PUBLISHING HOUSE
OF THE ROMANIAN ACADEMY

PROCEEDINGS OF THE ROMANIAN ACADEMY, Series A,
Volume 3, Number 3/2002.

ON THE ISOBARIC SPIN AND THE SCATTERING MATRIX

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The Isobaric Spin and the Scattering Matrix are fundamental Nuclear Physics Concepts invented by Werner Heisenberg. The Cardinal Impact, principial and methodological, of the Heisenberg Concepts on historical developement of Nuclear Physics and other Quantum and Classical Physics Fields is discussed in this communication.

1. INTRODUCTION

Heisenberg in Physics is synonimus to Monumental Scientific Creations:

-"Creation of Quantum Mechanics", (Nobel Prize, 1932)-"Heisenberg Relations", or "Heisenberg Inequalities" or "Uncertainty Principle" or "Indeterminacy Principle" - Basis for Copenhagen Interpretation of Quantum Mechanics-"World Formula" - Project for an Unitary Theory representing all existing particles; it would exhibit a Basic Set of Universal Symmetries in Nature Heisenberg does signify also Important/Cardinal Contributions to many Fields of the Physics:

-Hydrodynamical Theory of Turbulence, (Dissertation, Sommerfeld) -Theory of Ferromagnetism-Study of Cosmic Rays-Nuclear Physics Heisenberg has invented two Nuclear Physics Concepts: Isobaric Spin and Scattering Matrix. The Heisenberg Concepts became cornerstones of the two main Fields of the Nuclear Theory, namely the Nuclear Structure (Spectroscopy) and the Nuclear Reactions. This Communication intends to illustrate the Impact, both principial and methodological, of the Heisenberg Concepts on Developement of Nuclear Physics.

2. THE ISOBARIC SPIN

1. Nuclear Physics *a.* Heisenberg

Rutherford, studying the Structure of the Atom, has proposed the Planetay Model: a heavy Atomic Nucleus with electric positive charge and light Electrons with electric negative charges, gravitating around Nucleus. The Proton,a heavy particle (~ 2000 heavier than Electron), with electric positive charge, is a constituent of the Nucleus. Rutherford (in order to explain both the charge and the mass of Nucleus) has proposed also a hypothesis known as the Proton- Electron Model of the Atomic Nucleus. This model was confronted with principial difficulties in explaining experimental facts, as e.g. the Nitrogen Catastrophe.

In 1932 Chadwick has discovered the Neutron. But according to Chadwick the Neutron is a composite particle: a dipole Proton-Electron, obviously, electrical- neutral.

2. Proton -Neutron Model of Atomic Nucleus

It is merit of Heisenberg who realized that the Neutron is rather a "selbststaendinger Fundamentalbestandteil" of Atomic Nucleus. Heisenberg has proposed the Proton-Neutron Model of Atomic Nucleus: the Protons and Neutrons are the only constituents of the Nucleus.The Nucleus electric charge is given by Protons; the mass of the nucleus is given both by Protons and Neutrons. According to historians of Physics, this result "marks the begining of the Theory of Nuclear Structure"

Heisenberg has published his results in a work entitled "Ueber den Bau der Atomkerne" [Z. Phys. 77(1932)1]. The Proton-Neutron Model was also proposed by Soviet Physicist, Ivanenko ["The Neutron Hypothesis", Nature 129(1932)798]. But the Heisenberg's work was more complete and profound.

3. Nucleon

Moreover Heisenberg realized that the two constituents of the Nucleus, the Proton and the Neutron. might be regarded as two states (or substates) of an elementary particle, the Nucleon The dichotomic variable associated with the two substates was, later on, named Isospin, Isotopic Spin or Isobaric Spin. The Isospin hypothesis did assume that, except electric charge properties, all other Nucleon internal degrees of freedom, (as e.g. the mass, the intrinsec Spin), are same both for Neutron and Proton.

The mathematical apparatus for describing variables which do bifurcate was already to hand. Pauli used it to describe the Electron's Spin with the two "magnetic" (sub)states. The Heisenberg's Isobaric Spin is a labelling device which differentiates between the two charge (sub)states of Nucleon. The Isospin vector **t** does operate not in real space but rather in Isospin Space ; the Isospin Space is another concept introduced by Heisenberg. The Isospin vector **t**, operating in Isospace, has two "charge" projections correseponding either to Neutron (t_3 = + 1/2) or Proton (t_3 = - 1/2) The electric charge Q of Nucleon is related to Isospin projections by a linear relation $Q = 1/2 - t_3$, i.e. $Q = 0$ (n) and $Q = 1$ (p).

As the electric charge is conserved the Isospin projection is conserved too; nothing was said with respect to Isospin conservation. Moreover there is no physical meaning for Isospin states which are coherent superposition of basic Neutron and Proton states, feature in which Isospin is very different from intrinsec Spin. The state of a Nucleon is characterized by 5 degrees of freedom: 3 -spatial, one - intrinsec Spin (with two "magnetic" (sub)states) and one -Isospin with two "electric" (sub) states). The Pauli Principle was, later on, generalized to include the Isospin degree of freedom: the Nucleon system wavefunction is antisymmetric with respect to simultaneous permutation of five coordinates of pair of Nucleons.

4. Exchange Nuclear Forces

Heisenberg firstly introduced the ideea of Exchange Nuclear Forces. According to different nucleon degrees of freedom, 3 types of exchange nuclear forces were defined. The Heisenberg Exchange Force is related to permutation of charge coordinates of Nucleons. (The other two exchange nuclear forces do concern the spatial coordinates permutation and Spin coordinates permutation; Majorana and Bartlett forces).

5. Charge Symmetry Principle

Heisenberg firstly recognized the Charge Symmetric properties of nuclear forces. By disregarding electromagnetic interactions it follows the equality of p - p force to n - n one. The Charge Symmetry results into invariance of properties of nuclear system by interchanging Protons and Neutrons ("Isospin flip"). This principle was, later on, generalized as :

6. Charge Independence Principle, (Isotopic / Isobaric Invariance)

The properties of a nuclear system are invariant with respect to rotations in Isotopic Space. According to it n - n, p - p and n - p forces are equal. (However recent studies revealed that n - p force is more attractive than n - n or n - p ones). From Charge Independence Principle it follows that the Isospin is a good quantum number.

7. Isospin and Nuclear Symmetries

a. Isospin Multiplets

The Isospin vectors may be added as ordinary Spins. The Isospin **T** of a nuclear state having N Neutrons and Z Protons has "charge" projection $T_3 = 1/2\,(N - Z)$.

According to Charge Independence Principle, the Isospin is a good quantum number: the nuclear states are characterized by definite values of Isospin. States of given Isospin,(same spin, same parity, and , in limit of Charge Independence, same energy) do constitue members of an Isospin Multiplet. (An Isospin Multiplet is, formally, an irreducible representation of unitary transformations group in bidimensional Isospin Space.). Members of Isospin Multiplet correspond to states in a set of isobars, $A = N + Z$, defined by condition, $- T < 1/2(N-Z) < T$. States of same Isobaric Multiplet are degenerate,have same structure, differing only in numbers of Protons and of Neutrons. The actual states differ however by Coulomb energy, mass difference between Protons and Neutrons, etc.

b. Isospin and Shell-Model

Isospin **T** as good quantum number can be used in classification of nuclear states. The Isospin, for a given nucleus, is a measure of Permutation Symmetry in charge variables. But the permutation of charge variables are correlated, by Pauli Principle, (antisymmetry of wave function), with permutation of spatial (orbital)-spin variables. Spin-orbital Permutation Symmetry is adjoint to Isospin Symmetry. After removal of saturated pairs, the spin-orbital part of wave function is characterized by Seniority and its Isospin part by "Reduced Isospin".

The nuclear potential could be invariant to interchange of Spin- **S** and Isospin- **T** variables of all nucleons, i.e. to transformation in four dimensional Spin- Isospin space. The corresponding multiplets are known as **SU**(4) Wigner-Supermultiplets; they will break into pairs of Spin-Isospin (**S-T**) multiplets.

c. Isospin and Algebraic Models

Interacting Boson Models define Bosons (pairs of correlated Nucleons) in Isospin terms. A pair of Nucleons in same orbit can be described by sextets of "Isospin - Intrinsec Spin" labels (**T** = **1**, **S** = **0**: **T** = **0**, **S** = **1**). The "space part" of Interacting Bosons wave functions has same symmetry as the "Isospin - Intrinsec Spin" one. The symmetry of Boson microscopic structure does include the Wigner Supermultiplets. Extension to Interacting Boson-Fermion Models means to add the Fermions Isospin to that of Boson ones.

8. Isobaric Spin in Nature

The Physics is Philosophy of the Nature; a Concept is Physics provided it has a Nature Corresponding.

a. Isospin Purity

The usefulness of Isobaric Spin Concept and Formalism depends on Isospin Purity of nuclear states. Low-lying states of light nuclei are usually Isospin - pure. It was thought that in heavy nuclei states of different Isospin would mix; however it was found that the mixing is not strong and states with definite Isospin are observable. The Isospin remains a good quantum number for low-lying states of heavy nuclei.

b. Mirror Nuclei

They have an equal number of Protons and Neutrons and also an additional Nucleon either Proton or Neutron; they are Isobaric Doublets (**T** = 1/2). The lowest mass Mirror Nuclei are ${}^{3}{}_{1}H_{2}$ and ${}^{3}{}_{2}He_{1}$. Another example of Mirror Nuclei is ${}^{27}{}_{13}Al_{14}$ and ${}^{27}{}_{14}Si_{13}$. The states of Mirror Nuclei are in one to one correspondence; they have similar structure (quantum numbers) except Isospin "charge" projection. Their spectroscopic properties are nearly identical.

c. Isobaric Analog Resonances

The phenomenon of Isobaric Analog Resonances was a major surprise in Isospin Story. Isospin analogues of lowest states in heavy nuclei were found as highly-lying states which are unbound, or resonances; these states are excited by Compound Nucleus reactions. They were used to study Spectroscopy in an Isospin Multiplet by means of Proton Scattering, (cross-section and esp. polarization experiments).

d. Isospin Coupled Reactions

The simplest Isospin Coupled Reactions are those of Nucleon Scattering and Charge Exchange on Mirror Nuclei

$$p + C \rightarrow p + C$$

$$\rightarrow n + A$$

A and *C* are members of same Isodoublet (**T** = 1/2) as well as Proton and Neutron are an Isodoublet (*t* = 1/2). The two reactions are coupled by Isospin-Isospin interaction (*t* **T** - Lane potential).

Experimental evidences for Isospin Coupling of Mirror Channels, with Light-Medium Nuclei, via the "Isotopic Threshold Effect", was found with Bucharest Tandem Accelerator and in Soviet Laboratories.

Experimental evidence for Isospin Coupled Reactions was found with Medium-Heavy nuclei too. The first polarization study of this reaction was realized in Erlangen Tandemlabor. I was fortunate to join,as Humboldt Researcher, the Polarization Studies of Isospin Coupled Reactions, done in Erlangen-Nuernberg University, (Professor Jens Christiansen and Dozent Gerhard Graw).

9. On Isospin Physics

Heisenberg invented the Isospin Concept, representing a new internal degree of freedom in Atomic Nucleus.

The Isospin Concept Ramifications are : Heisenberg Exchange Nuclear Forces, Charge Symmetry Principle, Charge Independence Principle, Nuclear Symmetries related to Isospin.

The Ramifications together with Experimental Evidences for Nuclear Isospin are proofs of Isospin Cardinal Importance in developement of Nuclear Physics.

The Isospin Concept has also methodological importance both in Physics of Atomic Nucleus and Physics of Elementary Particles. Similar Concepts, dealing with other internal degrees of freedom, were later on introduced. To mention here the recent **F**- Spin in Algebraic Models for Nuclear Structure, describing Proton- and Neutron- Bosons.

3. THE SCATTERING MATRIX

Nuclear Physics has two main branches:

- Nuclear Structure (Spectoscopy) - (Study of nuclear bound states)
- Nuclear Reactions (Study of continuum: scattering and resonant states).

The Isobaric Spin is mainly related to Nuclear Structure. The Scattering Matrix is mainly related to Nuclear Reactions.

1. Nuclear Scattering/Reaction

A nuclear reaction means a transformation of a pair of nuclei into another pair of nuclei, as result of their interctions

$$a + A \rightarrow b + B, \qquad (\alpha \rightarrow \beta)$$

Formally it is a transition from an initial state alpha (called input channel) to a final state beta (called exit or output channel). The scattering process is completely described by the Transition Amplitude; it defines the "Complete Experiment", i.e. the scattering/reaction cross-sections and polarizations, which are measures of reaction probabilities and of Spin-orientations.

The description of a scattering process in terms of Transition Amplitude has to take into account both Reaction Dynamics as well as Reaction Quantum Kinematics (i.e. aspects originating in Symmetry Principles).

2. Scattering Matrix.

Heisenberg ["Die beobachtbaren Groessen in der Theorie der Elementar-teilchen", Z.P. 120(1943)513] has invented the concept of "Streu-Matrix" (**S**- or Scattering- or Collision-Matrix) in order to describe Dynamics of Quantum Scattering/Reaction. The Scattering Matrix **S** does connect possible reaction channels,

$$\| \mathbf{S}_{\alpha,\beta} \| = S \ .$$

The "Streu-Matrix" Approach to Dynamics of the Scattering Process takes into account only those physical quantities which are directly observable; it is congruent to Heisenberg's Positivistic Principle, ("to establish connection between facts").

The **S**- Matrix is used both in Stationary and Non-Stationary descriptions of Quantum Scattering. The "Time Delay" , specific to a scattering process, is evaluated via Wigner condition, relating energy dependance of **S**- Matrix to Causality Principle, $\Delta\tau = d(\arg \mathbf{S})/ dE$, (or $\Delta\tau = - i\, d \ln\mathbf{S}/ dE$).

3. Scattering Matrix Properties

The **S**- Matrix Invariance properties result from Symmetries of the Physical Laws for a Nuclear Reastion.

The conservation of total Angular Momentum J, Isospin **T** and Parity π do result into splitting of S-Matrix into distinct components with corresponding Quantum labels $S^{JT\pi}$.

The Time-Reversal results into Symmetry of **S**-Matrix elements, (Reciprocity Theorem), $S_{\alpha,\beta} = S_{\beta,\alpha}$.

Conservation of probability flux is a fundamental property of the Scattering- Matrix and is expressed by **S**- Matrix Unitarity, $\mathbf{S}^{+}\mathbf{S} = \mathbf{I}$.

The only Singularities of **S**- Matrix are Poles and Branch Points; physically they correspond , respectively, to Resonances and to Thresholds of Reaction Channels.

4. S- Matrix Fields

The Power of **S**- Matrix Concept can be illustrated just by displaying its penetration in various Fields of Quantum and Classical Physics:

- Nuclear Reactions
- Atomic Collisions
- Acoustics
- Optics (Light Scattering)
- Microwave Electrdynamics
- Mesoscopic Systems (Electronic Transport, Resonant Tunneling)

5. Microwave Electrodynamics/ Nuclear Reactions

The **S**- Matrix physical aspects could be illustrated by a paralel of two examples from Classical and Quantum Physics.

A Microwave Device consists from Wave Guides coupled to a Resonant Cavity. A Nuclear Reaction consists from Reaction Channels connected via a Compound Nuclear System. Wave Guides correspond to Reaction Channels; Resonant Cavity does correspond to Compound Nucleus. The Interior States of the two systems are Vibration Modes of Resonant Cavity and, respectively, Resonances of Compound Nucleus.
The **S**- Matrix of Microwave Electrodynamics, (called Distribution Matrix), describes distribution of the energy flux over different Waveguides. The S- Matrix for Compound Nucleus problems, (sometimes named Collision Matrix), does represent the distribution of probability flux over Reaction Channels. The **S**- Matrix Unitarity Defect is related to Quality Factor of Microwave Cavity, respectively, to "spreading" of flux via Statistical fluctuating Components of Nuclear Reactions.

6. S- Matrix and Open Systems

An Open System transfers energy or probability to its Environment, (although the Open System together with its Environment is a Closed System). Any Physical System with Input, Output and Interior States can be regarded as an Open System; examples are:- Quantum Scattering (Input/Output Channels, Interior States)- Decaying Systems (only Output Channels, Interior States)- Electronic Circuits (Input, Output) The Open Systems are related to Mathematical Theory of Non-Selfadjoint Operators. A Mathematical Theory of Open Systems (Livsic) defines, inspired from Heisenberg, a *S*- Function; it does coincide with a **S**- Matrix parametrization, at least for Compound Nucleus Reactions.

7. Reduced S- Matrix

It is an autochton contribution in Heisenberg's Spirit of Positivistic Principle.

The Multichannel Scattering Matrix describes both Reaction Dynamics (open channels) as well as Spectroscopy (of bound states in closed channels). The closed channels are "invisible" but they do influence on complementary open channels. The "unobserved" open channels, (specific to a large group of Multichannel Scattering Problems), do influence also on the observed complementary open channels. Both "invisible" closed and "unobserved" open channels are "eliminated" [e] from Reaction Systems but their effect on "retained" [r] channels is taken into account by means of different "Effective Operators" ("Effective Hamiltonian", "Reduced **R**- or **K**- Matrices"). But the Primary Object of Scattering Theory is, according to Heisenberg, Scattering Matrix and the concept of "Reduced" or "Effective" Operator has to be extended to **S**- Matrix too. The Reduced **S**- Matrix describes, in addition to observed channels, the transitions

$$\Delta S_{rr} = S_{re}(1 + S_{ee})^{-1} S_{er}$$

to and from eliminated channels.

The Reduced **S**- Matrix contains as limit cases several Scattering Problems:

- Nuclear Threshold Cusp
- Atomic Quantum Defect
- Quasi-Resonant Scattering (Coupled Channel Resonances).
- Wood Anomaly (Light Diffraction)

4. HEISENBERG THINKING ON PHYSICS

- Inventing New Concepts - trademark of Heisenberg's style of research. The Isospin and **S**- Matrix Concepts, invented by Heisenberg, became cornerstones of Nuclear Structure and Nuclear Reactions Physics.

- Simplicity as Criterion of Central Order of Nature. This criterion, shared also by Einstein, was guide in Heisenberg's Thinking: "besides invariance properties... the simplicity of equations". The Isospin and **S**- Matrix are simple and profound Concepts, with a determining impact in History of Nuclear Physics.

- Heisenberg meant his research to cover more than one discipline. Orizontal Extensions: **S**- Matrix Migrations in different Research Fields Vertical Developements: Isospin Descendents - Methodological Impact of Heisenberg's Concepts Similar concepts were developed for describing other Internal Degrees of Freedom; examples are **F**- Spin of Algebraic Models, many others in Physics of Elementary Particles, **S** - Function, etc.

- Philosophical Parentages

The **S**- Matrix Concept originates in Positivistic Principle, ("to establish coonection between facts not explainable in the present Physics"), (1925). The Nucleon and Isospin concepts are not pure descriptive ones but rather they do reflect a profound Understanding of Nature of Nucleus constituents.

- Heisenberg's Motivations

Heisenberg's motivations have been mainly of a philosophical nature; (philosophical motivations are found also at Einstein, Bohr and de Broglie) Heisenberg looked in Platons Philosophy to analogies for Theory of Symmetry. However the Proton-Neutron Nuclear Model and the Isospin were stimulated by experiments.

- Heisenberg - Great Physicist Thinker. The main characteristic of Heisenberg's Thinking on Physics is a profound Religiosite.
- Heisenberg's Attitudes on Science: (1) "Description" (only observables)-Positivistic Principle, and (2) "Understanding" (e.g. World Formula)

This Work was communicated to the Humboldt Symposium "Werner Heisenberg und die Wissenschaft, das Denken und die Kunst", (Goethe Institut, Bucharest, October 16, 2001).

The inaugural Lecture to this Humboldt Symposium of the LMU Professor Gerhard Graw on "Supersymmetry Evidence in Nuclear Spectra" is a deserved hommage paid to Werner Heisenberg, the brillant Representative of the Munich University.

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Received January 15, 2002