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CAPACITANCE MATRIX CALCULATION FOR INTEGRATED CIRCUIT INTERCONNECTS USING GREEN'S FUNCTION APPROACH

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In very high speed IC circuits, the extraction of the self and coupling capacitances of multilayer and multiconductor interconnects is usually required. A new approach for the accurate calculation of capacitance matrix of multilayered IC interconnects with very thick conductors is presented, based on a perturbation procedure to calculate the multilayer Green's function for planar interfaces. The validity of the technique is verified by comparing its results with accelerating capacitance calculations by Pade approximation technique and moment method for total charges in the structure, respectively.

Key words-Multilayer IC interconnect, Green's function method, Line capacitance

1. INTRODUCTION

Calculation of the capacitance matrix in multilayer IC interconnects is a well-known problem that can be solved by many analytical and numerical techniques [1-8]. Often these procedures were based on the integral equation formulation, differential equation formulation, or have been the results of extensive numerical simulations using adequate empirical corrections.

This letter proposes a new and more general formulation for computation of capacitance matrix of the most common 2-D interconnect structures using perturbation calculation of the Green's function of the structures.

2. ANALYSIS

In the formulation, 2-D L-layered interconnect structures with planar boundaries are considered. Each layer is linear, homogeneous, and isotropic, and has permittivity $\varepsilon^{(l)}$ and conductivity $\sigma^{(l)}$, where l = 1,...,L. For lossy medium the complex permittivity is $\underline{\varepsilon}^{(l)} = \varepsilon^{(l)} - j\sigma/\omega$. The point charge source is placed at $\mathbf{r}_s = (x_s, y_s, z_s)$, in the multilayered dielectric media (see Figure 1).

The Green's function of the problem $G(\mathbf{r}_s;\mathbf{r}_f)$ at the any point $\mathbf{r}_f = (x_f,y_f,z_f)$, satisfies the differential equation:

$$\nabla[e(\mathbf{r}_f)\nabla G(\mathbf{r}_f;\mathbf{r}_s)] = -d(\mathbf{r}_f - \mathbf{r}_s) \tag{1}$$

with standard boundary conditions on the interfaces, where $\varepsilon(\mathbf{r}_f)$ represents the dielectric permittivity $\varepsilon^{(m)}$ in the half-space with charge source and that $\varepsilon^{(l)}$ in the layer l of the multilayer media, respectively.

If we suppose that the whole space in the structure is homogeneous (no dielectric inhomogenities), we have: $\nabla^2 G_0(\mathbf{r}_f,\mathbf{r}_s) = -\delta(\mathbf{r}_f-\mathbf{r}_s)/\varepsilon_0$, where $G_0(\mathbf{r}_f,\mathbf{r}_s) = [\varepsilon_0|\,\mathbf{r}_f - \mathbf{r}_s|\,]^{-1}$. The dielectric function $\varepsilon(\mathbf{r}_f)$ = can be written as:

$$e(\mathbf{r}_f) = e^{(m)}\left[1 - \sum_{l=1}^{L} h^{(l)} h_l(\mathbf{r}_f)\right], \tag{2}$$

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where $\eta^{(l)} = 1 - \varepsilon^{(l)}/\varepsilon^{(m)}$ and $h_l(\mathbf{r}_f)$ equals unity when $\mathbf{r}_f$ is in layered medium (l) and zero otherwise. Let V_l and S_l be the volume and surface of layer l, respectively. For $\mathbf{r}_f \in V_l$, the Green's function is given by [9]:

$$(1-\eta^{(l)})G(\mathbf{r}_f;\mathbf{r}_s) = \frac{G_0(\mathbf{r}_f;\mathbf{r}_s)}{\varepsilon^{(m)}} + \frac{1}{4\pi}\sum_{j=1}^{L}\eta^{(j)}\oint_{S_j}\left[\mathbf{n}_j'\nabla' G_0(\mathbf{r}_f;\mathbf{r}')\right]G(\mathbf{r}';\mathbf{r}_s)\cdot \mathrm{d}S', \tag{3}$$

where $\mathbf{n}'_j$ is a unit normal to S_j while for $\mathbf{r}_f \in V_m$, however, we get:

$$G(\mathbf{r}_f;\mathbf{r}_s) = \frac{G_0(\mathbf{r}_f;\mathbf{r}_s)}{\varepsilon^{(m)}} + \frac{1}{4\pi}\sum_{j=1}^{L}\eta^{(j)}\oint_{S_j}\left[\mathbf{n}_j'\nabla' G_0(\mathbf{r}_f;\mathbf{r}')\right]G(\mathbf{r}';\mathbf{r}_s)\cdot \mathrm{d}S', \tag{4}$$

where V_m means the volume of the upper half-space of the structures. The image potential is given by:

$$V_{im}(\mathbf{r}_s) = \frac{1}{8\pi}\sum_{j=1}^{L}\eta^{(j)}\oint_{S_j}\left[\mathbf{n}_j'\nabla' G_0(\mathbf{r}_f;\mathbf{r}')\right]G(\mathbf{r}';\mathbf{r}_s)\cdot \mathrm{d}S'. \tag{5}$$

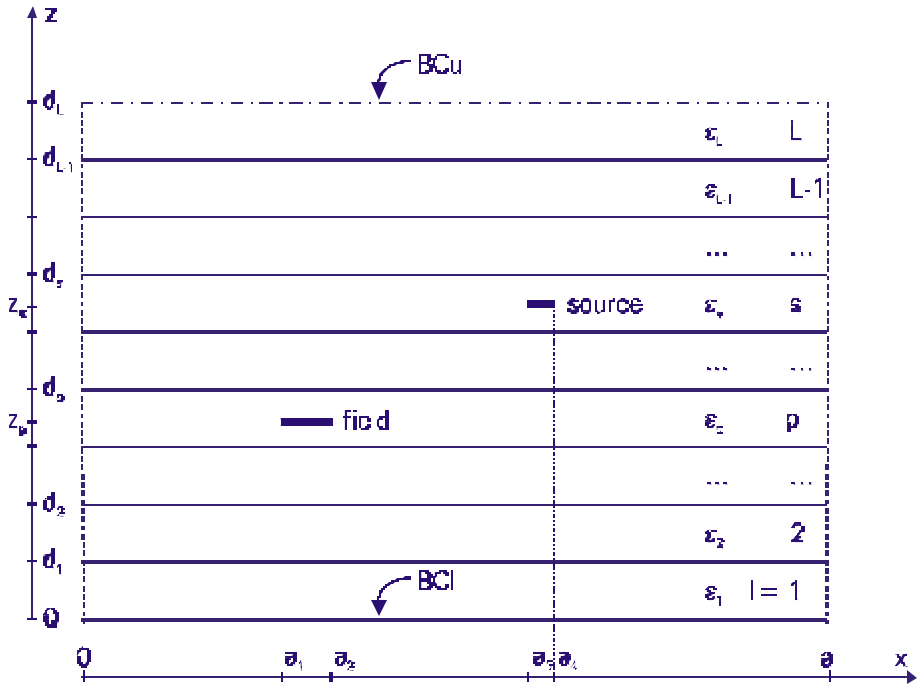

Fig. 1. Geometry of a layered structure with multiconductor interconnection lines.

The above described procedure provides a general method for determining the multilayered Green's function of the IC interconnects.

In order to determine the per unit length capacitance of the multiconductor multi-layer interconnect, the potential on any conductor in the structure resulting from the charge distribution $\rho(\mathbf{r}_s)$ is computed using a convolution of the form:

$$\varphi(\mathbf{r}_f) = \sum_{j=1}^{N}\int_{V_s}\rho(\mathbf{r}_s)\cdot G(\mathbf{r}_f;\mathbf{r}_s)\cdot \mathrm{d}V_s \tag{6}$$

where $G(\mathbf{r}_f;\mathbf{r}_s)$ is the Green's function determined using new defined approach in this paper.

The Galerkin's technique is used to set up the final matrix equation. The unknown charge distribution are expanded in terms of known basis function, namely:

$$r(z) = \sum_{m=1}^{K_b} a_m\, r_m(z) \tag{7}$$

where a_m's are unknown constants to be solved, and K_b is the number of basis functions used to model the charge density on each conductor.

3. NUMERICAL RESULTS

In our approach suggested in this paper, regions of different permittivities are modeled by modifying the homogeneous Green function (charge point) to incorporate the effects of dielectric interfaces (perturbation formalism). The advantages of using such an approach are that only conductor surfaces need to be discretized and that there is no need to truncate wide ground conductors and dielectric layers. Therefore the system matrix is reduced in size. The Green's function may be easily found for planar layered interconnect configurations and other VLSI and MCMs structures in which all dielectric interfaces and ground planes are planar, parallel and infinite in extent.

Example 1

Let us consider the system of three conductors embedded in three-layered dielectric region with structure as shown in Figure 2, where the conductors are numbered from left to right as 1, 2, and 3, respectively (all dimensions are in μm). Numerical values for the capacitance matrix elements, generated by the proposed approach, by Pade approximation formalism [1] and total charge boundary element procedures [2], respectively, are given in Table 1. Note that the discrepancies between the values generated by our approach and one by [2] are practically smaller than 0.2%.

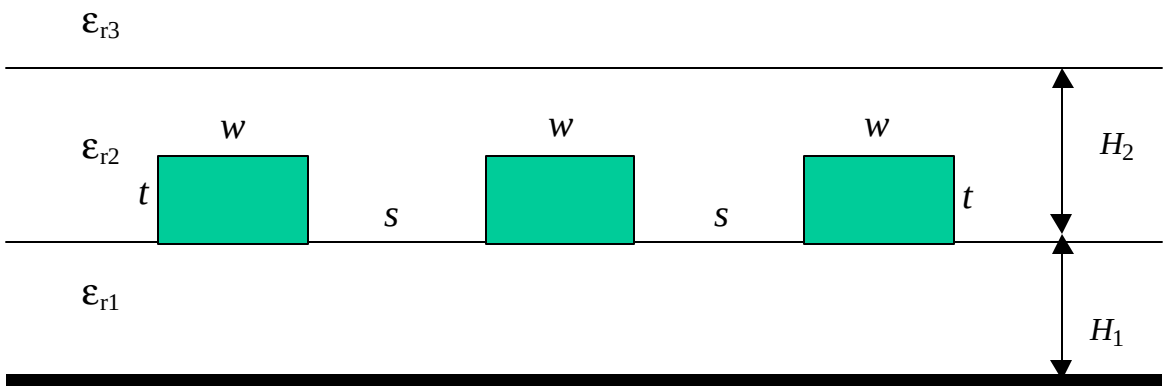

Fig. 2. Geometry of the structure from example 1 (w/H_1= w/t= 8/6, s/H_2 = 10/11, ε_{r1}= 9.5 and ε_{r2}= 4.0 and ε_{r3}= 1.0).

Table 1. Capacitance matrix of the structure of Fig. 2.

Capacitance (pF/m)	[1]	[2]	This letter
C_{11}	266.459	269.520	268.930
$C_{12}=C_{21}$	-34.812	-34.868	-34.770
$C_{13}=C_{31}$	-1.302	-1.256	-1.259
C_{22}	274.743	277.750	276.998
$C_{23}=C_{32}$	-34.812	-34.868	-34.770
C_{33}	266.459	269.520	268.930

Example 2

Numerical values of C_{11}, C_{12}, C_{21}, and C_{22}, generated by the proposed approach, for a pair of coupled interconnection lines (see Fig. 3), are shown in Table 2. Table 2 compares our results with those of [1] and [5,8], respectively. It is clear from this table that our approach can provide very high accuracy even if we have very thick interconnect conductors.

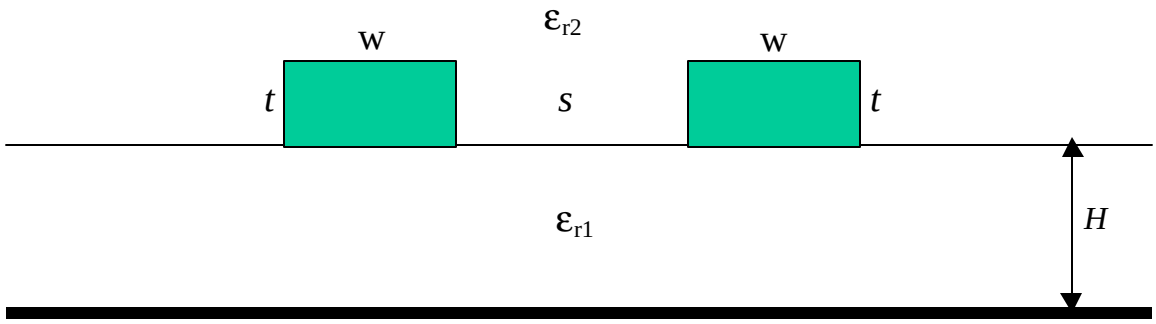

Fig. 3. Coupled parallel interconnection lines (w/H = w/t = 3, s = 2, ε_{r1}= 2.0 and ε_{r2}= 1.0).

Table 2. Comparison of results for example 2, Fig. 3.

Capacitance (F/m)	[1]	[5,8]	This letter
C_{11}	0.9264×10^{-10}	0.9165×10^{-10}	0.9189×10^{-10}
C_{12}	-0.8305×10^{-11}	-0.8220×10^{-11}	-0.8245×10^{-11}
C_{21}	-0.8305×10^{-11}	-0.8220×10^{-11}	-0.8245×10^{-11}
C_{22}	0.9264×10^{-10}	0.9165×10^{-10}	0.9189×10^{-10}

4. CONCLUSION

A new perturbation procedure for the analysis of multilayer IC interconnects is proposed in this letter and applied to for the case of very thick conductors. The derived algorithm is tested in the numerical calculation of capacitance per unit length of some inhomogeneous IC interconnects. The proposed method leads to an accurate and efficient computer code that permits one to analyze a variety of interconnect IC circuits.

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