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RECENT ANISOTROPIC YIELD CRITERIA FOR SHEET METALS

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Biaxial tensile deformation tests were carried out on cruciform specimens of AA3103-0, 1.2 mm thick sheet samples using a CNC stretch-drawing facility designed and built at the Institute for Metal Forming Technology, Stuttgart University. The beginning of plastic yield was monitored by temperature measurements according to the method of Sallat. The observed plastic anisotropy was modeled using the phenomenological descriptions developed by Banabic et al. (BBC200) [1], Barlat et al. (Barlat2000-2d) [2] and by Cazacu and Barlat [3]. In Banabic et al [1] and Barlat et al. [2] the anisotropy is introduced by a means of a linear transformation of the Cauchy stress tensor applied to the material whereas Cazacu and Barlat's [3] approach is based on representation theorems of tensor functions. Comparison with data show that the criteria can successfully describe the anisotropy of both the plastic strain ratio and yield of AA3103-0 aluminum thin sheets.

Key words: yield criteria, anisotropy, plane stress , aluminium alloy.

1. INTRODUCTION

For computer simulation of sheet metal forming processes, a quantitative description of plastic anisotropy by the yield locus of the material is required. The generalization of von Mises's yield criterion to orthotropy was proposed by Hill [4]. Examples of non-quadratic anisotropic yield functions can be found in Barlat et al [5] and Banabic [6]).To obtain new yield functions, Vegter et al [7] have directly used the test results and Bezier's interpolation. The Casteljau's graphical procedure and the biaxial anisotropy coefficient (an index introduced, independently, by Barlat et al [2] and Pöhlandt, Banabic, Lange [8])) has been used by Pöhlandt, Banabic and Lange [9] to improve the accuracy of yield criteria. In Banabic et al. [1] anisotropy was introduced by means of a linear transformation of the Cauchy stress tensor acting on the material (a method introduced by Karafillis and Boyce [10]). A new plane stress yield function that describes well the anisotropic behavior of sheet metals, in particular, aluminum alloy sheets was proposed by Barlat et al [2]. The anisotropy was introduced in the formulation using two linear transformations associated to two different isotropic yield functions. An alternate method to extend any isotropic yield criterion such as to include any type of anisotropy was proposed by Cazacu and Barlat [11]. They used representation theorems to construct generalizations to anisotropic conditions of the invariants of the deviatoric stress and then substitute these generalized invariants in the expression of the given isotropic criterion. An illustration of this approach was given by extending Drucker's [12] isotropic yield criterion to orthotropy. In this paper, a comprehensive set data on aluminum AA3103-0, 1.2 mm thick sheet samples are reported. The observed anisotropy is then modeled using three recent anisotropic yield criteria: the Banabic et al [1], Barlat et al [2], Cazacu and Barlat [3].

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2. RECENT YIELD CRITERIA FOR ORTHOTROPIC SHEET METALS

2.1. BBC2000 yield criterion

A yield surface is generally described by an implicit equation of the form:

$$\Phi(\bar{\sigma}, Y) := \bar{\sigma} - Y = 0 \tag{1}$$

where $\bar{\sigma}$ is the equivalent stress and Y is a yield parameter. In practice, Y may be chosen as one of the following parameters of the sheet metal: $\sigma_0^{\exp}$ (uniaxial yield stress along the rolling direction), $\sigma_{90}^{\exp}$ (uniaxial yield stress along the transverse direction), $\sigma_{45}^{\exp}$ (uniaxial yield stress at 45° from the rolling direction), an average of $\sigma_0^{\exp}$, $\sigma_{90}^{\exp}$ and $\sigma_{45}^{\exp}$, or $\sigma_b^{\exp}$ (equi-biaxial yield stress). Banabic et al [1], proposed the following expression of the equivalent stress:

$$\bar{\sigma} = \begin{bmatrix} a\left(b\Gamma + c\Psi\right)^{2k} + a\left(b\Gamma - c\Psi\right)^{2k} + \\ +\left(1-a\right)\left(2c\Psi\right)^{2k} \end{bmatrix}^{\frac{1}{2k}} \tag{2}$$

where a, b, c, and k are material parameters, while Γ and Ψ are functions of the second and third invariants of a transformed stress tensor $\boldsymbol{s}' = \boldsymbol{L}\boldsymbol{s}$, where $\boldsymbol{L}$ is a 4th order tensor. In this formulation anisotropy is described by means of the tensor $\boldsymbol{L}$, which satisfies: (i) the symmetry conditions $L_{ijkl} = L_{jikl} = L_{jilk} = L_{klij}$ (i, j, k, l =1…3), (ii) the requirement of invariance with respect to the symmetry group of the material, and (iii) the three conditions $L_{1k} + L_{2k} + L_{3k} = 0$ (for k = 1, 2, and 3), which ensures that $\boldsymbol{s}'$ is traceless (see Karafillis-Boyce[10]). Hence, in the reference system associated with the directions of orthotropy, the tensor L has 6 non-zero components for 3 D conditions and 4 components for plane stress state, respectively.

Let define $(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z})$, the reference frame associated with orthotropy. For a rolled sheet, $\boldsymbol{x}$, **y**, and **z** represent the rolling direction, the long transverse direction, and the short transverse direction, respectively. In the reference system $(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z})$:

$$\begin{aligned} s'_{xx} &= d\sigma_{xx} + e\sigma_{yy} \\ s'_{yy} &= e\sigma_{xx} + f\sigma_{yy} \\ s'_{zz} &= -(d+e)\sigma_{xx} - (e+f)\sigma_{yy} \\ s'_{xy} &= g\sigma_{xy} \\ s'_{xz} &= s'_{yz} = 0 \end{aligned} \tag{3}$$

where d, e, f, and g are the four independent components of the tensor **L**. The expressions of Γ and Ψ in terms of the stress components are:

$$\begin{aligned} \Gamma &= M\sigma_{xx} + N\sigma_{yy} \\ \Psi &= \sqrt{\left(P\sigma_{xx} + Q\sigma_{yy}\right)^2 + R\sigma_{xy}^2} \end{aligned} \tag{4}$$

where :

$$M = d + e \ ; \ N = e + f \ ; \ P = \frac{d-e}{2} \ ; \ Q = \frac{e-f}{2}$$

and $R = g^2$ (for more details see Banabic et al [1]). Let us note that the expression (2) of the equivalent stress is derived from the one proposed by Barlat and Lian [13] for plane-stress conditions. Two additional parameters, namely b and c, have been introduced in order to allow a better representation of the plastic behavior of the sheet metal. The convexity of the yield surface described by (2) is ensured if a $\in$ [0, 1] and k is a strictly positive integer number. If s_q is the yield stress in uniaxial tension along an axis at orientation q to the rolling direction $\boldsymbol{x}$, it follows that:

$$\sigma_\theta = \frac{Y}{\begin{bmatrix} a\left(bA_\theta + cB_\theta\right)^{2k} + a\left(bA_\theta - cB_\theta\right)^{2k} + \\ (1-a)\left(2cB_\theta\right)^{2k} \end{bmatrix}^{\frac{1}{2k}}}, \tag{5}$$

where:

$$\begin{aligned} A_\theta &= M\cos^2\theta + N\sin^2\theta \\ B_\theta &= \sqrt{\left(P\cos^2\theta + N\sin^2\theta\right)^2 + R\sin^2\theta\cos^2\theta} \end{aligned}$$

According to this criterion, the equibiaxial yield stress is:

$$\sigma_b = \frac{Y}{\begin{bmatrix} a\left(bA_b + cB_b\right)^{2k} + a\left(bA_b - cB_b\right)^{2k} + \\ +(1-a)\left(2cB_b\right)^{2k} \end{bmatrix}^{\frac{1}{2k}}} \tag{6}$$

where:

$$A_b = M + N,\ B_b = \left|P + Q\right|$$

The coefficient of plastic anisotropy associated to a direction inclined at an angle $\theta \in [0, 90°]$ with the rolling direction is predicted as follows:

$$r_\theta = \frac{Y}{\sigma_\theta \left[\frac{\partial\Phi}{\partial\Gamma}\left(\frac{\partial\Gamma}{\partial\sigma_{xx}} + \frac{\partial\Gamma}{\partial\sigma_{yy}} \right) + \frac{\partial\Phi}{\partial\Psi}\left(\frac{\partial\Psi}{\partial\sigma_{xx}} + \frac{\partial\Psi}{\partial\sigma_{yy}} \right) \right]} - 1 \qquad (7)$$

The shape of the yield surface is defined by the material parameters a, b, c, d, e, f, g, and k. Among these parameters, k has a distinct status. More precisely, its value is set in accordance with the crystallographic structure of the material [14]: k = 3 for BCC alloys, and k = 4 for FCC alloys. The other 7 parameters are determined such that the model reproduces as well as possible the following experimental characteristics of the orthotropic sheet metal: s_0^{exp}, s_{90}^{exp}, s_{45}^{exp}, s_b^{exp}, r_0^{exp}, r_{90}^{exp} and r_{45}^{exp}. It is possible to obtain their values by solving a set of seven non-linear equations. However, this set of equations have multiple solutions. We have concluded that the best solution is to avoid the strict enforcement of the restrictions mentioned above. A more effective strategy of identification is to impose the minimization of the following error function:

$$F(a,\ b,\ c,\ d,\ e,\ f,\ g) = \left(\frac{\sigma_0}{\sigma_0^{exp}} - 1 \right)^2 + \left(\frac{\sigma_{90}}{\sigma_{90}^{exp}} - 1 \right)^2 + \left(\frac{\sigma_{45}}{\sigma_{45}^{exp}} - 1 \right)^2 + \left(\frac{\sigma_b}{\sigma_b^{exp}} - 1 \right)^2 + \left(\frac{r_0}{r_0^{exp}} - 1 \right)^2 + \left(\frac{r_{90}}{r_{90}^{exp}} - 1 \right)^2 + \left(\frac{r_{45}}{r_{45}^{exp}} - 1 \right)^2 \qquad (8)$$

where $\sigma_0, \sigma_{90}, \sigma_{45}, \sigma_b, r_0, r_{90}$ and r_{45} are the uniaxial yield stresses, the equi-biaxial yield stress and the coefficients of plastic anisotropy predicted by the constitutive equation.

Eqns (5) to (7) are used in order to evaluate the quantities involved in the error function F. For the numerical minimization, the downhill simplex method proposed by Nelder and Mead [15] has been adopted because it does not need the evaluation of the gradients.

2.2 Barlat 2000-2d yield criterion

In general, a yield function written in terms of the deviatoric stress tensor fulfills the pressure independence condition. Therefore, another linear transformation could be:

$$\mathbf{X} = \mathbf{C.s} \qquad (9)$$

where **s** is the deviatoric stress tensor and **X** the linearly transformed stress tensor. Since there are 5 independent deviatoric stress components, the most general linear transformation from a five-dimensional space to a six-dimensional space, assuming orthotropic symmetry, with axes **x**, **y** and **z**, can be written as:

$$\begin{bmatrix} X_{xx} \\ X_{yy} \\ X_{zz} \\ X_{yz} \\ X_{zx} \\ X_{xy} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & 0 & & & \\ C_{21} & C_{22} & 0 & & & \\ C_{31} & C_{32} & 0 & & & \\ & & & C_{44} & & \\ & & & & C_{55} & \\ & & & & & C_{66} \end{bmatrix} \begin{bmatrix} s_{xx} \\ s_{yy} \\ s_{zz} \\ s_{yz} \\ s_{zx} \\ s_{xy} \end{bmatrix} \qquad (10)$$

with no conditions on the C_{ij} [2]. This gives 9 independent coefficients for the general case and 7 for plane stress. However, applied to plane stress conditions, only one coefficient (C_{66}) is available to account for σ_{45} and r_{45}. As pointed out in Barlat et al [2] additional coefficients in the context of linear transformations can be obtained by using two transformations associated to two different isotropic yield functions, respectively. A plane stress state can be described by two principal values only, say s_1 and s_2. The expressions of the two isotropic yield functions considered in Barlat et al [2] are:

$$\phi' = \left|s_1 - s_2\right|^a$$
$$\phi'' = \left|2s_2 + s_1\right|^a + \left|2s_2 + s_2\right|^a \qquad (11)$$

The resulting anisotropic yield function, Φ, is thus given by

$$\Phi = \Phi'(X') + \Phi''(X'') = 2\bar{s}^{\,a} \qquad (12)$$

where $\bar{\sigma}$ is the effective stress, a is a material coefficient and:

$$\mathbf{X}' = \mathbf{C}'.\mathbf{s} = \mathbf{C}'.\mathbf{T}.\text{ó} = \mathbf{L}'.\text{ó}$$
$$\mathbf{X}'' = \mathbf{C}''.\mathbf{s} = \mathbf{C}''.\mathbf{T}.\text{ó} = \mathbf{L}''.\text{ó} \qquad (13)$$

with :

$$\mathbf{T}=\begin{bmatrix} 2/3 & -1/3 & 0 \\ -1/3 & 2/3 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

C' and C'' being the linear transformations. In the reference frame associated with the material symmetry, and :

$$\begin{bmatrix} X'_{xx} \\ X'_{yy} \\ X'_{xy} \end{bmatrix}=\begin{bmatrix} C'_{11} & C'_{12} & 0 \\ C'_{21} & C'_{22} & 0 \\ 0 & 0 & C'_{66} \end{bmatrix}\begin{bmatrix} s_{xx} \\ s_{yy} \\ s_{zz} \end{bmatrix}$$
$$\begin{bmatrix} X''_{xx} \\ X''_{yy} \\ X''_{xy} \end{bmatrix}=\begin{bmatrix} C''_{11} & C''_{12} & 0 \\ C''_{21} & C''_{22} & 0 \\ 0 & 0 & C''_{66} \end{bmatrix}\begin{bmatrix} s_{xx} \\ s_{yy} \\ s_{xy} \end{bmatrix} \quad (14)$$

The principal values of $\mathbf{X}'$ and $\mathbf{X}''$ are:

$$X'=C'\cdot s=C'\cdot T\cdot\sigma=L'\cdot\sigma$$
$$X''=C''\cdot s=C''\cdot T\cdot\sigma=L''\cdot\sigma \quad (15)$$

with the appropriate indices (prime and double prime) for each stress. The anisotropic yield function is given by (12) where:

$$\phi'=\left|X'_1+X'_2\right|^a$$
$$\phi''=\left|2X''_2+X''_1\right|^a+\left|2X''_1+X''_2\right|^a \quad (16)$$

It reduces to the isotropic expression when the matrices $\mathbf{C}'$ and $\mathbf{C}''$ are both taken as the identity matrix so that $\mathbf{X}'=\mathbf{X}''=\mathbf{s}$. Because ϕ' depends on $X'_1-X'_2$, only three coefficients are independent in $\mathbf{C}'$. In Barlat et al. [2], the condition $C'_{12}=C'_{21}=0$ is imposed, but it is worth noting that $C'_{12}=C'_{21}=1$ is an acceptable condition that leads to $\mathbf{X}'=\boldsymbol{\acute{o}}$ if $C'_{11}=C'_{22}=2$. For convenience in the calculation of the anisotropy parameters, the coefficients of $\mathbf{L}'$ and $\mathbf{L}''$ are expressed as follows

$$\begin{bmatrix} L'_{11} \\ L'_{12} \\ L'_{21} \\ L'_{22} \\ L'_{66} \end{bmatrix}=\begin{bmatrix} 2/3 & 0 & 0 \\ -1/3 & 0 & 0 \\ 0 & -1/3 & 0 \\ 0 & 2/3 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_7 \end{bmatrix}$$
$$\begin{bmatrix} L''_{11} \\ L''_{12} \\ L''_{21} \\ L''_{22} \\ L''_{66} \end{bmatrix}=\frac{1}{9}\begin{bmatrix} -2 & 2 & 8 & -2 & 0 \\ 1 & -4 & -4 & 4 & 0 \\ 4 & -4 & -4 & 1 & 0 \\ -2 & 8 & 2 & -2 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \alpha_3 \\ \alpha_4 \\ \alpha_5 \\ \alpha_6 \\ \alpha_8 \end{bmatrix} \quad (17)$$

where all the independent coefficients α_k (for k from 1 to 8) reduce to 1 in the isotropic case.

The coefficients α_k for k = 1…6 can be determined using as input the values of the uniaxial tension along the rolling and the transverse directions, and the balanced biaxial stress state and r_b , where r_b defines the slope of the yield surface at the balanced biaxial stress state ($r_b=\dot{\varepsilon}_{yy}/\dot{\varepsilon}_{xx}$). Uniaxial tension tests loaded at 45° to the rolling direction give two data points, $\sigma=\sigma_{45}$ and $r=r_{45}$ from which α_7 and α_8 can be computed.

2.3 Cazacu-Barlat yield criterion

Characterization of the plastic response requires the specification of yield function and a flow rule by which the subsequent inelastic deformations can be calculated for specified loadings and displacements. Assuming that yielding is insensitive to hydrostatic pressure, for an isotropic material the yield function depends on stress through $J_2=tr\,\boldsymbol{S}^2/2$ and $J_3=tr\,\boldsymbol{S}^3/3$, the second and third invariants of the stress deviator $\boldsymbol{S}$, respectively. To introduce orthotropy in the expression of an isotropic criterion, Cazacu and Barlat [3] proposed generalizations of the invariants of the stress deviator. The generalization of J_3 was required to be a homogeneous function of degree three in stresses that reduces to J_3 for isotropic conditions, is insensitive to pressure, and is invariant to any transformation belonging to the symmetry group of the material. Similarly, the generalization of J_2 was required to be a homogeneous function of degree two in stresses that reduces to J_2 for isotropic conditions, is insensitive to pressure, and is invariant to any transformation belonging to the symmetry group of the material. Hence, relative to $(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})$, J_3^o, the generalization of J_3, must be of the form (see Cazacu and Barlat [11]):

$$
\begin{aligned}
J_3^o = {} & \frac{1}{27}\left(b_1+b_2\right)s_x^3 + \frac{1}{27}\left(b_3+b_4\right)s_y^3 + \frac{1}{27}\left[2\left(b_1+b_4\right)-b_2-b_3\right]s_z^3 \\
& -\frac{1}{9}\left(b_1 s_y + b_2 s_z\right)s_x^2 - \frac{1}{9}\left(b_3 s_z + b_4 s_x\right)s_y^2 \\
& -\frac{1}{9}\left[\left(b_1-b_2+b_4\right)s_x + \left(b_1-b_3+b_4\right)s_y\right]s_z^2 \\
& +\frac{2}{9}\left(b_1+b_4\right)s_x s_z s_y - \frac{s_{xz}^2}{3}\left[2b_9 s_y - b_8 s_z - \left(2b_9-b_8\right)s_x\right] \\
& -\frac{s_{xy}^2}{3}\left[2b_{10} s_z - b_5 s_y - \left(2b_{10}-b_5\right)s_x\right] - \frac{s_{yz}^2}{3}\left[\left(b_6+b_7\right)s_x - b_6 s_y - b_7 s_z\right] \\
& + 2b_{11} s_{xy} s_{xz} s_{yz} .
\end{aligned}
\tag{18}
$$

where the coefficients b_k (k = 1…11) reduce to unity for isotropic conditions. Similarly, J_2^o, the generalization of J_2, is expressed relative to (x, y, z) as:

$$J_2^o = \frac{a_1}{6}\left(s_x - s_y\right)^2 + \frac{a_2}{6}\left(s_y - s_z\right)^2 + \frac{a_3}{6}\left(s_x - s_z\right)^2 + a_4 s_{xy}^2 + a_5 s_{xz}^2 + a_6 s_{yz}^2 \tag{19}$$

where the coefficients a_k (k = 1…6) reduce to unity in the isotropic case. Note that J_2^0 is Hill's [4] quadratic yield function. Using these generalized invariants any isotropic yield criterion can be extended such as to describe orthotropy. In Cazacu and Barlat [11] , this approach was used to extend Drucker's [12] isotropic yield criterion:

$$f = J_2^3 - c\, J_3^2 = k^2 \tag{20}$$

where c and k are constant. Hence, the expression of the proposed orthotropic criterion is:

$$f^O = \left(J_2^o\right)^3 - c\left(J_3^o\right)^2 = k^2. \tag{21}$$

For 3-D stress conditions the criterion involves 18 material parameters. In the case of a sheet, where the only non-zero stress components are the in-plane stresses $\left(s_x, s_y, s_{xy}\right)$, the criterion may be written as:

$$
\begin{aligned}
f_2^O \equiv {} & \left[\frac{1}{6}\left(a_1+a_3\right)s_x^2 - \frac{a_1}{3}s_x s_y + \frac{1}{6}\left(a_1+a_2\right)s_y^2 + a_4 s_{xy}^2\right]^3 - \\
& - c\left\{\begin{array}{l} \frac{1}{27}\left(b_1+b_2\right)s_x^3 + \frac{1}{27}\left(b_3+b_4\right)s_y^3 - \frac{1}{9}\left(b_1 s_x + b_4 s_y\right)s_x s_y - \\ -\frac{1}{3} s_{xy}^2\left[\left(b_5 - 2b_{10}\right)s_x - b_5 s_y\right] \end{array}\right\}^2 = k^2.
\end{aligned}
\tag{22}
$$

If s_q is the yield stress in uniaxial tension along an axis at orientation q to the rolling direction x, it follows that:

$$s_q = k^{\frac{1}{3}} \left\{ \begin{array}{l} \left[\frac{1}{6}(a_1+a_3)\cos^4 q + (a_4 - a_1/3)\cos^2 q \sin^2 q + \frac{1}{6}(a_1+a_2)\sin^4 q \right]^3 - \\ -c \left[\begin{array}{l} \frac{1}{27}(b_1+b_2)\cos^6 q + \frac{1}{27}(b_3+b_4)\sin^6 q - \\ -\frac{1}{9}\left[\begin{array}{l} (b_1 + 3b_5 - 6b_{10})\cos^2 q + \\ +(b_4 - 3b_5)\sin^2 q \end{array} \right] \sin^2 q \cos^2 q \end{array} \right]^2 \end{array} \right\}^{-1/6} \tag{23}$$

Yielding under equibiaxial tension occurs when s_x and s_y are both equal to:

$$s_b = k^{\frac{1}{3}} \left[\left(\frac{a_2 + a_3}{6} \right)^3 - c \left(\frac{-2b_1 + b_2 + b_3 - 2b_4}{27} \right)^2 \right]^{-\frac{1}{6}} \tag{24}$$

Yielding under pure shear parallel to the orthotropic axes occurs when s_{xy} is equal to:

$$t = k^{\frac{1}{3}} (a_4)^{-\frac{1}{2}} \tag{25}$$

The 10 anisotropy coefficients and the value of c can be determined from the measured uniaxial yield stresses s_q and strain ratios r_q in 5 different orientations and s_b, the value of the equibiaxial tensile stress (see more details in [3]).

3. EXPERIMENTAL INVESTIGATION

By varying the longitudinal and transverse force acting on a cross tensile specimen (see Figure 1) any point of the yield locus in the range of biaxial tensile stress can be obtained. A description of the cross tensile specimen, which has been optimised by means of stress optical methods such as to obtain a zone of homogeneous stress can be found in Kreissig [16].

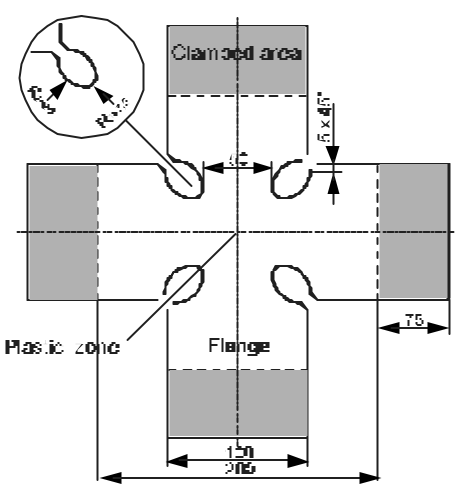

Fig.1 Cruciform specimen for the biaxial tensile test

Starting from this geometry, a further optimization was carried out [17] whereby, besides a zone of homogeneous stress, a large strain is obtained before instability occurs in the notches. For this purpose, the geometrical parameters shown in Figure 1 were varied. Since the optimum geometry depends to some extent on the material properties, it was verified by stress optical experiments that for the dimensions used there is a large zone of homogeneous biaxial stress [17]. The problem of the „equivalent cross section" of the specimen, i.e. the cross section by which the acting force has to be divided for obtaining the true stress has also been addressed in [17]. It was shown that a good accuracy in the determination of the yield loci of materials in the initial state without pre-straining can be achieved by using the nominal cross section from the workshop drawing.

In this paper, we report the results of a series of biaxial tensile tests carried out on an aluminium alloy sheet metal AA3103-0, 1.2 mm thickness using a CNC stretch-drawing facility designed and built at the Institute for Metal Forming Technology, Stuttgart University (Figure 2).

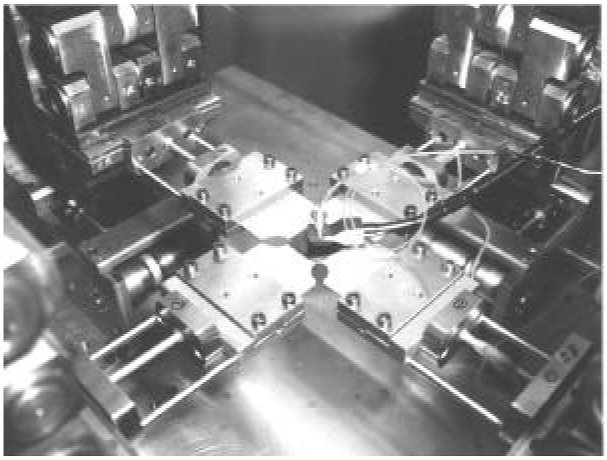

Fig. 2 Cruciform specimen and the temperature measurement device

The beginning of plastic yielding was monitored by temperature measurements using the method of Sallat [18]. The temperature of the specimen was measured by an infrared thermo-couple positioned at an optimized distance from the specimen. During elastic straining, the specimen's temperature decreases by a fraction of a degree due to thermo-elastic cooling. When plastic flow begins, the temperature rises strongly – as it can be seen in Figure 3.

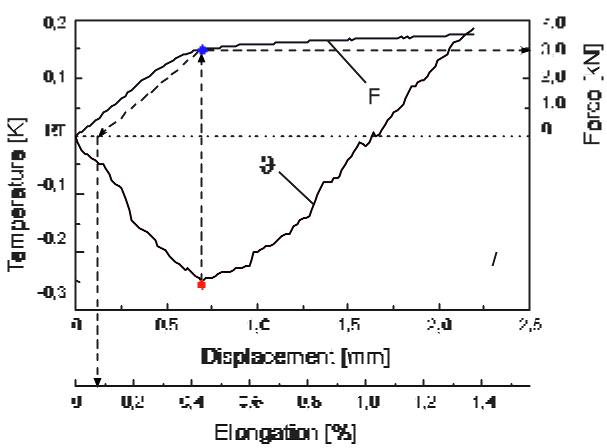

Fig.3. Temperature vs. elongation obtained for standard tensile test specimen [17]

At difference with the definition of the yield point given in standards, the definition based on the minimum of the temperature vs. elongation has the advantage of ruling out any arbitrariness. In general, the values of the yield stresses obtained with this method are larger than the values obtained using the classical method, i.e. stress gauges measurements. The yield limits corresponding to seven different ratios of the applied stresses: 1:0, 4:1, 2:1, 1:1, 1:2, 1:4, 0:1 were measured. All these points are in the first quadrant.

The experimental values used as input data for the numerical identification of the material parameters involved in the expression of the Banabic et al.[1], Barlat et al.[2] and Cazacu-Barlat [3] yield criteria are [19, 20]: σ_0=55 MPa, σ_{30}=56 MPa, σ_{45}=58 MPa, σ_{75}=61 MPa, σ_{90}= 61 MPa, σ_b=60 MPa, r_0=0.639, r_{30}=0.555, r_{45}=0.513, r_{75}=0.581and r_{90}=0.605. All the other experimental points were used for validation purposes. Table 1 shows the values of the coefficients involved in the Banabic et al.[1], Barlat et al.[2] and Cazacu-Barlat [3] yield functions.

Table 1. Coefficients used in the BBC2000, Barlat 2000-2d and Cazacu-Barlat yield criteria

BBC2000		Barlat 2000-2d		Cazacu-Barlat	
				a1	0.060
A	0.786	α_1	1.056989	a2	0.600
b	0.750	α_2	0.755749	a3	0.872
c	0.408	α_3	0.907686	a4	0.268
M	0.574	α_4	0.928370	b1	0.174
N	0.582	α_5	0.972375	b2	-1.247
P	1.239	α_6	0.789798	b3	-0.706
Q	-1.243	α_7	0.882467	b4	0.192
R	5.692	α_8	1.065600	b5	-0.164
				b6	-0.170
				c	1.400

4. COMPARISON WITH EXPERIMENTS

The yield surfaces predicted by the yield criteria described in the §2 for the AA3103-0 sheet metal are presented in Figure 4.

The experimental data obtained at Institute for Metal Forming Technology are also plotted on the diagrams. A very good agreement has been found between predicted and experimental yield loci for all tested yield criteria. The predicted distribution of the uniaxial yield stress and the r-ratios with respect to the angle with the rolling direction are shown in Figures 5 and 6, respectively. A very good agreement has been found between predicted and experimental distribution of the r-ratio for all tested yield criteria. A better prediction of the uniaxial yield stress distribution has been found by using Barlat 200-2d and Cazacu-Barlat yield criteria (the deviations between theory and experiments are ~3% by using BBC2000 yield criterion). In [21] it was found that the BBC2000 yield criterion was capable of reproducing the general trend of the anisotropic behaviour of the steel sheets with different r-values too.

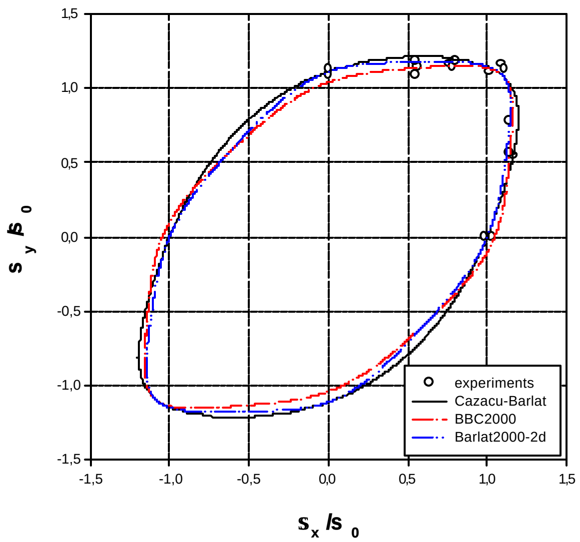

Figure 4. Experimental and theoretical yield loci for AA3103-0 aluminium alloy

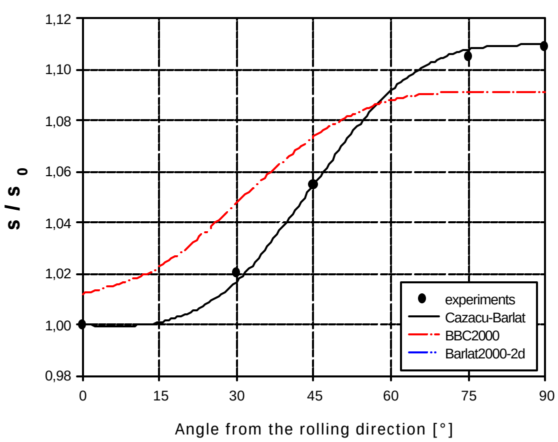

Figure 5. Distribution of the uniaxial yield stress

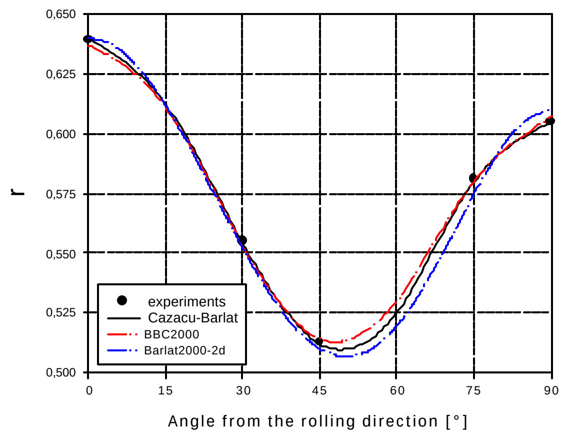

Figure 6. Distribution of the r ratios

CONCLUSIONS

The anisotropic plastic behaviour of AA3103-0 aluminium alloy was modelled using the phenomenological description proposed by Banabic et al. (BBC2000) [1], Barlat et al. (Barlat2000-2d) [2] and the yield criterion developed by Cazacu and Barlat [3]. In BBC2000 and Barlat2000-2d the anisotropy is introduced by a means of a linear transformation of the Cauchy stress tensor applied to the material whereas Cazacu and Barlat approach is based on representation theorems of tensor functions. Biaxial tensile deformation tests were carried out on cruciform specimens using a CNC stretch-drawing facility. The beginning of plastic yield was monitored by temperature measurements. Comparison with data show that the tested criteria can successfully describe anisotropic behaviour of AA3103-0 aluminium sheets.

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