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ON THE ANALYTICAL CALCULATION IN MECHANICAL MODELLING

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In this paper are studied the possibilities of replacing the analytical calculation, that intervene in the mechanical modelling, by an isomorphic numerical calculation which can be performed on digital computers. Is described an algorithm for performing the greatest common divisor of two polynomials with several variables that may be used to determine the analytical inverse matrix for a matrix of such polynomials that are used in a mathematical modelling of mechanical phenomena. The model of calculus for revolution shells is deduced by coding, from the Goldenveizer thin plates model. We appeal the codified summation and multiplication operations for the set of polynomial of several variables with real coefficients and the codified differentiation for the vectors attached to any middle surface point of the shell.

Key words: mechanical modelling, shells, analytical calculation, coding of operations, matrix of polynomials.

1. CODING THE OPERATIONS IN THE SET OF POLYNOMIALS WITH SEVERAL VARIABLES

For coding the summing and the product operations in the set of polynomial of several variables with real or integer coefficients we start with the coding of the algebraic operations for two monomials as follows:

$$Ca_1...a_m\ \ X_1^{a_1}...X_m^{a_m}\ +\ Cb_1...b_m\ \ X_1^{b_1}...X_m^{b_m}\ \rightarrow \{(\ Ca_1...a_m\ ,\ a_1,...,\ a_m)\ ,\ (\ Cb_1...b_m\ ,\ b_1,...,\ b_m)\}$$
$$Ca_1...a_m\ \ X_1^{a_1}...X_m^{a_m}\ *\ Cb_1...b_m\ \ X_1^{b_1}...X_m^{b_m}\ \rightarrow (\ Ca_1...a_m * Cb_1...b_m\ ,\ a_1+b_1,...,\ a_m+b_m)$$
$$C\ *\ Ca_1...a_m\ \ X_1^{a_1}...X_m^{a_m} \rightarrow\ (\ C\ *\ Ca_1...a_m\ ,\ a_1,...,\ a_m)$$

The inverse matrix of the matrix $\left[p_{ij}\right],\ i,\ j=1,...,n$ (where n is non zero and non negative integer and p_{ij} is polynomial with several variables) is denoted by $\left[q_{ij}/q_i\right],\ i,\ j=1,\ ...,n,$ and is deduced by reduce the fractions of polynomials that intervene in the algorithm. It is necessary to know the algorithm for performing the greatest common divisor of two polynomials with several variables as follows and to codify this algorithm.

2. INTRODUCTIONAL NOTIONS FOR THE SET OF POLYNOMIALS

A unitary and commutative ring K without divisors of zero is named an integral domain. We write briefly K i.d.

Let K a factorial ring, therefore an integral domain with the property that every non zero and non invertible element of K is a product of prime elements of K . We write K f.r.

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If $a,b \in K$ we say that a divide b if $b = a \cdot c$ with $c \in K$ and will write a/b.

A non zero and non invertible element $p \in K$ is named "prime" if for any $a,b \in K$ with p/ab it follows p/a or p/b.

An element $c \in K$ (if exist) is named a greatest common divisor of a and b if c/a, c/b and if d/a, d/b then d/c. Is denoted $c = (a,b)$.

If $d1 = (a,b)$, $d2 = (a,b)$ then exists $u \in K$, invertible such that $d1 = ud2$.

Two elements $d1, d2 \in K$ such that exists $u \in K$ invertible, with $d1 = u\, d2$ are named adjoints in divisibility.

The elements $a, b \in K$ such that $(a, b) = 1$ are named relatively prime.

The ring of polynomials of one variable with coefficients in K is denoted by $K[X]$ and the ring of polynomials of several variable $X_1,..., X_n$ with coefficients in K is denoted by $K[X_1,..., X_n]$.

If K i.d. and $f \in K[X]$ of the form

$$f = a_o + a_1 X + ... + a_n X^n \tag{2.1}$$

is denoted by $c(f) \in K$ the greatest common divisor (g.c.d.) for the coefficients $a_i \in K$, $(i = 1,...,n)$ of polynomial f .

If $f \in K[X]$ is of the form (2.1) and $a \in K$ with a/f then a/a_i , $i = 1,...,n$ where $a_i \in K$.

If $g \in K[X]$ and $c(g) = 1$ we say that g is primal polynomial.

Is denoted by $K_0[X_m]$ the ring of polynomials in indeterminate X_m over ring $K_0 = K[X_1,..., X_{m-1}, X_{m+1},..., X_n]$, $m \le n$.

A polynomial $g \in K_0(X_m)$ is of the form

$$g = b_0 + b_1 X_m + ... + b_n X_m^n \tag{2.2}$$

where $b_0, b_1,..., b_n$ are polynomials from the ring $K[X_1,..., X_{m-1}, X_{m+1},..., X_n]$

If K i.d. then $K[X]$ i.d. and if K f.r. then $K[X]$ f.r.

If K f.r.; $f, g, h \in K[X]$ and f, g relatively prime such that f/gh then f/h.

We will use the following property [1]:

Theorem 1

Let $f(X)$ and $g(X) \neq 0$ be polynomials in $R[X]$, R a ring, and let p be degree and b_p the leading coefficient of $g(X)$. Then there exists $k \in N$ and polynomials $q(X)$ and $r(X) \in R[X]$ with $\deg r(X) <$ $\deg g(X)$ such that

$$b_p^k f = q\, g + r \tag{2.3}$$

where $k = \max(0, \deg f - \deg g + 1)$

3. A DIVISION WITH A REMAINDER THEOREM FOR $K[X_1,..., X_n]$

Let K factorial ring and $0 < m \le n$, with $m, n \in N$

We formulate bellow the following:

Theorem 2

If a polynomials $p_1, p_2 \in K[X_1,..., X_n]$, $p_1 \neq 0$, $p_2 \neq 0$, for fixed m, exists a polynomials $q_1, q_2, r \in K[X_1,..., X_n]$, unique without a adjointly in divisibility, such that

$$p_1\, q_1 = p_2\, q_2 + r \tag{3.1}$$

where $r = 0$ or $\deg\, r < \deg\, p_2$, with degree referred to variable X_m .

The polynomials $q_1,\ q_2,\ r$ are relatively prime and $q_1 \neq 0$.

Proof. In the following all polynomials are considered as polynomials in the variable X_m. If $\deg\, p_1 < \deg\, p_2$ the relation (3.1) is determined by considering $q_1 = 1,\ q_2 = 0,\ r = p_1$.

For $\deg\, p_1 \geq \deg\, p_2$ we use the relation (2.3) of the theorem 1, where $R[X]$ is the ring $K_0[X_m]$ of polynomials with variable X_m with coefficients from K_0 .

$$b_p^k\, p_1 = q\ p_2 + r^* \tag{3.2}$$

where b_p is a leading coefficient of p_2 , therefore is a polynomial from the ring K_0 , otherwise the ring $K[X_1,..., X_{m\text{-}1}, X_{m+1},..., X_n]$, and where $k = \max(0,\ \deg\, p_1 - \deg\, p_2 + 1)$

Let d the greatest common divisor of polynomials b_p^k and q as the polynomials of the ring $K[X_1,..., X_n]$. Because b_p^k is a polynomial no more than $n-1$ variables then d is a polynomial no more than $n-1$ variables. Polynomial d is also divisor of polynomial r^* because

$$b_p^k\ p_1 - q\ p_2 = r^* \tag{3.3}$$

We simplify the relation (3.2) with polynomial d and it follows:

$$q_1\ p_1 = q_2\ p_2 + r \tag{3.4}$$

where are denoted by q_1 , q_2 and r the polynomials $b_p^k,\ q$ respectively r^* divided by d .

The polynomials $q_1,\ q_2,\ r$ are relatively prime from your deduction and $q_1 \neq 0$ because $p_2 \neq 0$.

We study the uniqueness of the relationship (3.1). Suppose the existence of the second division relationship of the polynomials p_1 and p_2 such that

$$p_1\ q_1' = p_2\ q_2' + r' \tag{3.5}$$

where the polynomials $q_1',\ q_2',\ r'$ are relatively prime and $q_1' \neq 0$.

From (3.1) and (3.5) it follows that

$$p_2(q_1'q_2 - q_1q_2') = r'q_1 - rq_1' \tag{3.6}$$

If $q_1'q_2 - q_1q_2' \neq 0$ then $\deg(r'q_1 - rq_1') \geq \deg\, p_2$ as polynomials in $X_{m.}$

But $\deg\, r < \deg\, p_2$ and $\deg\, r' < \deg\, p_2$ then $\deg(r'q_1 - rq_1') < \deg\, p_2$.

Contradiction. It follows $q_1'q_2 - q_1q_2' = 0$ and $r'q_1 - rq_1' = 0$.

Because $q_1 / q_1'q_2$ and $q_1,\ q_2$ are relatively prime it follows that q_1 / q_1' . Analogue, from $r'q_1 = rq_1'$ and $q_1' / r'q_1$ with $q_1',\ r'$ relatively prime, we deduce that q_1' / q_1 such that q_1 and q_1' are adjointly in divisibility.

From $r'q_1 = rq_1'$ and $q_1,\ q_1'$ adjointly in divisibility, it follows that $r,\ r'$ are adjointly in divisibility.

4. THE EUCLID'S TYPE ALGORITHM IN THE FACTORIAL RING $K[X_1,..., X_n]$

We suppose that K is factorial ring and $0 < m \leq n$, with $m,\ n \in N$

Let $p_1,\ p_2 \in K(X_1,..., X_n),\ p_1 \neq 0,\ p_2 \neq 0.$ From the second theorem, for fixed m exists a polynomials $q_1,\ q_2,\ r \in K[X_1,..., X_n]$, unique without a adjointly in divisibility, such that

$$p_1\ q_1 = p_2\ q_2 + r \tag{4.1}$$

where $r = 0$ or $\deg\ r < \deg\ p_2$, with degree referred to variable m.

The polynomials $q_1,\ q_2,\ r$ are relatively prime and $q_1 \neq 0$.

By $D(p_1,\ p_2)$ is denoted the set of divisors with zero remainder for both polynomials p_1 and p_2. Is named briefly the set of divisors for p_1 and p_2.

There is the following property:

Theorem 3

In the conditions of second theorem, is true the equality $D(p_1,\ p_2) = D(p_2,\ r),$ where r is the remainder of the division of the polynomials p_1 and p_2, for fixed m .

Proof. We suppose, for beginning, that p_1 and p_2 are primal polynomials. It is sufficiently to provide the property for the set of prime divisors.

Let $d \in D(p_1, p_2)$, d prime polynomial and $d / p_1,\ d / p_2$. But $r = p_1\ q_1 - p_2\ q_2$. Then d / r and thus $d \in D(p_2,\ r)$, such that $D(p_1,\ p_2) \subseteq D(p_2,\ r)$.

Inversely, let d prime polynomial, $d \in D(p_2,\ r)$. Then d / p_2 and d / r . Thus $d / p_1 q_1$ because $p_1\ q_1 = p_2\ q_2 + r$. But d prime polynomial, therefore d / p_1 or d / q_1. Because d / p_2 and p_2 primal polynomial it follows d primal polynomial. If d / q_1 than d is polynomial independent of X_m and because d / p_2 it follows d divide the coefficients of p_2. Contradiction, because p_2 is primal polynomial. Then d / p_1, such that $d \in D(p_1, p_2)$. Thus $D(p_1,\ p_2) \supseteq D(p_2,\ r)$

We denote by $D'(p_1, p_2)$ the set of polynomials common divisors of coefficients for p_1 and p_2.

If $p_1,\ p_2$ are not primal polynomials and d , prime polynomial, divide the coefficients of polynomials p_1 and p_2 then d divide the polynomial r and thus the coefficients of polynomial r, such that $D'(p_1,\ p_2) \subseteq D'(p_2,\ r)$. If d divide the coefficients of polynomials p_2 and r then d divide $p_1 q_1$. If d / q_1 then q_1 and r are not relative prime. It follows d / p_1, such that d divide the coefficients of p_1, thus. $D'(p_1,\ p_2) \supseteq D'(p_2,\ r)$

This theorem permits to give an Euclid's type algorithm for performing the greatest common divisor of two polynomials of several variables with coefficients in factorial ring.

We suppose that $\deg\ p_1 \geq \deg\ p_2$. From the third theorem applied to polynomials p_1 and p_2 we obtain that $D(p_1,\ p_2) = D(p_2,\ r)$, where r is the remainder of division of p_1 and p_2. If $r = 0$ then $(p_1,\ p_2) = p_2$. If $r \neq 0$ then $\deg\ r < \deg\ p_2$.

Apply the third theorem polynomials p_2 and r . We can write:

$$p_2\ q_1' = r\ q_2' + r_1 \tag{4.2}$$

If $r_1 = 0$ then $(p_1,\ p_2) = (p_2,\ r) = r$. If $r_1 \neq 0$ then:

$$\deg\ p_1 \geq \deg\ p_2 > \deg\ r > \deg\ r_1 > \ldots \tag{4.3}$$

and $(p_1,\ p_2) = (p_2,\ r) = (r,\ r_1) = \ldots$ such that after a finite number of steps is obtained a zero remainder. The latest none zero divisor in the row (4.3) is the greatest common divisor of polynomials p_1 and p_2.

In the next place we describe the expression of an inverse matrix of a matrix of polynomials with several variables that intervene in the mechanical modelling of the plane shapes.

The inverse matrix of the matrix $\left[p_{ij} \right],\ i, j = 1,...,8$, is denoted by $\left[q_{ij} / q_i \right],\ i, j = 1, ...,8$, and is deduced by reduce the fractions of polynomials. The expression of the elements is:

$$\begin{gathered} p_{14} = -b,\ p_{16} = a,\ p_{25} = a,\ p_{26} = -b,\ p_{37} = 2ab,\ p_{48} = -2ab,\ p_{51} = 1,\ p_{53} = -b,\ p_{54} = -a, \\ p_{55} = p\ a,\ p_{56} = b\ (1+p),\ p_{62} = 1,\ p_{63} = -a,\ p_{64} = -bp,\ p_{66} = -a(1+p),\ p_{73} = b,\ p_{74} = -a, \\ p_{75} = a\ p,\ p_{76} = -b(1+p),\ p_{77} = a^2 + b^2,\ p_{83} = -a,\ p_{84} = b\ p,\ p_{85} = -b,\ p_{86} = -a(1+p), \end{gathered} \tag{2.1}$$

$p_{88} = -(a^2 + b^2)$.

In the rest, the values of p_{ij} are zero.

$$q_{11} = -4a^2b^2(1+p)(2a^2+b^2-b^2p),\ q_{12} = -4a^3b^3(1+p)^2,$$
$$q_{13} = (a^2+b^2)(a^4-2a^2b^2-b^4-2a^2b^2p),\ q_{14} = 2ab(a^2+b^2)(a^2-b^2p),$$
$$q_{15} = 2ab(a^2+b^2)^2,\ q_{17} = -2ab(a^4-2a^2b^2-b^4-2a^2b^2p),\ q_{18} = -4a^2b^2(a^2-b^2p),$$
$$q_{21} = -4a^3b^3(1+p)^2,\ q_{22} = -4a^2b^2(1+p)(2b^2+a^2-a^2p),\ q_{23} = 2ab(a^2+b^2)(-b^2+a^2p),$$
$$q_{24} = (a^2+b^2)(2b^2+a^2-a^2p),\ q_{26} = 2ab(a^2+b^2)^2,\ q_{27} = -4a^2b^2(a^2p-b^2),$$
$$q_{28} = -2ab(a^4+2a^2b^2-b^4+2a^2b^2p),\ q_{31} = 2a^2b(1+p),\ q_{32} = 2ab^2(1+p)$$
$$q_{33} = b(a^2+b^2),\ q_{34} = -a(a^2+b^2),\ q_{37} = -2ab^2, q_{38} = 2a^2b,\ q_{41} = -2b^2(2a^2+b^2+a^2p),$$
$$q_{42} = -2ab(b^2-a^2p),\ q_{43} = a^2(a^2+b^2),\ q_{44} = ab(a^2+b^2),\ q_{47} = -2a^3b$$
$$q_{48} = -2a^2b^2,\ q_{51} = -2ab(a^2-b^2p),\ q_{52} = -2a^2(a^2+2b^2+b^2p),\ q_{53} = -ab(a^2+b^2),$$
$$q_{54} = -b^2(a^2+b^2),\ q_{57} = 2a^2b^2,\ q_{58} = 2ab^3,\ q_{61} = -2a(a^2-b^2p),\ q_{62} = 2b(b^2-a^2p),$$
$$q_{63} = -a(a^2+b^2),\ q_{64} = -b(a^2+b^2),\ q_{67} = 2a^2b,\ q_{68} = 2ab^2,\ q_{73} = 1,\ q_{84} = 1.$$

In the rest the values of q_{ij} are zero.

$$q_1 = 2ab(a^2+b^2)^2,\ q_2 = 2ab(a^2+b^2)^2,\ q_3 = -2ab(a^2+b^2),\ q_4 = 2b(a^2+b^2)^2,$$
$$q_5 = -2a(a^2+b^2)^2,\ q_6 = -2(a^2+b^2)^2,\ q_7 = 2ab,\ q_8 = -2ab.$$

5. THE MODEL OF CALCULUS FOR THIN PLATES AND CODING

The thin plate is supposed homogeneous, isotropic and elastic linear. The thickness "h" of the plate is constant and satisfy the relation

$$2h/R_m < 1/20 \tag{5.1}$$

where R_m is the minimum of curvature radius of the plate points.

Is considered also the hypothesis Love-Kirchoff of the "indeformed normal element".

The revolution thin plate is described by the vector of position for the point situated on the middle surface:

$$\vec{R}\,(\theta,\ z) = r(z)\cos(\theta)\vec{i} + r(z)\sin(\theta)\vec{j} + z\,\vec{k} \tag{5.2}$$

The vector of displacements with his applied point on the surface is considered of the form:

$$\vec{U} = u\,\vec{t}_z + v\,\vec{t}_\theta - w\vec{n} \tag{5.3}$$

which $\vec{t}_z$, $\vec{t}_\theta$ are the unitary vectors attached to coordinate curves concerning point $P(\theta,\ z)$ (fig.1).

The first fundamental form of the middle surface is : $ds^2 = r(z)^2 d\theta^2 + (1+r_z^{\,2})\,dz^2$

with the coefficients $A = r(z),\ B = (1 + r_z^{\,2})^{1/2}$

The coefficients of the second fundamental form are: $D = -r(z)/B,\ D' = 0,\ D'' = r_{z2}/B,$ where $r_z^{\,2} = d^2r(z)/dz^2$

The building of the thin plate model on computer is based on coding of the differentiation operation.

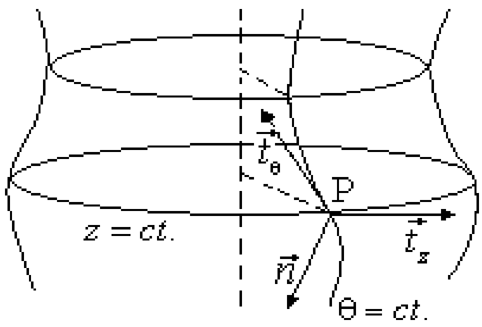

Fig.1

For deduction of the codified differentiation of the vector $\vec{U}$.

firstly are deduced the codified formulas of differentiation for the unitary vectors $\vec{t}_z$, $\vec{t}_\theta$, $\vec{n}$. For example:

$$\partial \vec{t}_z / \partial \theta = -r_z / B^2 \ \vec{t}_\theta \ - \ 1./B^2 \ \vec{n} \tag{5.4}$$

where $r_z = \partial r / \partial z$.

The vector equilibrium equations of the Goldenveizer model, attached to any middle surface point of the plate and to appropriate three dimensional local system of axis, are:

$$- \frac{\partial \ (\ r \ \vec{F}^{(z)})}{\partial z} - \frac{\partial \ (\ B \ \vec{F}^{(\theta)})}{\partial \theta} + \ r \ B \ \vec{P} \ = \ 0. \tag{5.5}$$

$$- \frac{\partial \ (\ r \ \vec{C}^{(z)})}{\partial z} - \frac{\partial \ (\ B \ \vec{C}^{(\theta)})}{\partial \theta} - r \ B \ \vec{F}^{\ \theta} \ X \ \ \vec{t}_z \ - r \ B \ \vec{F}^{\ z} \ X \ \ \vec{t}_\theta \ + \ rB \ \vec{C} = \ 0 \tag{5.6}$$

with $\vec{P}$ - the external force which action on the plate, $\vec{C}$ - the external resultant moment and

$$\vec{F}^{(z)} = N^{z\theta}\vec{t}_z - N^z\vec{t}_\theta + Q^z\vec{n} \quad , \quad \vec{C}^z = M^z\vec{t}_z - M^{z\theta}\vec{t}_\theta$$

$$\vec{F}^{(\theta)} = -N^{\theta} \ \vec{t}_z - N^{\theta z} \ \vec{t}_\theta \ + \ Q^{\theta} \ \vec{n} \quad , \quad \vec{C}^{\theta} = -M^{\theta z} \ \vec{t}_z - M^{\theta} \ \vec{t}_\theta \quad \text{(see fig. 2)}$$

By minimizing the energy of the forces and of the moments which action on the plate we deduce the expression of the generalized forces which are utilized to impose the boundary conditions at the end z = const. :

$$N^{z*} = \ N^z, \ N^{z\theta *} = \ N^{z\theta} + \frac{1}{r \ B} \ M^{z\theta}, \ Q^{z*} = \ Q^z + \frac{1}{r}\frac{\partial \ M^{z\theta}}{\partial \ \theta}, \ M^{z*} = \ M^z \tag{5.7}$$

From the hypothesis of linearity are deduced the constitutive equations :

$$N^{\theta} = \frac{2Eh}{1-\nu^2} \ (\ \varepsilon^{z} + \nu \ \ \varepsilon^{\theta}), \ N^{z} = \frac{2Eh}{1-\nu^2} \ (\ \varepsilon^{\theta} + \nu \ \ \varepsilon^{z}), \ N^{\theta z} = -N^{z\theta} = \frac{Eh}{1+\nu}\omega$$

$$M^{\theta} = -\frac{2Eh^3}{3\ (1-\nu^2)} \ (\ \chi^z \ + \ \nu \ \chi^{\theta}), \ M^{z} = -\frac{2Eh^3}{3\ (1-\nu^2)} \ (\ \chi^{\theta} \ + \ \nu \ \chi^{z}), \ M^{\theta z} = - \ M^{z\theta} = \frac{2Eh^3}{3\ (\ 1 \ - \ \nu^{\ 2})} \ \tau$$

,

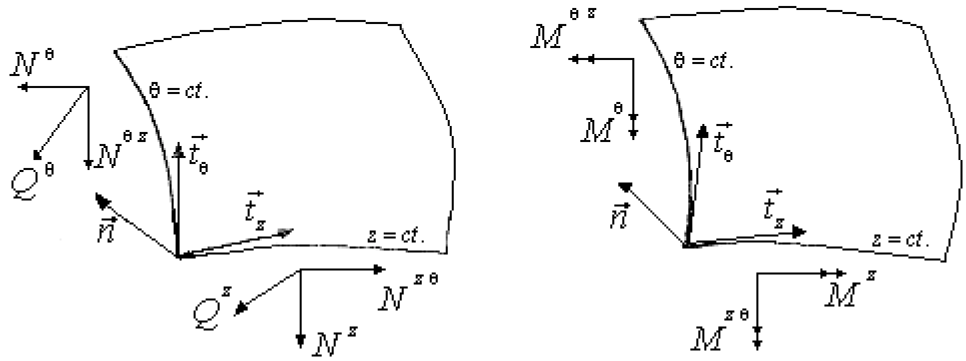

Fig. 2

The coding of the operations for the deduction of the model of the revolution thin plates, in displacements, is performed about the following successive variable which intervene in the model :

$$E,\ h,\ \frac{1}{1+\nu},\ \frac{1}{1-\nu^2},\ \frac{1}{r},\ r,\ r_z\ ,...,r_{z5},\ \frac{1}{B},\ B,\ u,\ u_\theta,\ u_z,\ u_{\theta\theta},\ u_{\theta z},\ u_{zz},\ u_{\theta\theta\theta},...,u_{\theta zzz},\ u_{zzzz},\ v,...,\ v_{zzzz},\ w,\ ...,w_{zzzz}$$

where E is modulus of elasticity, h is the shell half-thickness, ν is Poisson's coefficient and u, v, w are the components of the displacement of a point of the middle surface of shell.

Vector and scalar addition, multiplication and differentiation subroutines have been performed.

We describe some of the results deduced on the computer :

$$\begin{aligned}
A\,Q^z = {} & 1.33\,E2\,h^2r^{-2}r_zB^{-2}u_\theta + 0.66\,E2\,h^2r^{-1}r_zr_{z2}B^{-4}u_\theta - 0.66\,E2\,h^2r^{-1}r_zr_{z2}B^{-4}u_\theta - \\
& - 0.66\,E2\,h^2r^{-2}r_zB^{-2}u_\theta - 1.33\,E1\,h^2r^{-1}r_zB^{-2}u_\theta - 0.66\,E1\,h^2r^{-1}\,r_zr_{z2}B^{-4}u_\theta + E1\,h^2r^{-2}r_zB^{-2}u_\theta - \\
& - 0.66\,E2\,h^2r^{-1}B^{-2}u_{\theta z} + 0.33\,E1\,h^2r^{-1}B^{-2}u_{\theta z} + 0.66\,E2\,h^2r\,r_{z4}B^{-5}v - 0.66\,E2\,h^2r\,r_zr_{z2}r_{z3}B^{-7}v - \\
& - 2E2\,h^2\,r\,rz2^3B^{-7}v + 12.E2\,h^2\,r\,r_z^{\,2}r_{z2}^{\,3}B^{-9}v - 0.66E2\,h^2r^{-1}r_z^{\,2}rz2\,B^{-5}v + 0.66\,E2\,h^2rz2^2\,B^{-5}v + \\
& + 0.66E2\,h^2r_zr_{z3}B^{-5}v - 2.66\,E2\,h^2rz^2rz2^2B^{-7}\,v - 0.66E2\,h^2r^{-1}r_z^{\,2}r_{z2}B^{-5}\,v - 0.66\,E2\,h^2r^{-2}r_z^{\,2}B^{-3}v + \\
& + 0.66\,E1\,h^2r^{-1}r_z^{\,2}r_{z2}B^{-5}v - 0.66\,E1\,h^2r_{z2}^{\,2}B^{-5}v - 0.66\,E1\,h^2r_zr_{z3}\,B^{-5}v + 0.66\,E1\,h^2r^{-1}r_zr_{z2}r_{z3}B^{-7}v + \\
& + 2.66\,E1\,h^2r_z^{\,2}r_{z2}^{\,2}B^{-7}v + 0.66\,E1\,h^2r^{-1}r_{z3}B^{-5}v - 2\,E1\,h^2\,rz^2r_{z2}^{\,2}B^{-7}v - 0.66\,E1\,h^2r^{-1}r_z^{\,2}r_{z2}B^{-5}v + \\
& + 0.33\,E1\,h^2r^{-2}B^{-1}v_{\theta\theta} + 0.66\,E1\,h^2r^{-1}r_{z2}B^{-3}v_{\theta\theta} + 0.66\,E2\,h^2r_zr_{z2}B^{-5}v_z - 2\,E2\,h^2r\,r_zr_{z2}^{\,2}B^{-7}\,v_z + \\
& + 0.66\,E2\,h^2r\,r_{z3}B^{-5}v_z + 0.66\,E2\,h^2r_z\,r_{z2}B^{-5}v_z - 0.66\,E1\,h^2\,r_zr_{z2}B^{-5}v_z + 0.66\,E1\,h^2r_zr_{z2}B^{-5}v_z\ - \\
& - 0.66\,E2\,h^2B^{-3}v_{\theta z} - 0.66\,E2\,h^2r_{z3}B^{-5}w + 2.\,E2\,h^2r_zr_{z2}^{\,2}B^{-7}w - 1.33\,E2\,h^2\,r\,r_{z2}r_{z3}B^{-7}w + \\
& + 4E2\,h^2r\,r_zr_{z2}^{\,2}B^{-9}w + 0.66\,E2\,h^2r^{-1}\,r_zr_{z2}B^{-5}w + 0.66\,E2\,h^2r^{-2}r_zB^{-3}w + 1.33\,E2\,h^2r^{-1}r_zB^{-1}w_{\theta\theta} - \\
& - 1.33\,E1\,h^2r^{-2}r_zB^{-1}w_{\theta\theta} + 1.33\,E1\,h^2r^{-2}r_zB^{-1}w_{\theta\theta} - 0.66\,E2\,h^2r_{z2}B^{-5}w_z + 2.E2\,h^2r\,r_{z2}^{\,2}B^{-7}w_z + \\
& + 0.66\,E2\,h^2r\,r_zr_{z3}\,B^{-5}w_z - 2E2\,h^2r\,r_{z2}^{\,2}B^{-5}w_z + 0.66\,E2\,h^2r^{-1}r_z^{\,2}B^{-3}w_z - 0.66\,E2\,h^2r_{z2}B^{-3}w_z + \\
& + 1.33\,E2\,h^2r_z^{\,2}r_{z2}B^{-5}w_z - 0.66\,E2\,h^2r^{-1}r_z^{\,2}B^{-3}w_z + 0.66\,E1\,h^2r_{z2}B^{-3}w_z - 1.33\,E1\,h^2r_z^{\,2}r_{z2}B^{-5}w_z + \\
& + 0.66\,E1\,h^2r_z^{\,2}r_{z2}B^{-5}w_z + 0.66\,E1\,h^2r^{-1}r_z^{\,2}B^{-3}w_z - 0.66\,E2\,h^2r^{-1}B^{-1}w_{\theta\theta z} + 2\,E2\,h^2r\,r_zr_{z2}B^{-5}w_{zz} - \\
& - 0.66\,E2\,h^2r_zB^{-3}w_{zz} + 0.66\,E1\,h^2r_zB^{-3}w_{zz} - 0.66\,E2\,h^2r\,B^{-3}w_{zzz}.
\end{aligned}$$

The boundaries conditions for the generalized displacements and forces are applied to:

$$u,\ v,\ w,\ \gamma^\theta,\ N^{z*}\ ,\ N^{z\theta*},\ Q^{z*},\ M^{z*}\quad \text{where:}\ \ \gamma^\theta = r_{zz}B^{-3}v\ -\ B^{-1}w_z,$$

$$N^{z*} = -2\,E1\,r^{-1}u_\theta + 2\,E2\,r^{-1}u_\theta - 0.66\,E2\,h^2 r_{z2} r_{z3} B^{-7} v + 2\,E2\,h^2 r_z {r_{z2}}^3 B^{-9} v -$$
$$-2\,E1\,r^{-1} r_z B^{-1} v + 2\,E2\,r^{-1} r_z B^{-1} v - 0.66\,E2\,h^2 r^{-1} r_{z3} B^{-5} v + 2\,E2\,h^2 r^{-1} r_z {r_{z2}}^2 B^{-7} v +$$
$$+ 2\,E2\,B^{-1} v_z + 0.66\,E2\,h^2 {r_{z2}}^3 B^{-9} w - 2\,E2\,r^{-1} B^{-1} w + 2\,E2\,r_{z2} B^{-3} w + 0.66\,E2\,h^2 r^{-1} {r_{z2}}^2 B^{-7} w -$$
$$- 0.66\,E2\,h^2 r^{-1} r_z r_{z2} B^{-5} w_z - 0.66\,E2\,h^2 r_z {r_{z2}}^2 B^{-7} w_z + 0.66\,E2\,h^2 r^{-1} B^{-3} w_{zz} +$$
$$+ 0.66\,E2\,h^2 r_{z2} B^{-5} w_{zz}.$$
$$M^z = 0.66\,E1\,h^2 r^{-2} B^{-1} u_\theta - 0.66\,E2\,h^2 r^{-2} B^{-1} u_\theta + 0.66\,E2\,h^2 r^{-1} r_z\, r_{z2} B^{-4} v - 0.66 E1\,h^2 r^{-1} r_z r_{z2} B^{-4} v +$$
$$+ 0.66\,E2\,h^2 r_{z3} B^{-4} v - 2E2\,h^2 r_z {r_{z2}}^2 B^{-6} v - 0.66\,E2\,h^2 r^{-1} B^{-2} v_z - 0.66\,E2\,h^2 r^{-1} r_{z2} B^{-4} w -$$
$$- 0.66\,E2\,h^2 {r_{z2}}^2 B^{-6} w + 0.66\,E1\,h^2 r^{-2} w - 0.66\,E2\,h^2 r^{-2} w_{\theta\theta} + 0.66\,E2\,h^2 r_z r_{z2} B^{-4} w_z +$$
$$+ 0.66\,E1\,h^2 r^{-1} r_z B^{-2} w_z - 0.66\,E2\,h^2 r^{-1} r_z B^{-2} w_z - 0.66\,E2\,h^2 B^{-2} w_{zz}.$$

The codified scalar equilibrium equations in displacements deduced on the computer are used as input data for the program of static or dynamic calculus of the thin plates.

The numerical method take into account the boundary conditions at the extremities $z = z_1$ and $z = z_2$ as well as the development in series of vector displacement components concerning coordinate variables which are considered of the form:

$$u = \sum_{n=0}^{n1} u_{ns}(z)\ \sin(n\theta),\ v = \sum_{n=0}^{n2} v_{nc}(z)\ \cos(n\theta),\ w = \sum_{n=0}^{n3} w_{nc}(z)\ \cos(n\theta)$$

and where $u_{ns}(z),\ v_{nc}(z),\ w_{nc}(z)$ are developed in series by a complete system of polynomials of single variable. The semi-analytical method used take into account a decomposition of the revolution surface in modules concerning direction of revolution axis.

6. CONCLUSIONS

The possibilities of replacing the manual analytical calculation that intervene in the mechanical modelling by an isomorphic numerical calculation, which can be performed on digital computers are investigated.. A division with a remainder theorem in the set of polynomials of several variables with coefficients in factorial ring (as the integers ring), proof here, permit us to perform on the computer the analytical expression of the inverse matrix of polynomials with several variables used in the modelling. Appear a question about how much can lead the analytical calculation in the modelling up to replacing with a numerical method of the solutions calculation. Any steps has analyzed here.

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