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ON THE MAGNETIC ENERGY AND REACTIVE POWER

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The magnetic energy is expressed as a volume integral of the magnetic energy density. The magnetic energy is found by integrating this expression in the magnetic field domain. Using the magnetic vector potential, in stationary state this expression may be transformed in the sum of a divergence term and a scalar product. The volume integral of the divergence term is transformed into a surface integral, which may vanish in some conditions. This approach is very inciting in applications for the reactive power calculation of field configurations in harmonic conditions.

Key words: Stationary condition, Magnetic energy, Reactive power, Boundary conditions.

1. INTRODUCTION

In this paper we limit us only to linear magnetic media that is without permanent magnetization. For these media the magnetic energy is a well-defined quantity.

Magnetic energy and reactive power are related quantities in harmonic, quasi-stationary conditions. In order to calculate the reactive power from field problems first the magnetic energy must be determined. This is calculated as the volume integral of magnetic energy density.

When the magnetic field occurs in wide or infinite spaces the calculation of the magnetic energy, starting from its volume density, becomes difficult and time consuming. But when the magnetic field is created by conduction currents, using some properties of the magnetic field, the magnetic induction being expressed by the curl of the magnetic vector potential and the curl of magnetic field strength by the conduction currents, and using some vectorial transformations, the magnetic energy may be presented as a volume integral extended only to the domains with current density and a surface integral [1 – 3], which can vanish in some boundary conditions. Then the calculation of the magnetic energy or of the reactive power becomes easy.

This paper attempts to establish the conditions in which the magnetic energy, respectively the reactive power, may be calculated by an integral extended only to the regions with density of conduction currents.

2. EXPRESSIONS OF THE MAGNETIC ENERGY

The differential of magnetic field volume density of energy may be expressed in terms of the local magnetic field quantities [1 – 3]

$$\mathrm{d}\,w_m = \boldsymbol{H}\,\mathrm{d}\,\boldsymbol{B}. \tag{1}$$

By integrating in the space of field quantities, for magnetic characteristics that pass through the origin ($\boldsymbol{B}$ and $\boldsymbol{H}$ vanish simultaneously) the magnetic energy volume density is found

$$w_m = \int_0^{\boldsymbol{B}} \boldsymbol{h}\,\mathrm{d}\boldsymbol{b}, \tag{2}$$

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where $\boldsymbol{b}$ and $\boldsymbol{h}$ are the actual quantities. In linear magnetic bodies, with the constitutive relation $\boldsymbol{b} = \mu\, \boldsymbol{h}$, the volume density of the magnetic energy becomes

$$w_m = \tfrac{1}{2}\boldsymbol{BH}. \tag{3}$$

The magnetic energy of a domain D_Σ is expressed by the volume integral

$$W_m = \int_{D_\Sigma} w_m \,\mathrm{d}\,v. \tag{4}$$

Using the magnetic vector potential $\boldsymbol{A}$ and the Ampere's law in stationary or quasi-stationary conditions rot $\boldsymbol{H} = \boldsymbol{J}$, where $\boldsymbol{J}$ is the field of conduction current density, the magnetic energy volume density may be transformed successively

$$2w_m = \boldsymbol{BH} = \boldsymbol{H}\,\mathrm{rot}\,\boldsymbol{A} = \mathrm{div}(\boldsymbol{A}\times\boldsymbol{H}) + \boldsymbol{A}\,\mathrm{rot}\,\boldsymbol{H} = \mathrm{div}(\boldsymbol{A}\times\boldsymbol{H}) + \boldsymbol{AJ}. \tag{5}$$

Using the Gauss-Ostrogradski formula for the first term, the magnetic energy is expressed as the sum of two integrals

$$W_m = \tfrac{1}{2}\int_\Sigma (\boldsymbol{A}\times\boldsymbol{H})\boldsymbol{n}_\Sigma \,\mathrm{d}\,\Sigma + \tfrac{1}{2}\int_{D_\Sigma} \boldsymbol{AJ}\,\mathrm{d}\,v. \tag{6}$$

In free space the surface integral vanish on a surface at infinity, because of closing conditions of the magnetic field [1], p. 124, [2], p. 492-493, [3], p. 335, and the magnetic energy may be calculated only with the second integral, which spans the domains with current density.

Note. The term $\boldsymbol{AJ}/2$ may replace the magnetic energy density in the specified conditions, but it is not describing the actual spatial repartition of the magnetic energy. Its value depends also on the used vector potential $\boldsymbol{A}$, which – for a given magnetic field – is not unique (chosen of div$\boldsymbol{A}$ and of some boundary conditions). Therefore this term may be referred only as "pseudoenergy density".

The surface integral in (6) may vanish in some conditions on certain surfaces, that we will design as “separating surface”. Such conditions occur at least in the following cases.

1) In 2D problems the magnetic vector potential has only components normal to the transversal plane (in which is described the field), that does not depend on the coordinate normal to this plane. Considering a bounding surface Σ consisting from two transversal planes and an axial band at infinity closing between the two planes, the surface integral vanish, because the surface integrals are compensating each other on the transversal planes and are null on the axial band at infinity.

2) In 3D problems, if the sources which create the currents are contained in a bounded part of the volume where is studied the field, the closing conditions for the vector potential $\boldsymbol{A}$ and for the magnetic field strength $\boldsymbol{H}$**,** on a bounding surface Σ drawn at infinity, ensure the annulment of the surface integral.

3) In 3D problems, when the sources which create the currents are placed in another volume, not considered in the solving of the field problem, the transmission lines (from sources to the studied domain) must satisfy some conditions in order to ensure a "separation" surface $S_s \subset \Sigma$, on which the surface integral vanish. The necessary conditions result from the form of the integrand in (6): $(\boldsymbol{A}\times\boldsymbol{H})\boldsymbol{n}_\Sigma$ must be null that is $\boldsymbol{A}\times\boldsymbol{H}$ does not have components normal to the surface S_s.

These conditions are accomplished if the transmission lines are of coaxial cable type, the currents return on their shields and the separating surface cuts normally the cables. Truly, in each coaxial cable the vector potential is axial (to the cable) and the magnetic field strength is tangential, therefore their vector product is also tangential and the normal component to the separating surface is null. Out of the cables the magnetic field strength is null, what is annulling the integral on the remaining surface between the coaxial cables.

The form (6) of the magnetic energy integral is very useful in applications, even if it must be calculated the two integrals, because the separating surface may be chosen in a manner that simplifies the calculation of the non-vanishing part of the surface integral.

3. MAGNETIC ENERGY AND REACTIVE POWER IN HARMONIC CONDITIONS

In quasi-stationary inductive conditions, of immobile media without hysteresis, the electric powers in a domain D_Σ satisfy the instantaneous energy balance relation

$$\frac{\mathrm{d}W}{\mathrm{d}t} = P_j + \frac{\mathrm{d}W_m}{\mathrm{d}t} \tag{7}$$

where W is the instantaneous externally feeding power, P_J is the instantaneous power dissipated in the domain by Joule effect

$$P_J = \int_{D_\Sigma} \rho\, J^2\, \mathrm{d}v, \tag{8}$$

and W_m is the instantaneous magnetic energy in the domain, as defined previously.

In harmonic conditions, where the field quantities are harmonic functions of time, usually the quantities are expressed by their complex amplitudes. For a harmonic quantity v the correspondence being

$$v = V_m \sin(\omega t + \alpha) \Leftrightarrow \underline{V} = V_m \exp(\mathrm{j}\alpha). \tag{9}$$

where $\omega = 2\pi/T$ is the angular frequency of the harmonic quantity of period T.

The energy balance is expressed in terms of the complex power $\underline{S}$, whose real part is the active power P and the imaginary part is the reactive power Q, as used in electrical circuits

$$\underline{S} = P + \mathrm{j}Q. \tag{10}$$

The active power, defined as the mean value of the instantaneous power on a period T, is identified without difficulties with the mean power dissipated by the Joule effect

$$P = P_{J\,med} = \frac{1}{T}\int_0^T P_J\, \mathrm{d}t = \frac{1}{T}\int_0^T \left(\int_{D_\Sigma} \rho\, J^2\, \mathrm{d}v \right) \mathrm{d}t. \tag{11}$$

Inverting the order of the two integrals, the active power is expressed with the square of the r.m.s. of the current density J_{rms}

$$J_{rms} = \frac{1}{T}\int_0^T J^2\, \mathrm{d}t, \quad P = \int_{D_\Sigma} \rho\, J_{rms}^2\, \mathrm{d}t. \tag{12}$$

The reactive power is calculated with the mean value on a period T of the magnetic energy

$$W_{m\,med} = \frac{1}{T}\int_0^T W_m\, \mathrm{d}t, \tag{13}$$

multiplied by 2ω,

$$Q = 2\omega W_{m\,med} \tag{14}$$

In harmonic conditions of the field quantities the volume density of the magnetic energy (in linear media, with scalar permeability μ) is an oscillating quantity with the period $T/2$

$$w_m = \tfrac{1}{2} B_m \sin(\omega t + \alpha) H_m \sin(\omega t + \alpha) = \tfrac{1}{4} B_m H_m [1 - \cos(2\omega t + 2\alpha)], \tag{15}$$

and the mean value

$$w_{m\,med} = \tfrac{1}{4} B_m H_m = \tfrac{1}{2} w_{m\,\max} \tag{16}$$

Using the corresponding complex quantities $\underline{\boldsymbol{B}}$ and $\underline{\boldsymbol{H}}$, the volume density is expressed as

$$w_m = \tfrac{1}{4} \underline{\boldsymbol{B}}\underline{\boldsymbol{H}}^*, \tag{17}$$

where the (*) signifies the complex conjugate of the quantity.

The volume density of the magnetic energy may be transformed in the same manner, using the complex magnetic vector potential $\underline{\boldsymbol{A}}$ and the Ampere's law, to obtain finally

$$W_{m\,med} = \tfrac{1}{4}\int_{\Sigma}\left(\underline{\boldsymbol{A}}\times\underline{\boldsymbol{H}}\,^*\right)\boldsymbol{n}_{\Sigma}\,\mathrm{d}\Sigma + \tfrac{1}{4}\int_{D_{\Sigma}}\underline{\boldsymbol{A}}\,\underline{\boldsymbol{J}}\,^*\,\mathrm{d}v. \tag{18}$$

The first integral vanishes in the previously specified conditions and then the reactive power may be calculated with only the last term, what dramatically reduces the calculating time.

An illustrative example of the advantages of the established formula is in the case of the calculation of line parameters of a bus bar system, from the active and reactive power losses on unit length. Although the magnetic field is plane-parallel, it is extending to infinity and the use of formula (4) is time consuming. The new formula (18) limits the integrals only to the bus-bars sections.

Another illustrative example, in 3D field, is the case of induction oven without magnetic circuit or shield. In this configuration the magnetic field is extending to infinity and the calculation with the formula (4) is very difficult. By using the new formula (18), the calculations must be extended only to the domains with current density, that is to the space of the inductor winding and the space of the crucible containing the molten metal.

Frequently the eddy current problems are formulated with the integral equation of the magnetic vector potential, and then the use of the formula (18) is the shortest way to obtain the reactive power.

Also when the "separating" conditions are not fulfilled, the use of formula (18) may shorten the calculations, because a suitable choosing of the limit surface S_s may simplify the calculation of the first integral, what leads to more simple calculations than these for the volume integral (4). Therefore the established relation (18) has an important practical interest in applications.

4. **CONCLUSIONS**

The magnetic energy and the reactive power may be calculated from the field quantities, using the integral expressions (6), respectively (18), consisting from a surface integral and a volume integral restricted to the volume with conduction currents. In some "separating" conditions the surface integral may vanish and the calculation is restricted to the second term.

The "separating" conditions are fulfilled by all the 2D problems.

For 3D problems these conditions are satisfied when both the sources and the studied field domain are contained in the same closed surface which tends to infinity, or when a separating surface can be defined on which the integrand of the surface integral vanish. A possibility for such a separating surface is offered by leading the currents from the sources to field domain in coaxial cables with currents returning on the surrounding cable screens.

Even when the "separating" conditions are not fulfilled, choosing a suitable separating surface, on which the surface integral is easy to calculate, the calculation with the two term expressions (6) or (18) is more advantageous, than with the volume integral (4) extended to the whole field domain.

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