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SEISMIC BASE ISOLATION – NON-LINEAR IMPLICATIONS IN PERIOD-SHIFT CHOICE

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The main goal of the base-isolated technology is to shift the natural period of structure away from the dominant period of earthquake excitation. The objective of this paper is to analyses the effects on period-shift evaluation due to non-linear characteristics of devices and materials from isolation layer of the base-isolated structures. Using a linear one-degree-of-freedom model for structure with fixed base and a non-linear one-degree-of-freedom model for base-isolated structure one presents some remarks for optimum choice of the isolated natural period under non-linear properties of isolator layer.

1. INTRODUCTION

The main feature of the base-isolation technology is that it introduce between superstructure and his foundation a properly chosen flexible layer in order to shift the natural period of structure away from the dominant period of earthquake ground motion and thus to avoid the destructive effects given by the system resonance.

As a result of flexibilization, the natural period of the past fixed-base structure undergo a jump and the new base-isolation structure has a new natural period. The flexibility of the interposing layers between structure and its foundation lead to a bigger fundamental period for structural ensemble. From this reason, base isolation technology is not suitable for all buildings. Most suitable candidates are low to medium-rise rested on hard soil underneath, high-rise buildings or buildings rested on soft soil are not suitable for base isolation.

This “period-shift” due to isolation layer depends on strength and damping characteristics of the materials or devices from isolation layer. It is an indubitable experimental reality that all of devices and materials used in isolation layers of the base-isolated structures exhibit, more or less, a non-linear behaviour. Thus, in the period-shift evaluation these non-linear properties must take into account.

Apparently, large period shifts assure a better dynamic response. But, the excessive flexibilization of the structural ensemble may lead to the weak dynamic response. Thus, the shift amount must so design that assures a reasonable reduced dynamic magnification under a reasonable strength reduction.

Based on results presented in [14], in a previous author's paper [5] was discussed some reasons that enable us to modellig the structures with fixed base as linear single-degree-of-freedom system (sdof) and for base-isolated structures was proposed a sdof system base on non-linear Kelvin-Voigt model [2]. In present paper these results are accepted and used. The same models, linear sdof for fixed-base behaviour and non-linear sdof for isolated-base structure, will now used in order to evaluate the influence of the non-linear properties of the isolation layer on period-shift determination.

Two is the main objectives of this paper. First is to present a method for necessary period-shift determination based on linear and/or nonlinear magnification functions and second objective is to evaluate the effects of the non-linear strength and damping functions of the isolatory materials on period-shift design.

Recommended by Radu P.VOINEA, member of the Romanian Academy

2. LINEAR ASSESSMENT OF PERIOD-SHIFT

For a qualitative evaluation of the dynamic behaviour one can consider the structure with fixed base as a linear sdof subjected to harmonic abutment accelerations:

$$\ddot{x}_g(t) = \ddot{x}_g^0 \sin \omega t \tag{2.1}$$

where $\ddot{x}_g^0$ is the acceleration amplitude (usually relate to peak ground accelerations - *PGA*) and ω is the pulsation (circularly frequency) of the excitation.

In linear dynamics a usual description of such sdof behaviour is given by the Kelvin-Voigt model consisting of a mass *m* supported by a spring (with a stiffness *k*) and a dashpot (with a viscosity *c*) connected in parallel (fig.2.1). The governing equation of this system is [8], [10]:

$$m\ddot{x} + c \cdot \dot{x} + k \cdot x = -m\ddot{x}_g\,, \tag{2.2}$$

or:

$$\ddot{x} + 2\zeta\omega_0\dot{x} + \omega_0^2 x = -\omega_0^2\ddot{x}_g\,. \tag{2.3}$$

where

$$\omega_0 = \sqrt{\frac{k}{m}} \tag{2.4}$$

is undamped natural pulsation and

$$\zeta = \frac{c}{c_{cr}} = \frac{c}{2m\omega_0} \tag{2.5}$$

is the damping ratio.

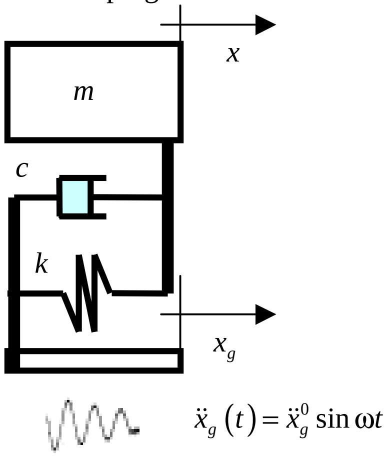

Fig. 2.1 Sdof system with abutment excitation

By using the change of variable $\tau = \omega_0 t$ and by introducing a new "time" function [2]:

$$\varphi(\tau) = x(t) = x\left(\frac{\tau}{\omega_0}\right) \tag{2.6}$$

one obtain for eq. (2.3) another form:

$$\varphi'' + C\varphi' + K\varphi = \mu \sin \upsilon\tau \tag{2.7}$$

where the superscript accent denotes the time derivative with respect to τ:

$$\varphi'(\tau) = \frac{\partial\varphi}{\partial\tau} = \frac{1}{\omega_0}\dot{x} \quad ; \quad \varphi''(\tau) = \frac{\partial^2\varphi}{\partial\tau^2} = \frac{1}{\omega_0^2}\ddot{x} \tag{2.8}$$

and:

$$C = \frac{c}{m\omega_0} = 2\zeta \quad ; \quad K = \frac{k}{m\omega_0^2} = 1 \quad ; \quad \mu = \frac{\ddot{x}_g^0}{\omega_0^2} \quad ; \quad \upsilon = \frac{\omega}{\omega_0} \tag{2.9}$$

Steady-state solution of the equation (2.7) read as:

$$\varphi(\tau,\upsilon,\zeta) = \mu\Phi(\upsilon,\zeta)\sin(\upsilon\tau - \psi) \tag{2.10}$$

where $\Phi(\upsilon,\zeta)$ is *the magnification function:*

$$\Phi(\upsilon;\zeta)=\frac{1}{\sqrt{\left(1-\upsilon^2\right)^2+\left(2\zeta\upsilon\right)^2}} \tag{2.11}$$

This function for $\zeta = ct.$ have the typical aspect depicted in fig. (2.2).

Usually, in structural dynamics is used the natural T_0 or impute T period instead of natural ω_0 or excitation ω pulsation. Because $T = 2\pi/\omega$ the dimensionless pulsation υ become:

$$\upsilon=\frac{\omega}{\omega_0}=\frac{T_0}{T} \tag{2.12}$$

and the magnification function can be expressed as:

$$\Phi(T;T_0,\zeta)=\frac{1}{\sqrt{\left[1-\left(\frac{T_0}{T}\right)^2\right]^2+\left[2\zeta\left(\frac{T_0}{T}\right)\right]^2}} \tag{2.13}$$

The typical aspect of this function for $\zeta = ct.$ and $T_0 = ct.$ is light different from (2.11) and read as in fig. 2.3.

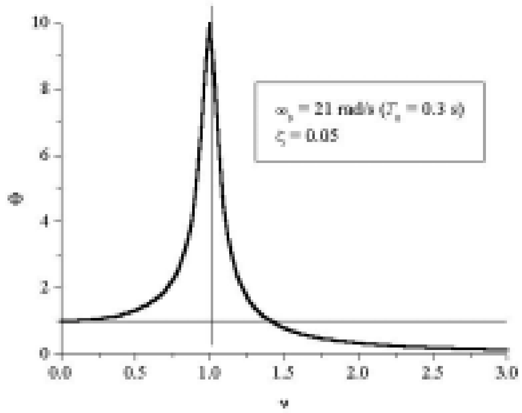

Fig. 2.2 $\Phi=\Phi(\upsilon)$

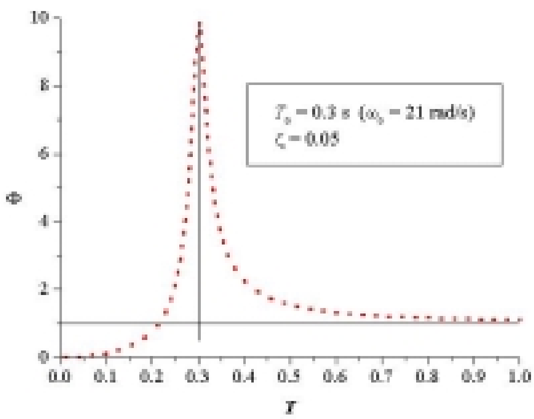

Fig. 2.3 $\Phi=\Phi(T)$

The magnification function can be used to illustrate the structural behaviour before and after period jump, that is to illustrate the behaviour differences between structure with fixed base and the same structure but with isolator layer. To illustrate this magnification functions ability, in the next, a case study will be presented.

Let a structure with following dynamic characteristics:

$$\begin{aligned}
&m = 30000\ \text{kg};\\
&T_0 = 0.3\ \text{s} \quad \Rightarrow \quad \omega_0 = \frac{2\pi}{T_0} \simeq 21\ \text{rad/s} \quad \Rightarrow \quad k_s = 13230000\ \text{N/m};\\
&\zeta = 0.05 \quad \Rightarrow \quad c = 2m\omega_0\zeta = 63000\ \text{Ns/m},
\end{aligned} \tag{2.14}$$

If a such structure is supported on a usual site, composed, for example, by consolidated aluvionary deposits with $T_0 \simeq 0.3 \div 0.5$ s, this structure become a proper candidate for isolated base technology.

Assuming this structure with fixed base as linear sdof system the magnification function (2.13) become (fig. 2.3):

$$\Phi(T)=\frac{1}{\sqrt{\left[1-\left(\frac{0.3}{T}\right)^2\right]^2+\left[2\cdot 0.05\left(\frac{0.3}{T}\right)\right]^2}} \tag{2.15}$$

As is expected the maximum dynamic magnification is meet at resonance, when excitation period T is nearly equal with natural period T_0, and before and after resonance the magnification is much shortened.

To avoid the resonance, the natural period T_0 must to modify in order to become farther from excitation period ($T_0 \neq T$). One can see in fig. 2.4 that natural period increase leads to reduce dynamic structural magnifications for the same excitation period T.

Due to vagueness in excitation period evaluation is it advisable to consider an excitation zone around natural period and new structural period must keep out from this zone. Also, the strong non-linear effect from site soils deposits lead to expanded resonant zone (This effect will be explain in chapter 4). In the case used for exemplification this excitation zone was considered beetwen $T = 0.2$ s and $T = 0.5$ s (fig. 2.4). Thus, the selection of the natural period may begin at least $T = 0.6$ s . But, from non-linear considerations, which be so far demonstrate, we choose now for isolated base structure the natural period $T = 1$ s (fig. 2.5).

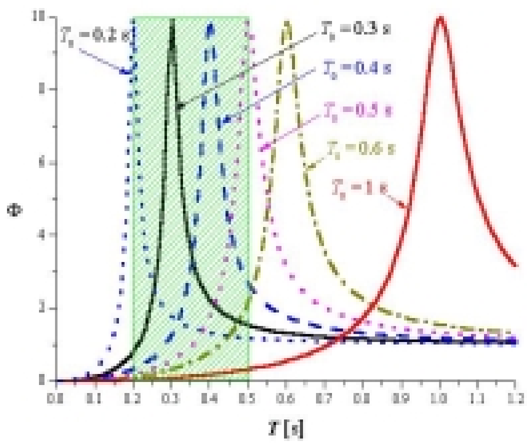

Fig. 2.4 Magnification functions for different natural periods

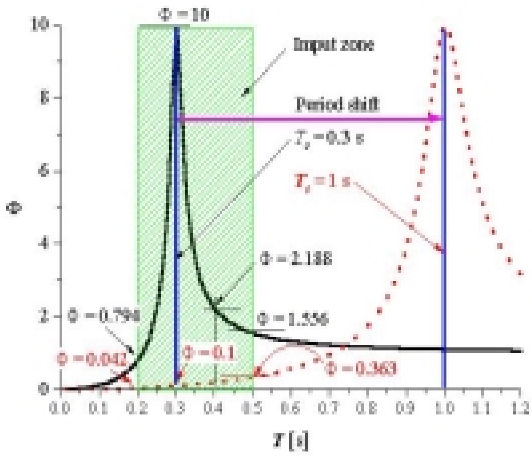

Fig. 2.5 Natural period shift from 0.3 s to 1 s

In fig. 2.5, as well as in table 2.1, one can see the isolated base efficiency, disclosed by drastic reduction of the dynamic magnification. Also, the efficiency of isolation emerge from displacements and accelerations reductions (fig.2.6 and 2.7).

Table 2.1 Isolated base efficiency

Impute period [s]	*Dynamic magnification*	
	Fixed base ($T_0 = 0.3$ s)	*Isolated base* ($T_0 = 1$ s)
0.2	0.794	0.042
0.3	**10.000**	**0.100**
0.4	2.188	0.234
0.5	1.556	0.363

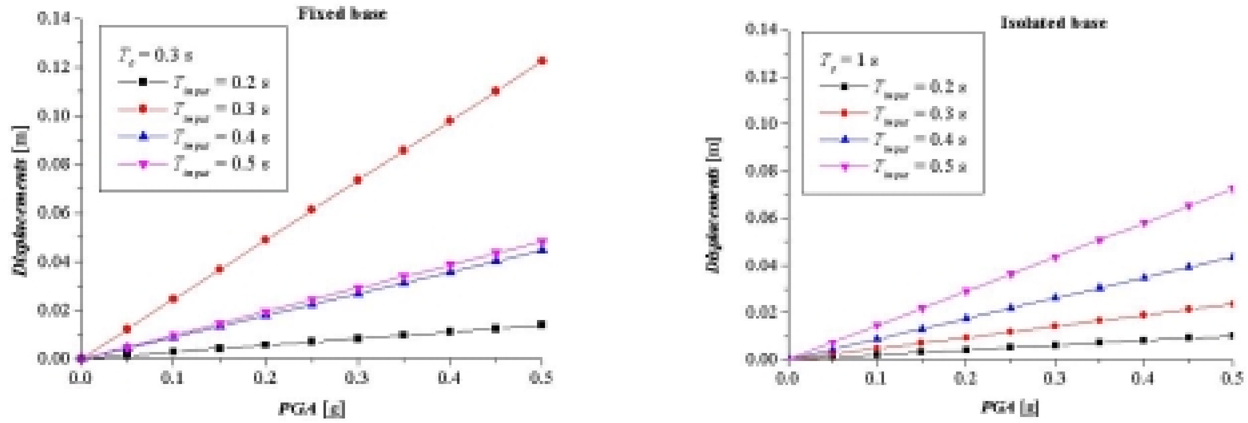

Fig. 2.6 Displacements of fixed and isolated base

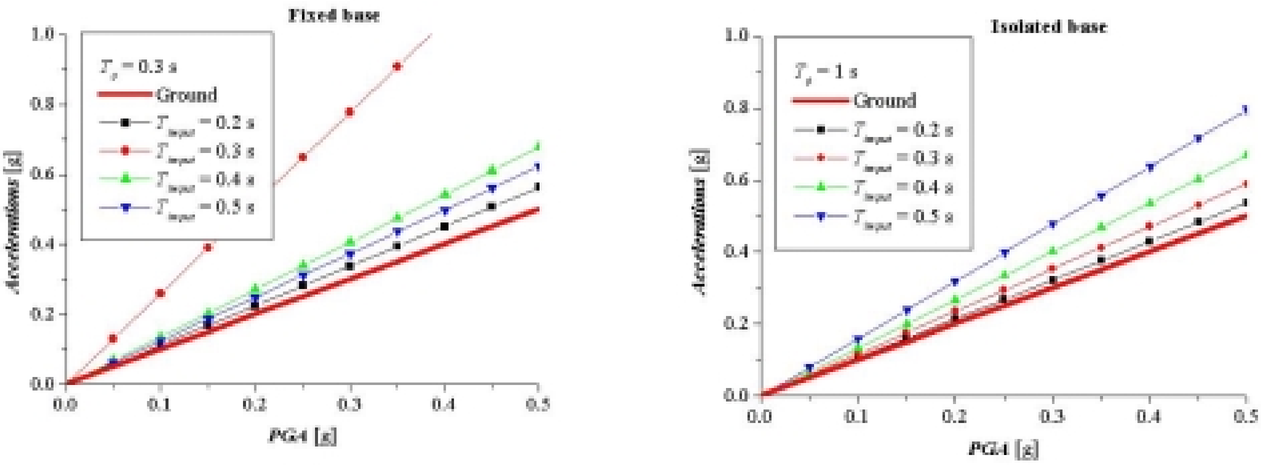

Fig. 2.7 Acceleration of fixed and isolated base

3. EFFCTS OF THE STRUCTURAL DAMPING ON PERIOD-SHIFT

Certainly, the structural damping capacity plays a topmost role in dynamic structural response. But, now examine the damping role in period-shift choice. For this let examine fig. 3.1 where is presents two kind of magnification function, one for damping ratio $\zeta = 0.05$ and other for $\zeta = 0.1$, both for $T_0 = 0.3$ s and $T_0 = 1$ s.

As can see from this figure the damping increase lead to severe reduction of the resonance magnification but outside resonance the damping effect become negligible. The magnification functions for both damping values and for both fixed and isolated structure are very closed outside resonance. Hence, a practical conclusion result – the combined seismic protection technologies, base isolation + dampers attach, appear to be not recommended.

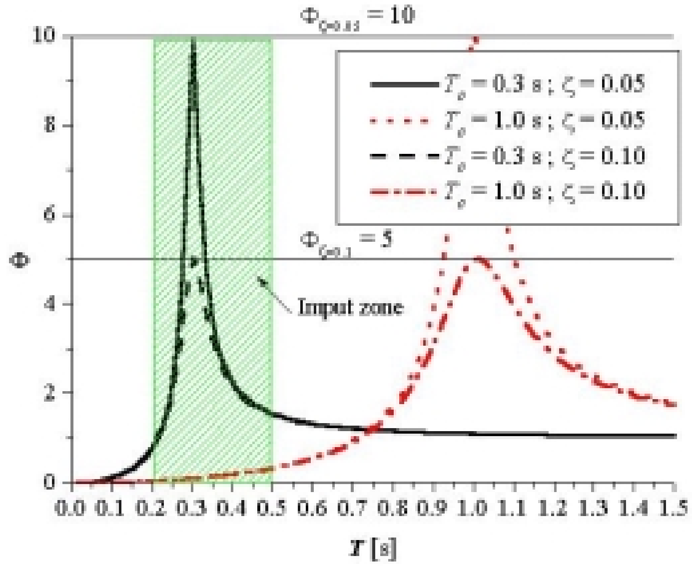

Fig. 3.1 – Effect of damping

4. EFFECTS OF NON-LINEARITY ON PERIOD-SHIFT

In chapter 2 the assessment of the period-shift amount was based on linear behaviour hypothesis of the fixed-base structure. But, an isolated-base structure has non-linear behaviour due to the nonlinearity of the isolator layer. This different behaviour may modify the linear estimate of the period-shift.

For make apparent these differences a numerical simulation study was performed using the non-linear Kelvin-Voigt model presented in [5]. This model is build up as an extension in non-linear domain of the linear Kelvin-Voigt model and is based on replacement of the dynamic linear characteristics – the spring stiffness *k* and the damper viscosity *c*, by functions in terms of displacements experimentally determinates $k = k(x)$ and $c = c(x)$ [2], [3], [4].

In order to compare the non-linear results with the linear calculus from chapter 2 the initial values of the non-linear damping function was scaled for coincide with constant values of the linear behaviour hypothesis. For this reason the initial values of the damping function was his linear values $\zeta_0 = \zeta(0) = \zeta(x)\big|_{x=0} = 0.05$. Also, the initial value of stiffness function was put in correspondence with the natural period of the isolated-base structure:

$$T_0 = 1\ \text{s} \quad \Rightarrow \quad \omega_0 = \frac{2\pi}{T_0} \simeq 6.28\ \text{rad/s} \quad \Rightarrow \quad k_0 = m\omega_0^2 = 30{,}000 \times 6.28^2 = 1{,}183{,}152\ \text{N/m} \tag{4.1}$$

For this simulation the non-linear aspect of these material functions was builder by extension starting to own test performed upon rubber [3] and experimental data performed upon rubber isolators given in [1], [6], [11], [17]. Also, the material function with middle non-linearity was adopted.

Thus, for softening nonlinearity type was chosen:

$$k_n(x) = 0.5 + 0.5\exp(-50x) \tag{4.2}$$

where $k_n(x) = k(x)/k(0)$ is the non-linear stiffness normalized in rapport with initial value $k_0 = k(0)$. For hardening nonlinearity type was adopted the following form:

$$k_n(x) = 0.5 + \left[1 - 0.5\exp(-50x)\right] \tag{4.3}$$

In both cases the damping function was the same (fig. 4.1):

$$\zeta(x) = 0.1 - 0.05\exp(-50x) \tag{4.4}$$

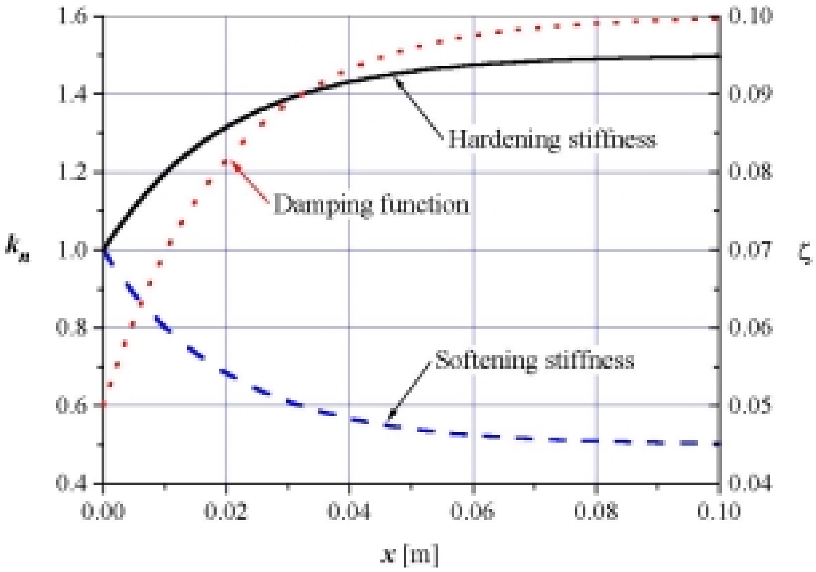

Fig. 4.1 Non-linear material functions

Using these dynamic material functions, $c(x)$ and $k(x)$, the differential equation of the non-linear sdof system can be write as an extension of eq. (2.2):

$$m\ddot{x} + c(x)\cdot\dot{x} + k(x)\cdot x = -m\ddot{x}_g\ , \tag{4.5}$$

or by extension of the eq. (2.3):

$$\ddot{x} + 2\omega_0\zeta(x)\cdot\dot{x} + \omega_0^2 k_n(x)\cdot x = -\ddot{x}_g\ . \tag{4.6}$$

Using the same change of variable $\tau = \omega_0 t$ and the "time" function:

$$\varphi(\tau) = x(t) = x\left(\frac{\tau}{\omega_0}\right) \tag{4.7}$$

one obtain for eq. (4.5) or eq. (4.6) another form:

$$\varphi'' + C(\varphi)\cdot\varphi' + K(\varphi)\cdot\varphi = \mu\sin\upsilon\tau \tag{4.8}$$

where the superscript accent denotes the time derivative with respect to τ as in linear case:

$$\varphi'(\tau) = \frac{\partial\varphi}{\partial\tau} = \frac{1}{\omega_0}\dot{x} \quad ; \quad \varphi''(\tau) = \frac{\partial^2\varphi}{\partial\tau^2} = \frac{1}{\omega_0^2}\ddot{x} \tag{4.9}$$

but in this nonlinear case the coefficients (2.8) of the eq. (2.7) are replaced by functions in term of φ:

$$C(\varphi) \equiv C(x) = \frac{c(x)}{m\omega_0} = 2\zeta(x) \quad ; \quad K(\varphi) \equiv K(x) = \frac{k(x)}{m\omega_0^2} = \frac{k(x)}{k(0)} = k_n(x) \tag{4.10}$$

and the transformed amplitude μ and the normalized pulsation υ was remained in the same linear form:

$$\mu = -\frac{\ddot{x}_g^0}{\omega_0^2} \quad ; \quad \upsilon = \frac{\omega}{\omega_0} \tag{4.11}$$

The solution of the equation (4.8) can be write like eq. (2.10):

$$\varphi(\tau,\upsilon,\mu,\zeta) = \mu\Phi(\upsilon,\mu,\zeta)\sin(\upsilon\tau - \psi) \tag{4.12}$$

but both solution $\varphi(\tau,\upsilon,\mu,\zeta)$ and the magnification function $\Phi(\upsilon,\mu,\zeta)$ become loads dependents functions, through transformed amplitude μ.

For given amplitude μ and relative pulsation υ, the non-linear equation (4.8) can numerically solved using a computer program [4] based on Newmark algorithm [13]. After this, from known solution and known excitation, the non-linear magnification functions was obtained first in term of normalized pulsation υ and then in term of period:

$$\Phi(\upsilon,\mu,\zeta) = \Phi(\upsilon;\zeta)\Big|_{\mu=ct.} \quad \Rightarrow \quad \Phi(T;\zeta)\Big|_{\substack{\mu=ct. \\ T_0=ct.}} \tag{4.13}$$

The non-linear magnification functions thus obtained can be used to evaluate the non-linear effects on resonant peak amplitudes due to non-linear mechanical characteristics of the materials from both isolated layer and site soil deposits. These effects will be expose with the aid of the numerical simulation results using the same base isolated structure situated by the same site (with natural period within 0.3 and 0.5 s).

If the excitation period is close to natural soil period, let $T = T_0 = 0.3$ s the resonance occur. For the linear magnification function this resonance is meet at 0.3 s period. But, taking into account the non-linear characteristics of the soils materials, the peak amplitude of the non-linear magnification functions depend on excitation amplitude μ and the resonance peaks occurs at different periods situated after the excitation period (usually soils has softening nonlinearity). This typical soils behaviour are illustrated in fig. 4.2 [4]. From this reason in chapter 2 was recommended the usage of impute zone.

Such peaks dispersion are meet already on isolatory materials or devices used for base-isolated structures [3], [4] and this non-linear behaviour affect the entire response of the base isolated system, inclusive the period-shift determination [5] (fig. 4.3).

5. CONCLUDING REMARKS

The results summarized in fig. 4.2 and 4.3 allow us the following remarks:

- The non-linear magnification functions have different shapes in comparison with the linear one. The resonant amplitude peaks are displaced towards high periods for softening stiffness and towards low periods for hardening stiffness.
- At resonance, the peak amplitudes of the non-linear magnification functions are inferior to the peak amplitude of linear magnification function, thus non-linear calculus makes apparently the damping capacity neglected by linear calculus.
- Because the structure was take out from dangerous impute zone the differences between linear and non-linear magnification of the same base-isolated structure are not significant for impute periods remained in initial impute zone.
- Whereas the linear calculus leads to the unique resonance value, the non-linear calculus lead to the multiple resonant values (in terms of excitation amplitudes) situated before or after linear resonant value.
- For period-shift jump choice these multiple resonant peaks may generate some difficulties. While in the softening nonlinearity case the peak displacements are added to linear jump in the hardening nonlinearity case the peak displacement decrease the linear estimated jump. Thus, in both cases the initial linear evaluation of necessary period-shift (as was presented in chapter 2) must be revalued.

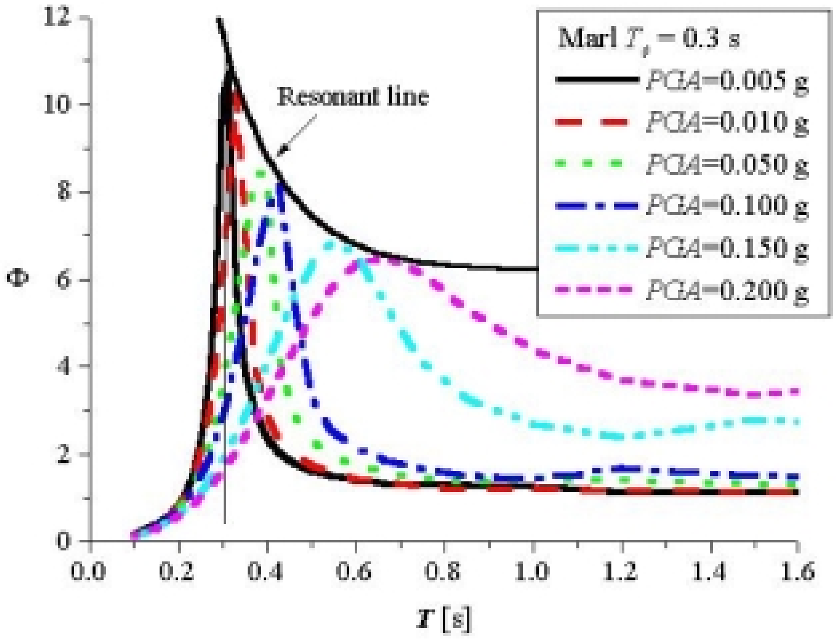

Fig. 4.2 Non-linear magnification functions for a site material

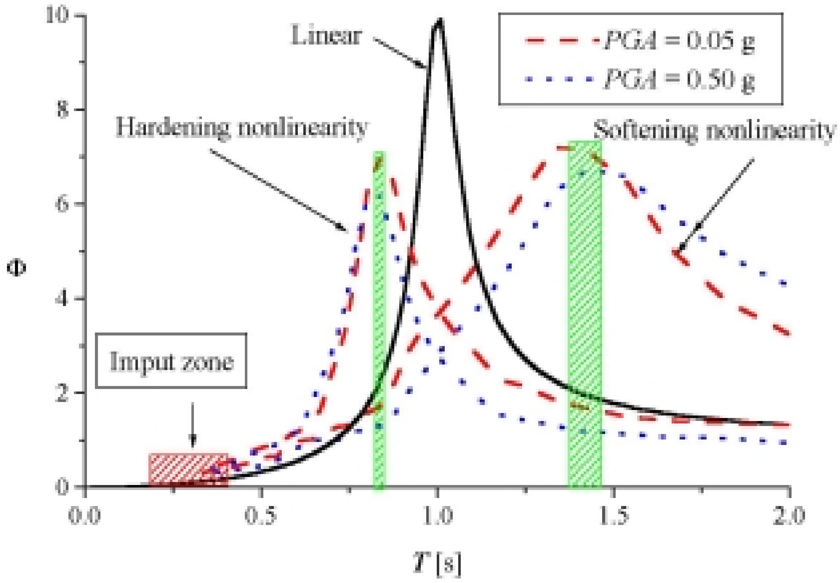

Fig. 4.2 Non-linear magnification functions for a base-isolated structure

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REFERENCES

1. AIKEN I.D., *Testing of seismic isolators and dampers - considerations and limitations,* Proc. Structural Engineering Congress, San Francisco, California, July, 1998.
2. BRATOSIN D., SIRETEANU T., *A nonlinear Kelvin-Voigt model for soils,* Proceedings of the Romanian Academy, *3*, 2002.
3. BRATOSIN D., *On dynamic behaviour of the antivibratory materials,* Proceedings of the Romanian Academy, **4**, 3, pp.205-210, 2003.
4. BRATOSIN D., *Nonlinear aspects in soils dynamics, cap.2* in *Topics in applied mechanics, vol.1,* (editors Veturia CHIROIU, Tudor SIRETEANU), Publishing House of the Romanian Academy, 2003.
5. BRATOSIN D., *Non-linear effects in seismic base isolation,* Proceedings of the Romanian Academy, **5**, 3, pp.297-309, 2004.
6. BURTSCHER S., DORFMANN A., BERGMEISTER K., *Mechanical aspects of high damping rubber,* 2nd Int. Symposium in Civil Engineering, Budapest, 1998.
7. CARNEIRO J.O., DE MELLO F.J.Q., JALALI S., CAMANHO P.P., *Analytical dynamic analysis of earthquake base-isolation structures using time-history superposition,* Proc. of the I MECH E Part K Journal of Multi-body Dynamics, vol.218, no.1, 2004.
8. CHEN Wai-Fah (ed.), *Structural Engineering Handbook,* CRC Press LLC, 1999.
9. DEB S.K., *Seismic base isolation - an overview,* CPFTEGE, December, 2003.
10. deSILVA, C., *Vibrations: Fundamentals and Practice*, CRC press, 2000.
11. JAIN S.K., *Quasi-static Testing of Laminated Rubber Bearings*, IE Journal, 84, August 2003.
12. KUNDE M.C., JANGID R.S., *Seismic behaviour of isolated bridges: A-state-of-art review,* Electronic Journal of Structural Engineering, 3, 2003
13. LEVY S., WILKINSON J.P.D., *The component element method in dynamics*, McGraw-Hill, 1976.
14. MIRANDA C.J., *Structural dynamics of base isolated buildings*, IDC Technical Papers, Presentation & Articles, August, 2001.
15. OLARIU I., *Passive control and base isolation : State-of-art lecture*, 10th European Conference on Earthquake Engineering, Vienna, 1995.
16. RAMALLO J.C., JOHNSON E.A., SPENCER B.F., *"Smart" Base Isolation Systems,* Journal of Engineering Mechanics, 1088, October, 2002
17. RAMORINO G., VETTURI D., CAMBIAGHI D., PEGORETTI A., ROCCO T., *Developments in dynamic testing of rubber compounds: assessment of non-linear effects*, Polymer Testing 22, pp.681-687, Elsevier, 2003.

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