THE PUBLISHING HOUSE OF THE ROMANIAN ACADEMY
PROCEEDINGS OF THE ROMANIAN ACADEMY, ***Series A,***
Volume 6, Number 3/2005, pp. 000-000

A POSSIBLE EXPLANATION OF THE BUILDING BREAKAGE BETWEEN 2nd AND 3rd FLOOR WHEN SUBJECTED TO STRONG EARTHQUAKES

Tudor SIRETEANU

Institute of Solid Mechanics – Romanian Academy 15 Constantin .Mille Street, RO-010141, Bucharest
E-mail: siret@imsar.bu.edu.ro

The dynamic response of a cantilever beam to imposed displacement of the clamped end is analyzed. It is shown that for the same values of amplification factor obtained below and above the first resonance frequency, the dynamic response displays two patterns, which lead to an important difference in the variation of the shear force along the beam. While for vibrations excited below resonance the maximum shear force is encountered at the clamped end, in the above resonance amplification range this force displays a maximum value at about one third of the beam length. This behaviour was obtained by both analytical and experimental approach. As a vertical cantilever beam with imposed displacement of the embedded base could be used as a simple mechanical model of a multiple storey building excited by seismic horizontal ground motion, this result could be a possible explanation of the breakages encountered after major earthquakes in many buildings between second and third floor.

Key words: *Earthquake, building damage, cantilever beam, first vibration mode, amplification factor.*

1. INTRODUCTION

Inspection of damaged buildings after major earthquakes reveals in many cases breakages of structural elements between second and third floor[1,2]. Usually, this phenomenon is attributed to the structural dynamic loads produced by seismic excitation of the above vibration modes. However, for medium height buildings (around 10 stories) the above vibration modes are unlikely to be excited by the seismic ground motion. Generally, even the second eigenfrequency of such buildings is too high (about 3 times greater than the frequency of the first vibration mode) to be excited by the ground motion, due to the spectral content of the seismic input.

Therefore, this breakage pattern has to be explained by the distribution of the dynamic shear forces developed along the building in the first bending vibration mode. It is well known that maximum bending moment and shear force in a cantilever beam with fixed clamped end, vibrating with frequencies within the amplification range of the first vibration mode, are encountered at the embedded end. These bending vibrations can be easily excited by applying a concentrated harmonic force, acting on transversal direction at the beam free end. Obviously, in this case, the beam cracks are supposed to develop at the fixed end, no matter the driving frequency is bellow or above the beam first eigenfrequency. This not anymore true when the beam vibration is excited by an imposed displacement of the embedded end, likewise a building structure is excited by the horizontal motion imposed to its foundation by the ground motion. There are two different vibration patterns when the excitation frequency of the base imposed displacement is below or above the frequency of the first vibration mode. From the practical point of view, the study of this dynamic behavior is meaningful, especially when beam vibrates in the neighborhood of the first resonance vibration mode

Reccomended by Radu P.VOINEA, member of the Romanian Academy

because in this case the dynamic structural output is strongly amplified by favored transfer of the kinetic energy from the ground motion to the structure and damaging shear forces can develop.

If the structural damping is neglected then the vibration amplitude tends theoretically to infinity when the driving frequency is equal to the first eigenfrequency. Practically, the vibration amplitude is finite and, unless the structure is damaged or destroyed, the amplitude reaches its maximum level when the excitation energy is balanced by the dissipated energy. The theoretical eigenfrequencies have irrational values and therefore they cannot be practically encountered in any real or simulated inputs.

Since the internal structural damping of buildings is small (damping ratio values of 0.05 are usually considered), in the subsequent analysis its value will be neglected. Therefore, in order to obtain pertinent results, the dynamic response of the analytical model will be investigated for base imposed displacements with frequencies below and above the first eigenfrequency as so to obtain reasonable values of the amplification factor. This factor is defined as the ratio between the vibration amplitude of the free end and the amplitude of the imposed displacement of the clamped base, for different values of the input frequency. The comparison of the dynamic behaviour of the vibrating beam below and above its first eigenfrequency is made for the same values of the amplification factor. Since the subsequent analysis is conducted only in the neighborhood of the first vibration mode, the contribution of the higher modes to the structural dynamic response is negligible.

2. EXPERIMENTAL RESULTS

The experiments were initially conducted to investigate the fatigue behaviour of some composite laminates by exciting the bending vibration of slender cantilever beams in the neighborhood of the first vibration mode. The experimental setup is shown in Fig.1.

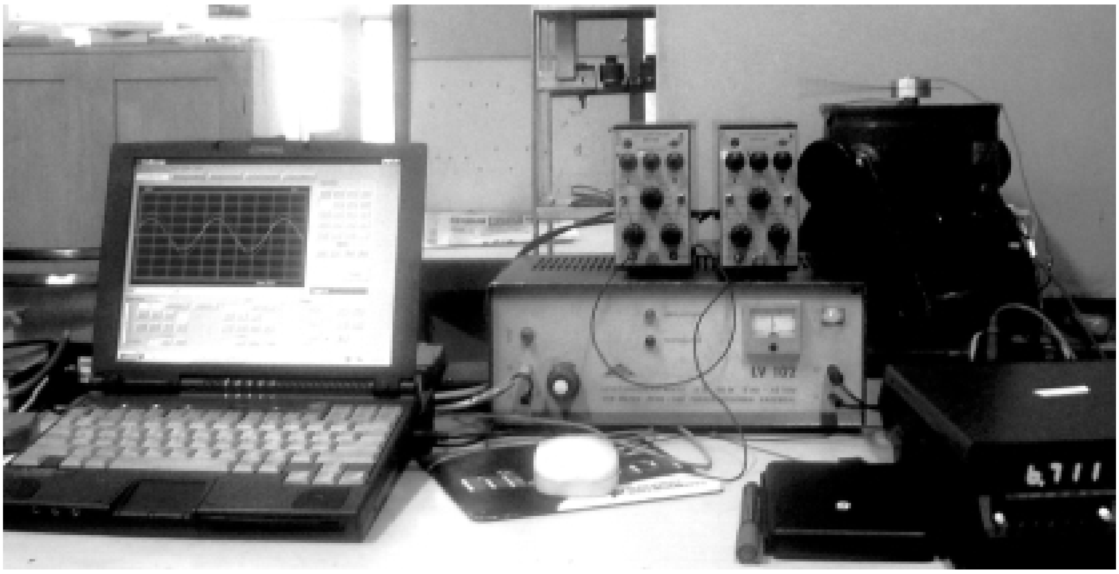

Fig.1 Experimental setup.

To obtain high bending stresses of the composite material the tests were performed by imposing harmonic displacements of the clamped end with frequencies close to the frequency of the first vibration mode. All tests started with forcing frequencies just below the resonance frequency. The input amplitude was then adjusted as to obtain initial desired amplitude of the beam free end. The beam stiffness degradation due to fatigue cyclic stresses was monitored by continuously measuring the interaction force between the clamping device and the platform of the shaking table. After the completion of a certain number of cycles, the frequency response function of the interaction force was determined by sweeping the excitation frequency. The vibration patterns at the start of the fatigue test and after a certain time interval from the start are shown in Figs.2 and 3. It is worth mentioning that in both cases the input frequency remained unchanged. The difference between these two types of beam dynamic response is due to the fact that the vibration shown in

Fig.2 takes place in the frequency range below resonance, while that pictured in Fig.3 is excited in the above resonance frequency range. This shifting of the vibration regime is a direct consequence of fatigue degradation of the composite material, which results in a certain decrease of the beam bending stiffness, and therefore of its eigenfrequencies, while the forcing frequency remained unmodified.

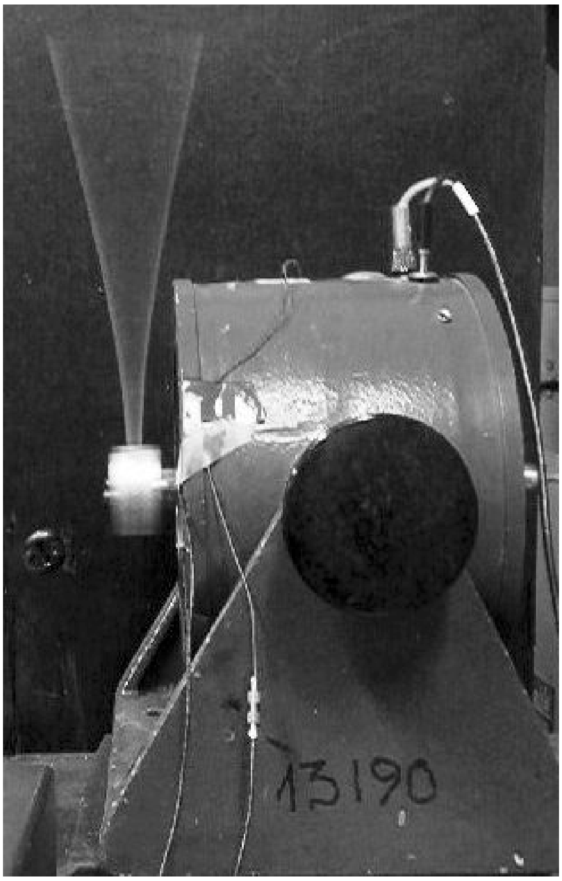

Fig. 2 Beam vibration below resonance (frequency 15Hz).

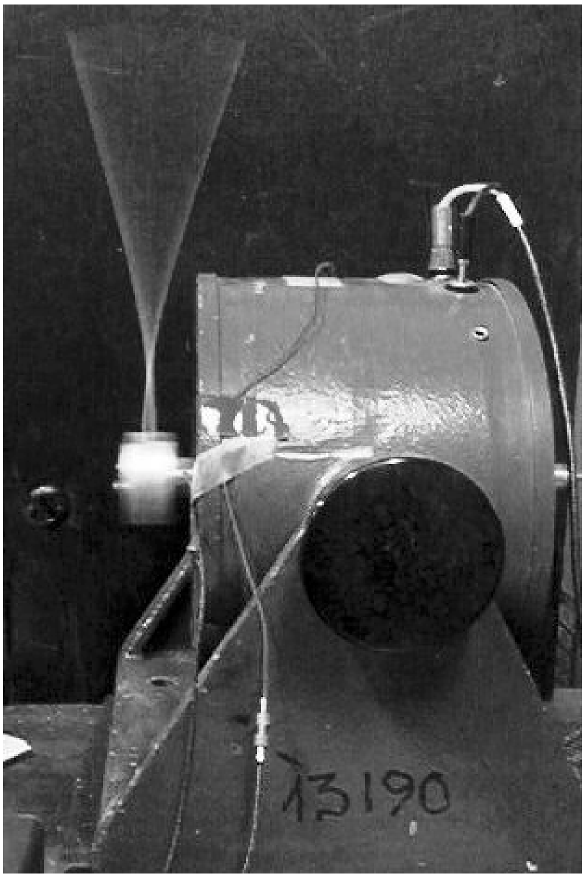

Fig 3 Beam vibration above resonance (frequency 17Hz).

3. ANALYTICAL MODEL

The transverse vibration excited by an imposed harmonic motion to the clamped end of an uniform slender cantilever beam with density ρ, cross section area A, length l and bending stiffness EI is governed by Euller-Bernoulli beam model [3]

$$\frac{\partial^2 y(x,t)}{\partial t^2} + c^2 \frac{\partial^4 y(x.t)}{\partial x^4} = 0, \quad c = \sqrt{\frac{EI}{\rho A}}. \tag{1}$$

The boundary and initial conditions are given by

$$y(0,t) = y_0 \sin \omega t, \ \frac{\partial y(0,t)}{\partial x} \equiv 0, \ \frac{\partial^2 y(l,t)}{\partial x^2} \equiv 0, \ \frac{\partial^3 y(l,t)}{\partial x^3} \equiv 0, \ y(x,0) \equiv 0, \ \frac{\partial y(x,0)}{\partial t} \equiv 0. \tag{2}$$

By defining

$$\omega_b = \frac{1}{l^2}\sqrt{\frac{EI}{\rho A}}, \tag{3}$$

and introducing the dimensionless parameters,

$$\tau = \omega_b t\,, \quad \xi = \frac{x}{l}\,, \quad \upsilon = \frac{\omega}{\omega_b}\,, \quad \eta(\xi, \tau) = \frac{y(l\xi, \tau / \omega_b)}{l}\,, \quad \eta_0 = \frac{y_0}{l} \tag{4}$$

the equation (1) and the boundary and initial conditions (2) become

$$\frac{\partial^4 \eta(\xi, \tau)}{\partial \xi^4} + \frac{\partial^2 \eta(\xi, \tau)}{\partial \tau^2} = 0\,, \tag{5}$$

$$\eta(0, \tau) = \eta_0 \sin \upsilon\tau\,, \ \frac{\partial \eta(0, \tau)}{\partial \xi} = 0\,, \ \frac{\partial^2 \eta(1, \tau)}{\partial \xi^2} = 0\,, \ \frac{\partial^3 \eta(1, \tau)}{\partial \xi^3} = 0\,, \ \eta(\xi, 0) = 0\,, \ \frac{\partial \eta(\xi, 0)}{\partial t} = 0\,. \tag{6}$$

For the sake of simplicity the dimensionless quantities will be given the same names as the corresponding physical ones: τ - time, ξ - length, υ - frequency, $\partial^2\eta/\partial\xi^2$ - bending moment, $\partial^3\eta/\partial\xi^3$ -shear force, etc.
A separation-of-variables solution of (5) is assumed of the form

$$\eta(\xi, \tau) = \eta_0 \varphi(\xi) \sin \upsilon\tau \tag{7}$$

Substitution of (7) into (5) and (6), yields

$$\varphi'''' - \upsilon^2 \varphi = 0\,, \tag{8}$$

$$\varphi(0) = 1, \ \ \varphi'(0) = 0, \ \ \varphi''(1) = 0, \ \ \varphi'''(1) = 0. \tag{9}$$

The general solution of equation (8) can be calculated to be of the form

$$\varphi(\xi) = \kappa_1 \sin \alpha\xi + \kappa_2 \cos \alpha\xi + \kappa_3 \sinh \alpha\xi + \kappa_4 \cosh \alpha\xi\,, \tag{10}$$

In order the function (10) to be a solution of equation (8) one must take $\alpha^2 = \upsilon$. The four boundary conditions yield a nonhomogeneous linear algebraic system in the four unknown coefficients $\kappa_1, \kappa_2, \kappa_3, \kappa_4$ .The solution exists only if the determinant of this system does not vanish, i.e. if

$$\Delta = 2(\cos \alpha \cosh \alpha + 1) \neq 0\,. \tag{11}$$

The characteristic equation $\Delta = 0$ is satisfied for an infinite number of irrational values α_n (eigenvalues). The corresponding eigenfrequencies (natural frequencies) of the cantilever beam are given by

$$\omega_n = \frac{\alpha_n^2}{l^2} \sqrt{\frac{EI}{\rho A}}\,, \ n = 1,2,3,... \tag{12}$$

Since the approximate values of the first two eigenvalues are $\alpha_1 \cong 1.875$ and $\alpha_2 \cong 4.694$, the second natural frequency is almost 6.3 times higher than the first one.

The subsequent analysis is carried out for forcing frequencies situated in the neighborhood of the first natural frequency of the beam. In this case, the contribution of the higher vibration modes can be neglected and the beam motion is sufficiently well described by (7) and (10) for $\alpha^2 = \upsilon$.

Substitution of (10) into (9) and solving the obtained system for the unknown coefficients $\kappa_1, \kappa_2, \kappa_3, \kappa_4$, yields

$$\kappa_1 = \frac{\sin \alpha \cosh \alpha + \cos \alpha \sinh \alpha}{2(\cos \alpha \cosh \alpha + 1)}\,, \ \kappa_2 = \frac{\cos \alpha \cosh \alpha - \sin \alpha \sinh \alpha + 1}{2(\cos \alpha \cosh \alpha + 1)}\,, \ \kappa_3 = -\kappa_1, \ \kappa_4 = 1 - \kappa_2 \tag{13}$$

The displacement transmissibility, i.e. the ratio of the maximum response amplitude at the free end to the input displacement magnitude expressed as a function of the forcing frequency υ, is given by

$$\Phi(\nu) = \left| \kappa_1 \sin \sqrt{\upsilon} + \kappa_2 \cos \sqrt{\upsilon} + \kappa_3 \sinh \sqrt{\upsilon} + \kappa_4 \cosh \sqrt{\upsilon} \right| \tag{14}$$

Here this relation is used in the neighborhood of the first natural frequency of the beam $\upsilon_1 = \alpha_1^2 \cong 3.516$, to find the frequencies below and above resonance frequency for which the same displacement amplification is obtained (see Fig.4).

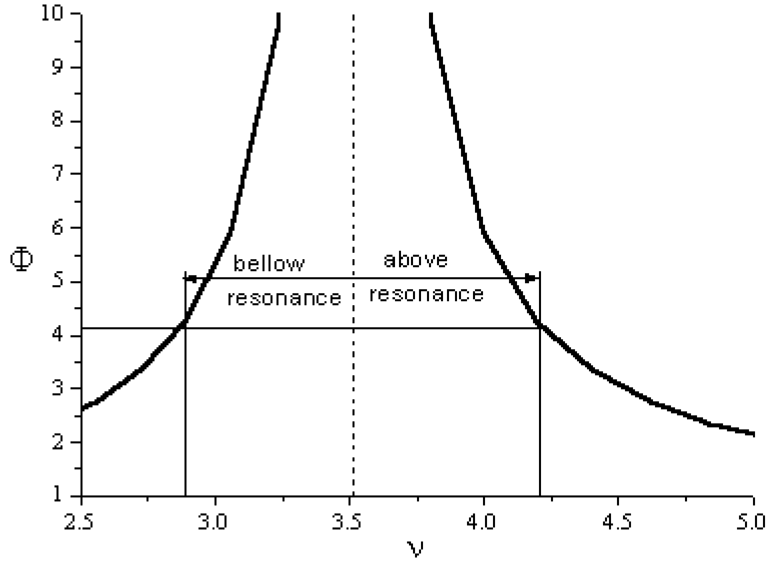

Fig. 4 Displacement transmissibility in the neighborhood of the first natural frequency.

4. NUMERICAL RESULTS

In this section are shown the results obtained for the equal values of the displacement transmissibility reached below and above the first natural frequency $\upsilon_1 \cong 3.516$:

$$\Phi(2.89) = \Phi(4.203) = 4.25 \tag{15}$$

If this shifting in the vibration regime is due only to the degradation of the beam bending stiffness and not to any modification of the forcing frequency, then it implies a 2.1 times decreasing of EI .

The maximum amplitude of the beam vibration below and above resonance, obtained for the excitation amplitude $\eta_0 = 1$, is shown in Fig.5. It is easily seen that the curves plotted in this figure are in good agreement to the vibration patterns obtained experimentally (see Fig.2). The beam motion above resonance display a node at approximately $0.36l$, where l is the beam length. The node position depends on the ratio of the forcing frequency to the first natural frequency of the beam, being shifted towards the beam free end when this ratio increases. The existence of fixed points of the vibrating beam in the above resonance range is due to the neglect of the beam internal damping in the analytical model. In practice, a pure node does not exists because it migrates continuously within a small length interval as function of the instantaneous values of the beam displacement. This is due to the beam hysteretic damping, which depends essentially of the shear strain and produces a displacement dependent phase shift.

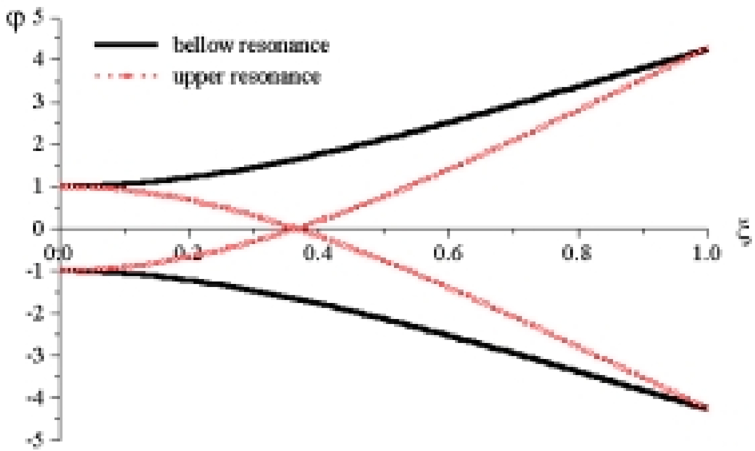

Fig. 5 Maximum amplitudes of beam vibration below and above resonance.

The maximum values of the bending moment and the shear force along the beam are proportional to $\varphi''(\xi)$ and $\varphi'''(\xi)$, respectively. These two functions are plotted in Figs.6,7.

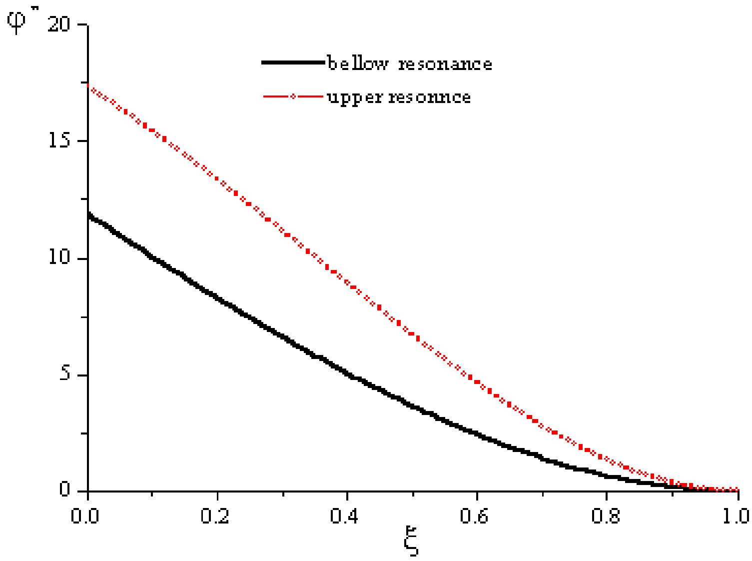

Fig. 6 Variation of the bending moment along the beam.

As expected, the maximum value of the bending moment is reached at the clamping point. However, it is worth noting that for equal amplitudes of the free end, the bending moment along the beam is bigger for vibration above resonance than below resonance.

The variation of the shear force along the beam displays two different patterns when the beam vibrates bellow or above resonance.

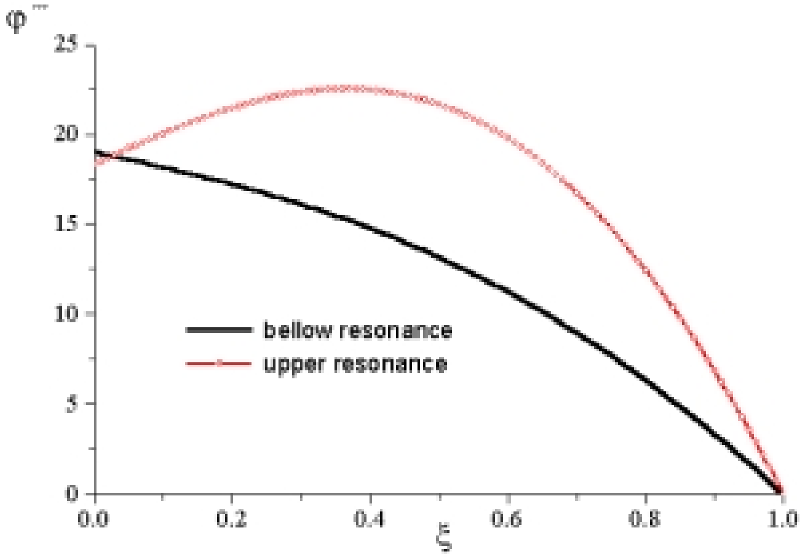

Fig. 7 Variation of the shear force along the beam.

Bellow resonance the shear force decreases along the beam, while for above resonance vibration it displays a maximum value at the node position. From the analytical point of view, this result is a direct consequence of the existence of a fixed point $\xi_1 \in (0,1)$ such that

$$\varphi(\xi_1) = -\upsilon^2 \varphi''''(\xi_1) = 0 \tag{3.16}$$

Physically, the presence of a node above resonance is due to the phase shift between the harmonic motions of the clamped base and the free end, produced mainly by the inertial properties of the beam and, in much less extent, by the internal damping of the beam material.

It should be mentioned that at the clamping point the shear force has practically the same value in the both cases. However, the maximum value of the shear force in the above resonance range is almost 18% higher than the maximum value reached bellow resonance at the clamping point. Comparing the shear force values reached at the node, one obtains even a higher difference (46%).

5. CONCLUSIONS

The vibrations of slender uniform cantilever beams with imposed harmonic motion of the clamped end were investigated both analytically and experimentally.

The analytical and experimental results showed different vibration patterns when the beam vibration is excited bellow or above the first natural frequency of the beam. While bellow resonance, the harmonic motions of the clamped and free ends end base are in phase, above resonance these motions are out of phase, displaying a node which shifts toward the free end, when the ratio of driving frequency to the beam natural frequency increases.

For same displacement transmissibility, for vibration regimes above resonance both bending moment and shear force along the beam are bigger than in the case of vibration bellow resonance. Moreover, above resonance the shear force has a maximum value at the node position, which for reasonable values of the displacement transmissibility is located around one third of the beam length. The maximum value of the shear force could be about 20% bigger than the value encountered at the clamped end (almost the same in both cases) and about 45-50% higher at the node position.

If a cantilever beam with base imposed motion is employed as a simplified model of a multiple storey building, then one can conclude that building response excited by the ground motion above resonance could be more damaging to the structural elements than if this response is excited bellow resonance. In both cases, it is assumed that the ground motion has spectral components with periods around the first natural period of the building.

In case of a building affected by a slow earthquake such as Vrancea intermediate earthquakes, or of a building founded on a soft soil and affected by a fast earthquake, the local damage of the building will lead to the increase of the building natural periods. By accepting the controlled damaging of a conventional building due to material degradation by over-loading (plastic hinges), the damaged building vibration periods can increase about 2 times (e.g. from 0.2 – 0.4 sec to 0.4 – 0.8 sec, [1,2]). In this case, the building vibration regime could change from the one bellow resonance to the one above resonance. This dynamic behaviour could be a possible explanation of the building breakages between the 2nd and 3rd floor.

Further work should consider the Timoshenko beam model, including the effect of rotary inertia, shear deformation and damping [3,4], which is more appropriate to portray the building dynamic response to earthquake inputs.

ACKNOWLEDGEMENT

The author would like to express his gratitude to the Romanian Academy for supporting this work through the Grant no.93/2005. The stimulating discussions with dr.Viorel Serban over this subject are also highly appreciated.

REFERENCES

1. SERBAN, V., PANAIT, A., SIRETEANU,T., STOICA, M., *SERB-B type devices for controlling the joints and the distortions of the spatial structure,* Proceedings of IASS 05, Bucharest, Sept. 6-pp.983-980, 2005.
2. SERBAN, V., PANAIT, A., SIRETEANU,T., CRETU, D., *SERB-B type devices for rehabilitation of historical monuments,* Proceedings of IASS 05,Bucharest, Sept.6- pp. 913-920, 2005.
3. INMAN, J.D., *Engineering Vibration*, Prentice Hall International, Inc., 1994.
4. PANKOVO, G.I., *Fundamentals of Applied Theory of Vibration and Waves* (in Russian), Machine Construction Printing House, Leningrad, 1976.

Received September 19, 2005