ \varepsilon^2\chi_\theta=-\mathrm{i}\frac{\partial^3\Phi}{\partial\theta^3};\ \zeta=-\mathrm{i}\frac{\partial^2\Phi}{\partial\theta^2}+\varphi\\
&N_z=\frac{\partial^2\varphi}{\partial\theta^2};\ N_\theta=\frac{\partial^2\varphi}{\partial z^2};\ N_{z\theta}=-\frac{\partial}{\partial z}\left(\frac{\partial\varphi}{\partial\theta}\right)\\
&M_z=-\mathrm{i}\left(\frac{\partial^2\varphi}{\partial z^2}+\mu\frac{\partial^2\varphi}{\partial\theta^2}\right);\ M_\theta=-\mathrm{i}\left(\frac{\partial^2\varphi}{\partial\theta^2}+\mu\frac{\partial^2\varphi}{\partial z^2}\right);\ M_{z\theta}=-\mathrm{i}(1-\mu)\frac{\partial}{\partial z}\left(\frac{\partial\varphi}{\partial\theta}\right).
\end{aligned} \tag{3}$$

Moreover, the dimensionless load parameter corresponding to tension load T is defined as $\sigma_T = T/(2\pi Rh\sigma_F)$ and the dimensionless characteristic functions mentioned above are given as follows,

$$\left(\overline{u}\ ,\ \overline{v}\ ,\ \overline{w}\right)=\frac{\sigma_F R}{E}\left(u\ ,\ v\ ,\ \varepsilon^{-2}w\right);\qquad \left(\overline{\chi_z}\ ,\ \overline{\chi_\theta}\ ,\ \overline{\zeta}\right)=\sigma_F R^2 h\varepsilon^2\left(\chi_z\ ,\ \chi_\theta\ ,\ \varepsilon^{-2}\zeta\right)$$
$$\left(\overline{N_z}\ ,\ \overline{N_\theta}\ ,\ \overline{N_{z\theta}}\right)=\sigma_F h\left(N_z\ ,\ N_\theta\ ,\ N_{z\theta}\right);\ \left(\overline{M_z}\ ,\ \overline{M_\theta}\ ,\ \overline{M_{z\theta}}\right)=\sigma_F Rh\varepsilon^2\left(M_z\ ,\ M_\theta\ ,\ M_{z\theta}\right). \tag{4}$$

Here, E is Young's modulus, σ_F is yield stress of the material and $\left(\overline{\ }\right)$ denotes dimensional quantity.

The complete solution Φ_c to the problem presented in this article can be expressed as

$$\Phi_c = \Phi_b + \Phi_s. \tag{5}$$

Here, Φ_b means the elementary solutions and Φ_s can be thought of as the solution incurred by the existence of the crack. The elementary solutions which is composed of simple axial tension solutions, rigid-body motion solutions (for which stresses disappear) and null solutions (for which displacements disappear) can be expressed as

$$\frac{\partial^2\Phi_b}{\partial\theta^2}=\left\{0.5\mathrm{i}\left[1+\mathrm{i}\varepsilon^2\left(2+\mu\right)\right]\theta^2+\left(1+\mathrm{i}\varepsilon^2\mu\right)\left(0.5\varepsilon^2 z^2-\mathrm{i}\right)\right\}\sigma_T+\mathrm{i}a-\varepsilon bz+\mathrm{i}c\cos\theta+\varepsilon dz\cos\theta. \tag{6}$$

Here, a, b, c, d are unknown constants.

As there is a static-geometric analogy between displacements and stress functions, if the boundary conditions on the stress measures can be equivalently expressed in terms of conditions on stress functions, the treatment of the boundary condition will be simplified. Expression for the stress functions in terms of integrals of the effective Kirchhoff edge resultants was derived by Sanders [10]. Expressions for the boundary values of the stress functions in terms of prescribed edge load T_z, T_θ, V and M_n acting on the edge $z = 0$ are as follows

$$\begin{aligned}
\varepsilon^2\chi_z &= \sin\theta\int^{\theta}\left(T_\theta\sin\eta+\varepsilon^2 V\cos\eta\right)\mathrm{d}\eta+\cos\theta\int^{\theta}\left(T_\theta\cos\eta-\varepsilon^2 V\sin\eta\right)\mathrm{d}\eta-\int^{\theta}T_\theta\mathrm{d}\eta\\
\varepsilon^2\chi_\theta &= \sin\theta\int^{\theta}\left(T_z+\varepsilon^2 M_n\right)\sin\eta\mathrm{d}\eta+\cos\theta\int^{\theta}\left(T_z+\varepsilon^2 M_n\right)\cos\eta\mathrm{d}\eta-\int^{\theta}T_z\mathrm{d}\eta\\
\zeta &= \sin\theta\int^{\theta}\left(T_z+\varepsilon^2 M_n\right)\cos\eta\mathrm{d}\eta+\cos\theta\int^{\theta}\left(T_z+\varepsilon^2 M_n\right)\sin\eta\mathrm{d}\eta\\
\frac{\partial\zeta}{\partial z} &= -\sin\theta\int^{\theta}\left(T_\theta\sin\eta+\varepsilon^2 V\cos\eta\right)\mathrm{d}\eta-\cos\theta\int^{\theta}\left(T_\theta-\varepsilon^2\sin\eta\right)\mathrm{d}\eta.
\end{aligned} \tag{7}$$

Here, η is infinitesimal angle of the cracked section. The cracked section is at the boundary between the horizontal brace and pontoon or column of the semi-submersible platform, illustrated as in Fig. 2. From the condition of symmetry, the cross section $z = 0$ is the border.

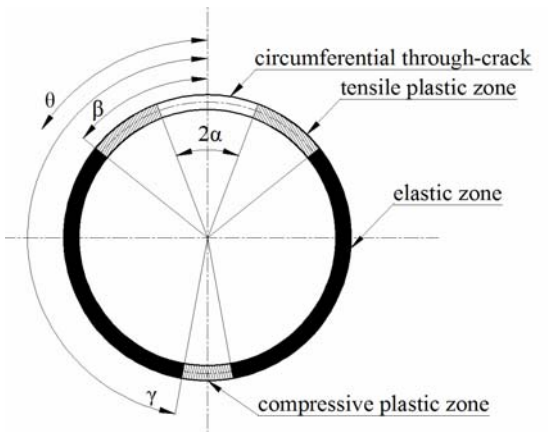

Fig. 2 – Circumferential through-cracked section of the horizontal brace.

On the cracked section, shown as in Fig. 2, the opening angle of the circumferential through-crack is 2α, while β–α and π–γ are the tensile and compressive plastic zones respectively. The boundary conditions can be obtained by means of methods given by Sanders [11]. Those and the displacement conditions are given as follows,

$$\begin{cases} T_z = T_\theta = V = M_n = 0 \quad (0 \le \theta < \alpha) \\ v = 0\ ;\ T_z = 1\ ;\ V = M_n = 0 \quad (\alpha \le \theta < \beta) \\ u = v = w = \dfrac{\partial w}{\partial z} = 0 \quad (\beta \le \theta < \gamma) \\ v = 0\ ;\ T_z = -1\ ;\ V = M_n = 0 \quad (\gamma \le \theta \le \pi). \end{cases} \tag{8}$$

The boundary condition mentioned above can be obtained in terms of the characteristic function Φ and φ following Eq. (3) and Eq. (7) as

$$\begin{cases} \mathcal{R}\left\{ \mathrm{i}\dfrac{\partial}{\partial z}\left(\dfrac{\partial^2 \Phi_C}{\partial \theta^2}\right)\right\} = 0; \quad \mathcal{R}\left\{ \mathrm{i}\dfrac{\partial^2 \Phi_C}{\partial \theta^2}\right\} = 0 \quad (0 \le \theta < \alpha) \\ \mathcal{R}\left\{ \dfrac{\partial^2 \Phi_C}{\partial \theta^2}\right\} = 0; \quad \mathcal{R}\left\{ \mathrm{i}\dfrac{\partial^2 \Phi_C}{\partial \theta^2}\right\} = -\cos(\theta-\alpha) - 0.5(\theta-\alpha)^2 + 1 \quad (\alpha \le \theta < \beta) \\ \mathcal{R}\left\{ \dfrac{\partial}{\partial z}\left(\dfrac{\partial^2 \Phi_C}{\partial \theta^2}\right)\right\} = 0; \quad \mathcal{R}\left\{ \dfrac{\partial^2 \Phi_C}{\partial \theta^2}\right\} = 0 \quad (\beta \le \theta < \gamma) \\ \mathcal{R}\left\{ \dfrac{\partial^2 \Phi_C}{\partial \theta^2}\right\} = 0; \quad \mathcal{R}\left\{ \mathrm{i}\dfrac{\partial^2 \Phi_C}{\partial \theta^2}\right\} = -\pi\sigma_T\theta - \cos\theta - G_1\cos\theta + \\ \qquad + 0.5(\pi-\theta)^2 - 1 + G_2 \quad (\gamma \le \theta \le \pi). \end{cases} \tag{9}$$

The symbol $\mathcal{R}\{\ \}$ denotes the real part of the expression in brackets and the subscript c means that the expression are in terms of the complete characteristic functions.

With Eq. (5), Eq. (6) and Eq. (9), the particular integral Φ_s at the boundary incurred by the existence of the crack turns into

$$\begin{aligned} &\mathcal{R}\left\{ \mathrm{i}\frac{\partial}{\partial z}\left(\frac{\partial^2 \Phi_s}{\partial \theta^2}\right)\right\} = -\varepsilon b_I + \varepsilon d_I\cos\theta; \quad \mathcal{R}\left\{ \mathrm{i}\frac{\partial^2 \Phi_s}{\partial \theta^2}\right\} = a_R + c_R\cos\theta + \left(1-0.5\theta^2\right)\sigma_T \quad (0 \le \theta < \alpha) \\ &\mathcal{R}\left\{ \frac{\partial^2 \Phi_s}{\partial \theta^2}\right\} = a_I + c_I\cos\theta; \quad \mathcal{R}\left\{ \mathrm{i}\frac{\partial^2 \Phi_s}{\partial \theta^2}\right\} = a_R + c_R\cos\theta + \left(1-0.5\theta^2\right)\sigma_T - \\ &\qquad\qquad - \cos(\theta-\alpha) - 0.5(\theta-\alpha)^2 + 1 \quad (\alpha \le \theta < \beta) \\ &\mathcal{R}\left\{ \frac{\partial^2 \Phi_s}{\partial \theta^2}\right\} = a_I + c_I\cos\theta; \quad \mathcal{R}\left\{ \frac{\partial}{\partial z}\left(\frac{\partial^2 \Phi_s}{\partial \theta^2}\right)\right\} = \varepsilon b_R - \varepsilon d_R\cos\theta; \quad (\beta \le \theta < \gamma) \\ &\mathcal{R}\left\{ \frac{\partial^2 \Phi_s}{\partial \theta^2}\right\} = a_I + c_I\cos\theta; \quad \mathcal{R}\left\{ \mathrm{i}\frac{\partial^2 \Phi_s}{\partial \theta^2}\right\} = a_R + G_2 + (c_R - G_1)\cos\theta + \left[\pi(\pi-\theta) + \left(0.5\theta^2 - 1\right)\right]\sigma_T - \\ &\qquad\qquad - \cos\theta + 0.5(\pi-\theta)^2 - 1 \quad (\gamma \le \theta \le \pi). \end{aligned} \tag{10}$$

Here, the subscripts R and I refer to the real and imaginary parts of these constants. Furthermore, any solution to the Eq. (1) satisfy the conditions [10]

$$\int_0^{\pi} \frac{\partial^2 \Phi_s}{\partial \theta^2}\bigg|_{z=0} \mathrm{d}\theta = \int_0^{\pi} \frac{\partial^2 \Phi_s}{\partial \theta^2}\bigg|_{z=0} \cos\theta \mathrm{d}\theta = 0 \tag{11}$$

$$\frac{\partial}{\partial z}\left(\frac{\partial^2 \Phi_s}{\partial \theta^2}\right) = -\mathrm{i}^{\frac{3}{2}}\varepsilon \frac{\partial^2}{\partial \theta^2}\left(\frac{\partial^2 \Phi_s}{\partial \theta^2} + \frac{1}{2}\Phi_s\right). \tag{12}$$

Now put

$$\frac{\partial^2 \Phi_s}{\partial \theta^2}\bigg|_{z=0} = F(\theta). \tag{13}$$

and use Eq. (11) and Eq. (12) to get

$$\int_0^{\pi}\left(F''+F\right)\mathrm{d}\theta = 0;\ \int_0^{\pi} F''\cos\theta \mathrm{d}\theta = 0 \tag{14}$$

$$\frac{\partial}{\partial z}\left(\frac{\partial^2 \Phi_s}{\partial \theta^2}\right)\bigg|_{z=0} = -\mathrm{i}^{\frac{3}{2}}\varepsilon\left(F''+\frac{1}{2}F\right). \tag{15}$$

Additionally, by using Eq. (15) and Eq. (15), Eq. (10) can be expressed in the equivalent form as follows,

$$F_R = \begin{cases} -\left(a_R + 2\sqrt{2}b_I\right) - \left(c_R + 2\sqrt{2}d_I\right)\cos\theta + S\cos\dfrac{\theta}{\sqrt{2}} - \left(0.5\theta^2 - 1\right)\sigma_T & 0 \le \theta < \alpha \\ a_I + c_I\cos\theta \quad \alpha \le \theta \le \pi \end{cases} \tag{16}$$

$$F_I = \begin{cases} -\left(a_R + c_R\cos\theta\right) - \left(0.5\theta^2 - 1\right)\sigma_T & 0 \le \theta < \alpha \\ -\left(a_R + c_R\cos\theta\right) - \left(0.5\theta^2 - 1\right)\sigma_T + \cos\left(\theta - \alpha\right) + 0.5\left(\theta - \alpha\right)^2 - 1 & \alpha \le \theta \le \beta \\ 2\sqrt{2}b_R - a_I + \left(2\sqrt{2}b_R - c_I\right)\cos\theta + 2\sqrt{2}P\cos\dfrac{\theta - \beta}{\sqrt{2}} + 2\sqrt{2}Q\cos\dfrac{\theta - \beta}{\sqrt{2}} & \beta \le \theta < \gamma \\ -\left[a_R + G_2 + \left(c_R - G_1\right)\cos\theta\right] - \left[\pi\left(\pi - \theta\right) + \left(0.5\theta^2 - 1\right)\right]\sigma_T + \cos\theta - 0.5\left(\pi - \theta\right)^2 + 1 & \gamma \le \theta \le \pi \end{cases}. \tag{17}$$

And S, P, Q, G_1 and G_2 are real constants of integration. Adding the undetermined β and γ there exit totally 15 unknown constants in the above equation. The continuity of the displacements and stress functions at the cracked section implies that F_R, F_R', F_R'', at α and F_I, F_I', F_I'', F_I''' at β and γ should be continuous (F_I, F_I', F_I'' at α are continuous from simply checking), from which 11 equations can be got. Four more equations can be found for the real parts and the imaginary parts equal to zero in Eq. (14). There are thus 15 conditions to determine the 15 constants and all the constants can be finally determined by means of algebraic methods.

And results for the 15 constants follow from

$$\begin{aligned} &\pi a_I = \mathrm{S}\left(\frac{\alpha}{2}\cos\frac{\alpha}{\sqrt{2}} - \frac{\sqrt{2}}{2}\sin\frac{\alpha}{\sqrt{2}}\right) - \frac{1}{3}\alpha^3\sigma_T \\ &2\sqrt{2}b_I = -a_R - a_I + 0.5S\cos\frac{\alpha}{\sqrt{2}} - 0.5\alpha^2\sigma_T \\ &2\pi c_I\cos a = S\cos\frac{\alpha}{\sqrt{2}}\left(\alpha - \sin\alpha\cos\alpha\right) - 2\left(2\alpha\cos^2\alpha - \alpha - \sin\alpha\cos\alpha\right)\sigma_T \\ &2\sqrt{2}d_I\cos\alpha = -c_R\cos\alpha - c_I\cos\alpha + 0.5S\cos\frac{\alpha}{\sqrt{2}} + \sigma_T \end{aligned} \tag{18}$$

$$
\begin{aligned}
\pi a_R &= (\pi-\beta)A + \left[2\sin\frac{\gamma-\beta}{\sqrt{2}} + \sqrt{2}(\pi-\gamma)\right]P + 2\left(1-\cos\frac{\gamma-\beta}{\sqrt{2}}\right)Q + \\
&+\left[\frac{1}{6}\left(2\pi^3-2\gamma^3-\beta^3\right)-\pi\gamma(\pi-\gamma)\right]\sigma_T + \frac{1}{6}\left[(\beta-\alpha)^3+2(\pi-\gamma)^3\right] \\
b_R &= \frac{\sqrt{2}}{4}\left(A-a_R+a_I\right) \\
\pi c_R &= -(\beta-\gamma+\sin\beta\cos\beta-\sin\gamma\cos\gamma)C + (\pi-\gamma-\sin\gamma\cos\gamma)G_1 + \\
&+4\left(\sqrt{2}\sin\gamma-\sqrt{2}\sin\beta\cos\frac{\gamma-\beta}{\sqrt{2}}-\cos\beta\sin\frac{\gamma-\beta}{\sqrt{2}}\right)P + 2(\sin\alpha-\sin\beta-\sin\gamma) + \\
&-4\left(\sin\gamma+\sqrt{2}\sin\beta\sin\frac{\gamma-\beta}{\sqrt{2}}-\cos\beta\cos\frac{\gamma-\beta}{\sqrt{2}}\right)Q - 2(\sin\gamma-\sin\beta)\sigma_T + (\beta-\alpha)\cos\alpha + \\
&+\cos\beta\sin(\beta-\alpha)-\sin\gamma\cos\gamma+\pi-\gamma \\
d_R &= \frac{\sqrt{2}}{4}\left(C-c_R+c_I\right)
\end{aligned}
\tag{19}
$$

$$
\begin{aligned}
A &= -\sqrt{2}\left(P\cos\frac{\gamma-\beta}{\sqrt{2}}+Q\sin\frac{\gamma-\beta}{\sqrt{2}}\right)-0.5\beta^2\sigma_T+0.5(\beta-\alpha)^2 \\
C\sin\beta &= -\beta\sigma_T+\sin(\beta-\alpha)+\beta-\alpha \\
G_1\cos\gamma &= C\cos\gamma+\sqrt{2}P-\sigma_T-\cos\gamma-1 \\
G_2 &= -A-\sqrt{2}P-\left[\pi(\pi-\gamma)+0.5\gamma^2\right]\sigma_T-(\pi-\gamma)^2 \\
P\sin\frac{\gamma-\beta}{\sqrt{2}} &= Q\cos\frac{\gamma-\beta}{\sqrt{2}}-\beta\sigma_T+\beta-\alpha \\
Q &= -(\pi-\gamma)\sigma_T-\pi+\gamma \\
S &= \frac{2(\sin\alpha-\alpha\cos\alpha)\sigma_T}{\sqrt{2}\cos\alpha\sin\frac{\alpha}{\sqrt{2}}-\sin\alpha\cos\frac{\alpha}{\sqrt{2}}}.
\end{aligned}
\tag{20}
$$

The following simultaneous transcendental equations are used to determine the extensions of the plastic zones β and γ with given α and σ_T. Here certain numerical techniques are required

$$
\begin{cases} B_1\sigma_T - N_1 = 0 \\ B_2\sigma_T - N_2 = 0 \end{cases}
\tag{21}
$$

where

$$
\begin{aligned}
B_1 &= (\sin\beta+\beta\cos\beta)\sin\frac{\gamma-\beta}{\sqrt{2}}+\sqrt{2}\left(\pi-\gamma+\beta\cos\beta\cos\frac{\gamma-\beta}{\sqrt{2}}\right)\sin\beta \\
B_2 &= \left[(\pi-\gamma)\cos\gamma-\sin\gamma\right]\sin\frac{\gamma-\beta}{\sqrt{2}}-\sqrt{2}\left[(\pi-\gamma)\cos\frac{\gamma-\beta}{\sqrt{2}}+\beta\right]\sin\gamma \\
N_1 &= \left[\sin\beta-\sin\alpha+(\beta-\alpha)\cos\beta\right]\sin\frac{\gamma-\beta}{\sqrt{2}}-\sqrt{2}\left[\pi-\gamma-(\beta-\alpha)\cos\frac{\gamma-\beta}{\sqrt{2}}\right]\sin\beta \\
N_2 &= \left[\sin\gamma-(\pi-\gamma)\cos\gamma\right]\sin\frac{\gamma-\beta}{\sqrt{2}}+\sqrt{2}\left[(\pi-\gamma)\cos\frac{\gamma-\beta}{\sqrt{2}}-\beta+\alpha\right]\sin\gamma
\end{aligned}
$$

From the above results displacements on the crack are obtained and the dimensionless crack tip opening displacement (CTOD) subjected to the tension load is

$$\delta = \frac{\sqrt{2}}{4}\varepsilon^{-1}\left[-A + C\cos\alpha - \left(0.5\alpha^2 + 1\right)\sigma_T\right]. \tag{22}$$

Further, for the present load circumstance the full plastic condition on the cracked section is

$$\sigma_T = \frac{2}{\pi}\left[\arccos\left(0.5\sin\alpha\right) - 0.5\alpha\right] \text{and } \beta = \gamma = 0.5\left(\pi + \alpha + \pi\sigma_T\right). \tag{23}$$

3. SOLUTIONS

The solution process of the analysis method, proposed to calculate the mechanical characteristic of semi-submersible platform's horizontal brace having a circumferential through-crack which lies at the boundary between the horizontal brace and column or pontoon loaded by tension in this article, is simple and the closed form solution is found. At the beginning of the solutions, basic parameters such as parameters of the horizontal brace, angle of the crack and the tension load should be input for the solution for plastic and CTOD.

Numerical method should be used to calculate of the plastic zones of the horizontal brace because it's hard to find explicit solution from Eq. (21). To start the calculation, assuming an initial β_0 and initial γ_0 is necessary. And then, define two more parameters F and λ according to Eq. (21) following from

$$F = \left[B_1(\beta_0, \gamma_0)\sigma_T - N_1(\beta_0, \gamma_0)\right]^2 + \left[B_2(\beta_0, \gamma_0)\sigma_T - N_2(\beta_0, \gamma_0)\right]^2$$

$$\lambda = \frac{F\ (\beta_0, \gamma_0)}{\left[\dfrac{F\ (\beta_0 + 0.01,\ \gamma_0) - F\ (\beta_0, \gamma_0)}{0.01\beta_0}\right]^2 + \left[\dfrac{F\ (\beta_0,\ \gamma_0 + 0.01) - F\ (\beta_0, \gamma_0)}{0.01\gamma_0}\right]^2}.$$

To guarantee to precision of the solution, verification is needed to see if $|F| < \varsigma$ is satisfied, where ς is a small specified quantity. If $|F| < \varsigma$ does not appear, the assumed initial β_0 and initial γ_0 should be modified as

$$\beta = \beta_0 - \lambda\frac{F\ (\beta_0 + 0.01,\ \gamma_0) - F\ (\beta_0, \gamma_0)}{0.01\beta_0} \qquad \gamma = \gamma_0 - \lambda\frac{F\ (\beta_0,\ \gamma_0 + 0.01) - F\ (\beta_0, \gamma_0)}{0.01\gamma_0}.$$

After $|F| < \varsigma$ is satisfied, the solution for plastic zones of the cracked horizontal brace is done. The output of β and γ can then input into Eq. (18), Eq. (19) and Eq. (20) to determine the 15 unknown parameters a_I, b_I, c_I, d_I, a_R, b_R, c_R, d_R, A, C, G_1, G_2, P, Q, S and the crack tip opening displacement(CTOD) can be obtained from Eq. (22) too. Till now, the solutions of the mechanical characteristics of semi-submersible platform's horizontal brace having a circumferential through-crack which lies at the boundary are done.

4. EXAMPLES

To illustrate the previous model with some examples, a typical semi-submersible platform with circumferential through-crack at the boundary between the horizontal brace and column is selected with parameters of the horizontal brace including the radius R = 0.915 m, thickness h = 0.0308 m, Poisson's ratio ν = 0.3, Young's modulus E = 210 GPa and yield stress of the material σ_F = 400 MPa. According to the transverse force transfer function and long term response of transverse force of longitudinal section in center plane of the chosen semi-submersible platform, shown as in Fig. 3, the largest tension load which the horizontal brace could be loaded is 65×10^6N. Different tension loads and angles of the circumferential through-crack which lies at the boundary between horizontal brace and column have been chosen to analyse the characteristic functions of the cracked section.

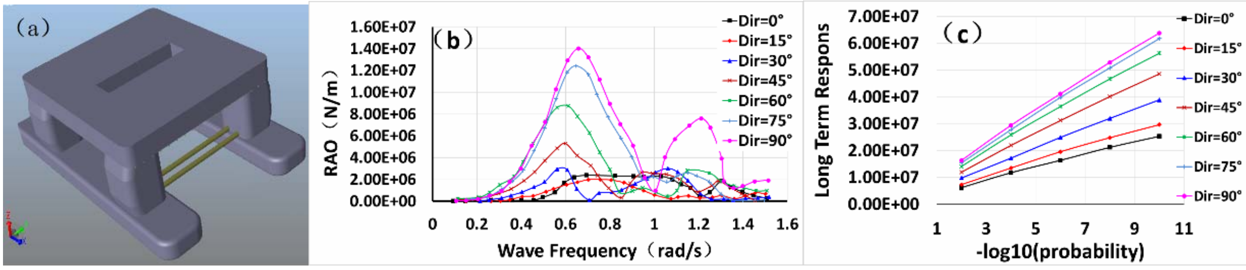

Fig. 3 – a) Semi-submersible platform; b) transverse force transfer function of longitudinal section in centre plane; c) long term response of transverse force of longitudinal section in centre plane.

The tensile plastic zone (β - α) on the circumferential through-cracked section of semi-submersible platform's horizontal brace vary with tension loads and angles of the crack (Fig. 4a). The tensile plastic zone of the cracked section increase sharply after a smoothly increase with the tension load. Additionally, the cracked section of the horizontal brace with greater circumferential through-crack has a larger tensile plastic zone while loaded a same tension and the compressive plastic zone appears (parameter $\gamma < \pi$) earlier. Same as the variation tendency of the tensile plastic zone, the crack tip opening displacement (CTOD) on the cracked section increase smoothly while the horizontal brace of the semi-submersible platform loaded by small tension loads and the crack tip opening displacement (CTOD) increase dramatically when loaded by larger tensions (Fig. 4b. Also, the cracked section has a larger CTOD while the angle of the crack lies at the boundary between the horizontal brace and column of the semi-submersible platform is greater.

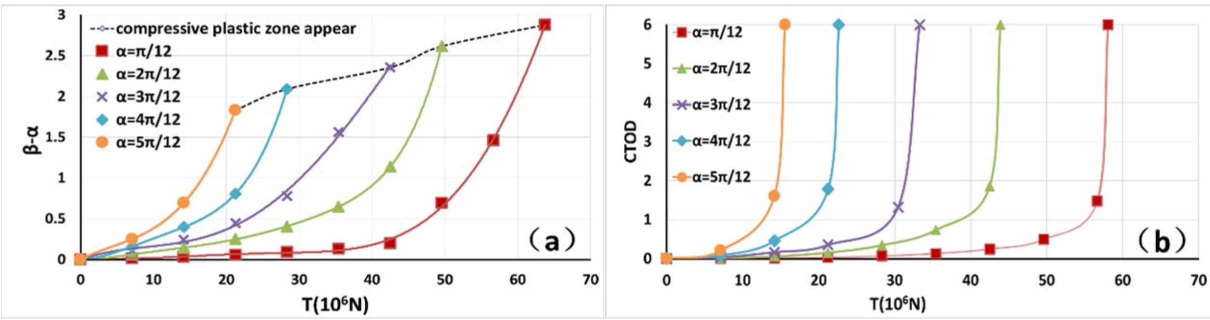

Fig. 4 – a) The tensile plastic zone on the cracked section under different tension loads with different half angles of the crack; b) the crack tip opening displacement (CTOD) on the cracked section under different tension loads with different half angles of the crack.

The elastic zone of the horizontal brace's cracked section, whose variation tendency is contrary to the variation tendency of plastic zone (shown as in Fig. 5a, decrease with the tension load until the cracked section is in the full plastic condition while contains cracked zone, tensile plastic zone and compressive plastic zone. Shown as in Fig. 5b, the tensile plastic zone decrease with the angle of the cracked section during the compressive plastic zone increase with the angle of the cracked section and the tensile plastic zone is always larger than the compressive plastic zone while the cracked section is in the full plastic condition. The load condition vary obviously with the angles of the cracked section, shown as in Fig. 5c, the tension load decrease with the angle of the circumferential through crack is almost linear.

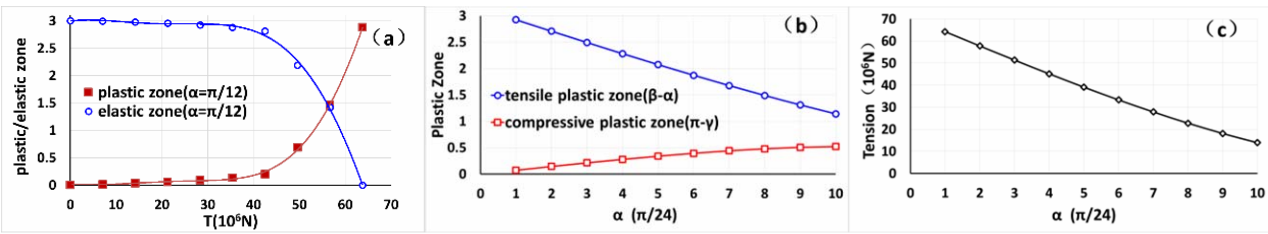

Fig. 5 – a) Tendencies of elastic zone and plastic zone of the cracked section; b) the plastic zones of the cracked section in different angles of the crack; c) the variation tendency of tension loads for different angles of the crack while cracked section is in the full plastic condition.

5. CONCLUSIONS

A simple elastic-plastic theoretical analysis method calculating the mechanical characteristics of semi-submersible platform's horizontal brace having a circumferential through-crack which lies at the boundary between the horizontal brace and the column loaded by tension is proposed in this article. The solution process is clear and closed form solution is found, which is especially suitable to solve problems with complicated boundary conditions. In practical Engineering application, it could give satisfactory precision. The presented model is also applied to analyse the cracked section, which is divided into crack zone, tensile plastic zone, elastic zone and compressive plastic zone in this article, as well as the influence of different tension loads and angles of the crack to them with a practical engineering application example in this paper and some variation tendency has been got:

(1) The tensile plastic zone and the crack tip opening displacement on the cracked section of the horizontal brace increase dramatically after a smoothly increase with the tension load and the horizontal brace with greater circumferential through-crack has a larger tensile plastic zone and crack tip opening displacement. Additionally, the cracked section of the horizontal brace with greater circumferential through-crack has a larger tensile plastic zone while loaded a same tension and the compressive plastic zone appears earlier.

(2) The elastic zone of the horizontal brace's cracked section, whose variation tendency is contrary to the variation tendency of plastic zone, decrease with the tension load until the cracked section is in the full plastic condition, in which the tensile plastic zone decrease with the angle of the cracked section during the compressive plastic zone increase with the angle of the cracked section and the tensile plastic zone is always larger than the compressive plastic zone. The load condition vary obviously with the angles of the cracked section, the tension load decrease with the angle of the circumferential through crack is almost linear.

ACKNOWLEDGMENTS

The authors would like to acknowledge the support of Grant No. 2012AA09A203 from the National High Technology Research and Development Program of China (863 Program) and Grant No. 2011STS04 from the Key Laboratory of Oil and Gas Equipment from Ministry of Education, China.

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Received January 13, 2015